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Students' Perceptions of Sense of Community in Abstract Algebra: Contributing Factors and Benefits

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In this phenomenological study, we explore how multiple assessments contribute to creating a sense of community (SOC) in an undergraduate abstract algebra course. Strike (2004) describes community as a process rather than a feeling and outlines four characteristics of community: coherence, cohesion, care, and contact. In this report, we describe contributing factors to and perceived benefits of SOC that students provided in an open-ended interview. Our findings indicate students viewed the teacher and the classroom environment as the primary sources for creating a SOC. Our findings also suggest students believed the SOC of the classroom increased classroom interaction and opened the doors of communication between students and with the instructor. The contributing factors align with Strike's and McMillan and Chavis'(1986) definitions of community, support social cognitive theory, and serve as a model for building a SOC in the classroom.

Keywords: Abstract Algebra, Multiple Assessments, Mathematics Classroom, Sense of Community

INTRODUCTION

In this qualitative study, we explore how multiple assessments contribute to creating a sense of community (SOC) in an undergraduate two-semester sequence abstract algebra course. Steen (1999) offers six guidelines to follow regarding undergraduate assessments in *Assessment Practices in Undergraduate Mathematics*. He claims assessment should (1) be a continuous cycle, (2) be an open process, (3) promote valid inferences, (4) employ multiple measures of performance, (5) measure what is worth learning, and (6) support every student's opportunity to learn important mathematics. With this in mind, mathematicians and mathematics educators began implementing a number of diverse assessments into their undergraduate mathematics courses including: collaborative assessments (Rouoviere, 1999), writing assignments (Blum, 1999), portfolios (Knoerr & McDonald, 1999), e-mail (Fried, 1999), and oral components through interviews or presentations (Heid, 1999). Although the literature pertaining to implementation of diverse assessments in undergraduate mathematics is plentiful, there is little research on the impact of various assessments implemented simultaneously into the same undergraduate mathematics course. In this report, we describe how multiple assessments meet other educational goals. Specifically we discuss how assessments contribute to the sense of community of the mathematics classroom. Our research questions are:

1. How do assessments contribute to a SOC in the mathematics classroom?
2. What are the benefits of creating a SOC in the mathematics classroom?

McMillan and Chavis (1986) define SOC as a perception where one feels (1) a sense of belonging, (2) influential, (3) nurtured, and (4) an emotional connection to the group. Hill (1996) suggests SOC goes beyond individual relationships and fluctuates from setting to setting, such as in a classroom. Strike (2004)

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further portrays community as a process rather than a feeling and outlines four characteristics of community: coherence, cohesion, care, and contact. Coherence refers to a shared vision; cohesion is the unity, which results from the shared vision; care is a necessity to initiate one into the vision, and contact refers to structural features of the community.

Much of the research related to SOC focuses primarily on adolescents (Pretty, Andrewes, & Collett, 1994; Sanchez, Colon, & Esparza, 2005; Strahan & Layell, 2006; Strike, 2004; Wighting, 2006). Pretty et al. argue SOC is significantly related to adolescent's loneliness. Sanchez et al. discuss the role of sense of belonging and academic outcomes of urban, Latino adolescents. Their results indicate a sense of school belonging is a good predictor for academic motivation, effort, and absenteeism. Strahan and Layell describe how struggling middle school students are able to progress in reading and mathematics under the tutelage of supportive teachers who provide an environment centered on the learner. Wighting shows how the use of computers in teaching may contribute to SOC and suggests SOC can be associated with academic success. Pretty and McCarthy (1991) ascertain the length of time a person spends in a setting and SOC do not have a consistent relationship. This suggests it is possible to create a SOC within a short- or long-term frame; our study supports this assertion.

More recently, researchers investigated the impact of SOC with college students, albeit the research is minimal. Jacobs and Dodd (2003) establish how burnout among undergraduate students can be predicted by factors such as social support, especially from friends. Students who feel a sense of nurturing from friends are less likely to experience burnout. Lounsbury and Loveland's (2003) research infers a psychological SOC is significantly related to extraversion, agreeableness, conscientiousness, and neuroticism in undergraduates enrolled in a lower-division psychology course. Thus, students who do not feel a SOC are less likely to interact with their peers. These results are crucial given collaborative work is the foundation of reform efforts (National Council of Teachers of Mathematics, 2000).

There is also research that addresses how graduate students build a community (Austin, 2002; Ferrer de Valero, 2001). Some of these inquiries focus specifically on mathematics graduate students (Carlson, 1999; Grevholm, Persson, & Wall, 2005; Herzig, 2002). Although the above-mentioned researchers do not use the term SOC, their results clearly indicate graduate students believe SOC is necessary for success in graduate programs. Austin describes the role of peer and faculty support in completing or continuing a graduate program. She also stresses the need for appropriate feedback and mentoring. Carlson characterizes good mentors as those who pose good questions, are non-intimidating, provide assistance in completing challenging problems, engage students in regular practice, and encourage students to discuss problems. Herzig emphasizes the importance of formal and informal interaction with faculty and the significance graduate students place on being viewed as junior colleagues. Research related to SOC and mathematics graduate students is scarce, and it is more limited at the undergraduate level. In this report, we add to the research knowledge of SOC in the undergraduate mathematics classroom.

Theoretical Perspective

This qualitative study is a phenomenological inquiry (Patton, 2002) because we explored students' lived experiences in a class with multiple assessments. Through the students' beliefs about, experiences with, and descriptions of the assessments, we uncovered how these assessments contributed to SOC. In an effort to implement assessments relevant to the literature and in line with Steen's (1999) criteria, we evaluated student's understanding of the content through homework, exams, oral interviews, projects, worksheets, and presentations. We attempted to promote valid inferences, allow for multiple measures of performance, measure what is worth learning, and support every student's opportunity to learn important mathematics through a variety of assessments. Social cognitive theory (Schunk, 2004) guided our implementation structure of the assessments since we believe social interaction influences what a student understands. This theory advocates the construction of knowledge, rules, skills, beliefs, and attitudes by observing others. The instructor made use of this theory by encouraging students to work together on all assessments except the exams. Since a majority of the students enrolled in the course were preservice secondary mathematics teachers, the course instructor valued the opportunity to model multiple assessments.

METHODOLOGY

The Course and Participants

The first-named author, who was the instructor of both courses, used the text *Abstract Algebra: A First Undergraduate Course*, by Hillman and Alexanderson (1994). The first semester centered on group theory and the second semester focused on rings and fields. Students successfully completed calculus I, II, and III, discrete mathematics, and linear algebra, before enrolling in the first semester course. Successful completion of the first course was a requirement for the second semester course. Eight male and twelve female

students (N = 20) who completed the abstract algebra sequence participated in this study. The students were primarily preservice secondary teachers; three students intended to pursue graduate school.

Assessments Implemented

Below we provide a brief description of the assessments; a rich description of the assessments can be found in Soto-Johnson, Dalton and Yestness (in press). The teacher graded all assessments for correctness and clarity. Homework was assigned daily, collected and graded on a weekly basis, and returned the class period after it was collected. The instructor encouraged students to work together on homework and provided solutions to the required exercises in the hope students would assess their own work. Frequently, the distributed solutions encompassed student work, which allowed students to examine their peers' proofs.

The primary purpose of the exams was to assess students' mastery of the content in a timed setting. The exams included in-class, take-home, and oral interview components. The teacher did not permit students to work together on take-home components; this was one of the few assessments where the teacher prohibited collaboration. The oral assessments served as an opportunity for students to individually express their knowledge orally.

The instructor designed the projects to assist students with discovering abstract algebra ideas, connecting abstract algebra and the high school mathematics they will teach in the future, and studying abstract algebra applications. Worksheets served as a method to connect several complex abstract algebra concepts through in-class group work. In the second semester, presentations served as a channel for students to communicate mathematics orally and to learn presentation and proof techniques from one another.

Research Instruments and Data Analysis

Our data came from semi-structured, audio-taped interviews (Patton, 2002) with each of the 20 students (see Appendix I). It is important to note the SOC questions (Questions 15 and 16) came at the end of the interview, but some students volunteered the idea of community in the classroom earlier in the interview, (e.g. questions one and nine). In order to best capture and summarize the students' perceptions about contributing factors of SOC and perceived benefits, we employed a pattern, theme, and content analysis (Patton) of the transcribed interviews. Two researchers performed the transcribing, coding and theme searching. This contributed to the validity of the research and improved the quality of research since it allowed for open discussion of findings. Our analysis allowed us to identify the contributing factors and benefits of SOC as perceived by the students.

RESULTS

Through our analysis, we found students mentioned teacher and environment as the primary contributors to SOC. Figure 1 displays these categories and their subcategories. The subcategories for teacher include teacher characteristics and teacher imposed structure of the classroom including assessments. The environment subcategories consist of the classroom setting and the students enrolled in the course. Below we elaborate on the characteristics and use student quotes to support our claims. All names are pseudonyms.

Contributors to SOC

Teacher. Fifteen of the twenty students remarked how the teacher's social and receptive aspects contributed to creating a SOC. Students shared how the example set by the teacher, the teacher's caring personality, and her flexibility contributed to SOC. Sarah stresses the importance of the teacher setting expectations and modeling those expectations.

Sarah: I think it's not only the people that we have in there but the attitude that you set for us. Like you set the example and everybody follows, and then everybody becomes comfortable with the example you set.

The teacher-imposed structure of the classroom refers to how the teacher set up the class especially how she implemented classroom activities. Assessments primarily contributed to a SOC through group work, as acknowledged by 17 of the 20 students. One student mentioned how the difficulty of tests pushed him and other students to work together to study for the tests. This supports the importance of social support as described by Jacobs and Dodd (2003). The difficulty of homework also allowed for both peer interaction and student interaction with the teacher. Students specifically noted how the difficulty of the projects required collaboration. The mini-presentations, while not considered group work, also required student interaction in the classroom. Students, such as Agustin, reported feeling supported by one another and a sense of respect from peers during their presentations.

Agustin I definitely liked it as a presenter because I felt like I had to write something good because it was going to be in front of my class. ...They were always really supportive. ... They provided helpful comments.

Environment. Students perceived the classroom setting made up of tables in the first semester and a smaller class size as well as smaller classroom in the second semester contributed to SOC. These observations were

commonly situated in a comment about growth of SOC from the first to the second semester, which eight students brought up during the interview. Half the students mentioned the two-semester sequence and common major as contributors to SOC. The fact the students were all mathematics majors is an example of coherence as described by Strike (2004); it translates, at least in this classroom, to a shared vision. The following statement by Melissa refers to the shared vision described by McMillan and Chavis (1986).

Melissa: Our class, I feel like we always run into each other. We're also all math majors, so most likely we've had other classes with each other. So with that, we can use each other as resources. I think that that's a huge part of our community, because we all have something in common.

Other students described an existing feeling of cohesion resulting from the SOC created in the first semester. Although Pretty and McCarthy's (1991) research suggests the length of time a person spends in a setting does not influence SOC, our students believe otherwise. It appears coherence and a sense of belonging to this classroom began in the first semester course and was strengthened in the second semester (Strike, 2004; McMillian & Chavis 1986). For some students, such as Bruno, the challenging assessments contributed to creating a SOC.

Bruno: For a lot of people it is the first time they are struggling in math and so if other people are also struggling in math it just automatically builds camaraderie.

Bruno's comment supports the literature (Jacobs & Dodd, 2003) related to SOC in graduate school; challenging assignments bring students together, to collaborate on the assignment. The assessments were designed to be challenging and required collaboration.

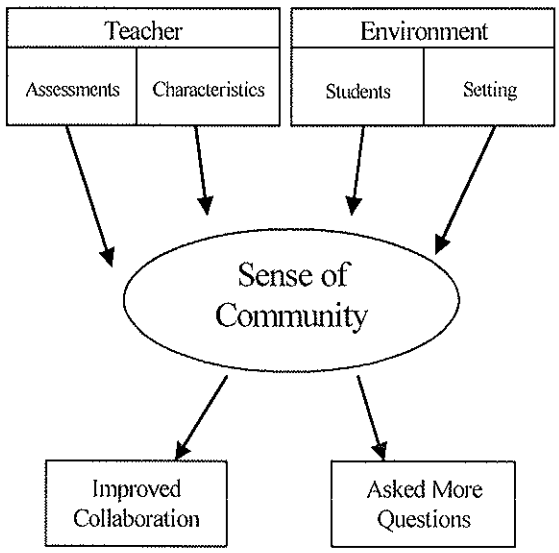

Figure 1. Benefits of SOC

Benefits of SOC

We expanded our SOC model, which only included contributing factors to include benefits of SOC as shown in the bottom portion of Figure 1. Through the coding process, we unveiled two important benefits of SOC as perceived by the students. Students believed a SOC improved collaboration and created an atmosphere where students felt comfortable asking questions. These benefits stem from an environment that endorses learning through increased comfort level among the students and between the students and the professor as illustrated by George.

George: We are totally different people and never would have become friends or associate if it wasn't for classes... The whole class, we can all discuss and ask each other question. It's a comfortable atmosphere.

Students reported they were more prone to ask questions in class and work with other students both in and out of class compared to other math classes.

Jayden: I think there was a group dynamic. I got to the point where I could even ask people that I wouldn't have talked to before how to solve a problem or work through things.

More importantly, the students described how SOC helped their learning. Students felt their grades reflected their involvement in the classroom community. For example, Melissa comments on how her lack of involvement in the community the second semester affected her grade and her confidence to work with the material.

Melissa: When we work outside of your office, I don't have time to do that anymore. People I used to converse with, I don't really talk to as much anymore. So it's a little different this semester. I think it has impacted my learning. Feeling not necessarily as big a part of the community as I was. I think it's made me less confident in the class, and with that obviously my grades are not nearly as good as they were last semester.

DISCUSSION

The model's two main categories emerged from the original research question regarding how assessment contributed to the SOC in the classroom. We broadened the assessments category to include teacher characteristics and named it Teacher since the teacher is responsible for the assessments. We also found students credited their physical setting as well as each other as contributors to the SOC. However, by studying the model it is evident both the teacher and environment categories have a human aspect, the teacher and the students. Studying the model through this lens helps us situate our results within the literature; see Table 1 below. The two subcategories, teacher characteristics

Table 1. Results Related to Literature

	Category: Teacher Subcategory: Characteristics	Category: Environment Subcategory: Students	Category: Teacher Subcategory: Assessments	Category: Environment Subcategory: Setting
Research Results				
Literature	Coherence – shared vision (Strike) Care – necessity to initiate once into the shared vision (Strike) Feels a sense of belonging (McMillan and Chavis) Feels influential (McMillan and Chavis) Feels nurtured (McMillan and Chavis)		Contact – structural features of community (Strike)	

(from teacher) and students (from environment), align with care and coherence from Strike (2004) as well as feeling a sense of belonging, nurturing and influential from McMillan and Chavis (1986). The assessments and setting subcategories from teacher and environment align more closely with contact from Strike.

Studying our model in relation to the literature indicates we can extend the human and environment aspects to Bandura's model of social cognitive theory. Bandura describes his model as "Human functioning is explained in terms of a model of triadic reciprocity in which behavior, cognitive and other personal factors and environmental events all operate as interacting determinants of each other" (as cited in Schunk 2004, p.84). The environment node includes the classroom setting as well as similarities among the student. Further, the teacher can constitute part of the environment because of the manner in which she implemented activities. Thus, the environment is composed of students, teachers and physical attributes of the classroom. Both the teacher and the students represent the behavior and the personal components of Bandura's model.

Furthermore, the teacher's behavior or the example she sets impacts how students interact with one another and creates an environment conducive for questions to the teacher and to other students. Thus, her behavior motivates the personal category as described by Carlson (1999). The teacher's personal qualities (or characteristics) influence the behavior in the classroom as described by students. Similarly, the students' behavior can sway the teacher's behavior. When students ask questions and engage in classroom activities the teacher may reflect on this and stimulate positive energy in the classroom. The students' personal interactions influence the behavior of the teacher and that of the entire classroom.

The results suggest creating a SOC in the classroom and the factors contributing to the SOC have some classroom implications. Our model and student comments illustrate transferable components as well as other components a teacher of any course can replicate. Some of the contributors such as teacher and student characteristics are not transferable. On the other hand, contributors such as teacher-imposed structure of the classroom and classroom setting are easily transferable into the classroom. The variety of assessments and their challenging nature provide a setting in which a class can experience a SOC.

Some students commented how the difficulty of some assignments influenced them to work with other people when in previous classes they worked by themselves because they did not feel the need to collaborate with other students. Multiple group assignments provided the opportunity for students to work with one another. Students referenced the emphasis on group work in the classroom and group assignments as a major contributor to the building of SOC. Other transferable contributors include environmental factors such as tables, small class size, small classroom, and a yearlong two-semester sequence.

We found facilitating a course that promotes interaction creates a SOC. Engaging students inside and outside the classroom through challenging assignments can enhance this learning perspective. We encouraged this collaboration outside the classroom with challenging assignments with the intention that students would seek the support of their classmates and collaborate outside the classroom. Miguel describes this for us from the perspective of a student.

Miguel: Especially because it's gone on all year. We've all taken this really, really hard class, or at least everyone says it's really hard, but maybe it's not that bad. We all had a chance to work with each other on at least something. I've worked with nearly everybody. It's a good community.

Limitations and Implications for Future Research

One limitation of this investigation is the instructor is the primary researcher and interviewer, thus the

research lacked anonymity. Although this can influence students to say what they believe the instructor wants to hear, the students did not hesitate to state pros and cons of the assessments. We also acknowledge that our data sources are limited, but our data is rich in description.

We have seen an increased amount of classroom collaboration in mathematics classrooms since the beginning of calculus reform. Many researchers have demonstrated how collaboration assists with learning mathematics. Social cognitive theory (Schunk, 2004) certainly champions this belief. However, we are not aware of the full impact of collaboration on other educational goals. Our students' perceptions demonstrate multiple assessments, which require collaboration can contribute to building a SOC, which is important to learning.

More research is needed to validate the findings of this study as well as to continue to discover and document benefits of collaborative work and alternative assessments. It is clear that in this course, the collaboration was effective and students learned not only from the teacher but also from one another. The impact of effective facilitation of collaborative work is of great value. This research can be expanded by investigating the influence that courses with multiple assessments have on pre-service teachers. Specifically, how do these courses impact their teaching and assessment styles?

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Appendix

Interview Questions

1. Did you learn from the presentations? Why or why not? Discuss from the standpoint of a presenter and evaluator.
2. Did you learn from the projects? Discuss in terms of the presentations and presenting?
3. What were the advantages of working on a project as a exam?
4. What were the disadvantages of working on a project as a exam?
5. Do you feel that it was valuable to try to read mathematics on your own as part of the exam #3 project? Why or why not?
6. What assessments do you feel reflected your knowledge of the material best? Why?
7. What assessments do you feel do not reflect your knowledge of abstract algebra? Why do you feel this way?
8. What assessments did you feel were the most challenging? Why?
9. What assessments did you enjoy the most? Why?
10. Did you enjoy having a practice midterm oral? Why or why not?
11. How do you feel about having an oral component to the final? Explain.
12. Have your feeling towards the oral component changed from last semester? If so, how? If not, why not?
13. Do you feel that your proof –writing skills have improved over the last two semesters? What do you attribute this too?
14. Is there anything that you would like to share with me about the assessments that have been used in the abstract algebra class?
15. Did you feel that there was a sense of community during this and last semester? Why or why not?
16. What do you feel contributed to this?