

# A productive math struggle: Questions and assessment

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## ARTICLE

**Abstract:** The focus is on productive math struggle questions. Questions given to pose during instruction, to have mathematical conversations, and to evaluate learning. A rubric is provided to evaluate student math conversations. An effective and efficient structure is created where students can delve deeper into mathematics through group discussions.

**Keywords:** Teaching and Learning, productive struggle, Standards for Mathematical Practice, SMP<sub>1</sub>

Students need time to rationalize mathematics, play with it, and make it their own. Through a productive math struggle, teachers can create opportunities for students to conceptualize mathematics. Hiebert and Grows (2007) define a productive struggle this way: “students expend effort in order to make sense of mathematics, to figure something out that is not immediately apparent” (p. 387). Students can contemplate mathematical ideas, take risks, justify their own thinking, and evaluate other student ideas. Creating this environment in the K-12 classroom takes time and practice to establish. The purpose of this article is to focus on productive math struggle questions to enrich learning and guide assessment.

A productive math struggle can take place when students are answering problem solving questions within groups. In a productive math struggle, teachers focus students’ attention on ideas and sense-making and develop their math confidence. Also, teachers allow entry and exit points for a wide range of students, provide extensions and elaborations, and make connections between ideas, concepts, strategies, and procedures. Teaching responsibilities specific to a productive struggle include selecting high-quality tasks (some examples: Illuminations <https://illuminations.nctm.org/>, YouCubed <https://www.youcubed.org/tasks/>, or Exemplars <https://www.exemplars.com/>), asking high-level math questions, allowing students to verify and relate their strategies, listening to student responses, examining their work to extend and formalize their thinking,



and providing targeted feedback. This structure of learning puts several of the Standards for Mathematical Practice into action, including: make sense of problems and persevere in solving them, construct viable arguments and critique the reasoning of others, use appropriate tools strategically, and attend to precision (Common Core State Standards Initiative, 2010).

I organized this article with questions and a rubric to support the structure of a productive math struggle. Teachers may use the first set of questions to support and challenge students' understanding of mathematics content as they move between groups of students. The second set of questions guide student discussion, giving concrete stems to help students share their mathematics ideas and critique the ideas of others. The third set of questions model how to have students reflect upon their learning. Finally, the rubric demonstrates a specific way to evaluate students during a productive math struggle.

The teacher can extend learning by deciding what type of guidance is needed for students and then using their questions to scaffold mathematics thinking. Teachers can ask students questions to clarify students' ideas, emphasize reasoning, and encourage student-to-student dialogue.

*Table 1: Teacher Questions: For a Productive Math Struggle*

Clarify Students' Ideas	Did you use the red trapezoid as your whole? What parts of your drawing/diagram/web relate to the problem? Who could share what Julia just said, using your own words?
Emphasize Reasoning	Why does it make sense to start with these particular numbers? Can you give me an example? What connections do you see between Sara's idea and Sam's idea?
Encourage Student-Student Dialogue	Who has a question for Juan? Turn to your partner and explain why you agree or disagree with Shelly. Talk with Scott about how your strategy relates to his.

During a productive math struggle, teachers can give question stems to guide student discussion during math conversations. These question stems give structure to promote students' participation. Because the stems help everyone to share their ideas, this can help more students to be heard, regardless of their background or status.

I suggest introducing one or two discussion stems each day that students participate in a productive struggle. Teachers can create a classroom anchor chart to hang in the room for student reference. Here are some sample math discussion stems:

- Explain why/how...
- What would happen if \_\_\_\_\_?
- How could \_\_\_\_\_ be used to \_\_\_\_\_?
- Why is \_\_\_\_\_ important?
- Did anyone think of this in a different way?
- Describe \_\_\_\_\_ in your own words.
- What are you thinking now?
- I agree/disagree with \_\_\_\_\_ because...
- That is good thinking because...
- I got different results because...
- My strategy is like yours because...
- My strategy is different than yours because...
- What I hear you say was...

By posing reflection questions for summative assessment, teachers can focus on the learning happening during the productive math struggle. This can exist in the form of math journals, exit tickets out the door, student self-evaluation, or group discussions to have students evaluate their own learning. Here are some sample math reflection questions:

- What were the main concepts or ideas you learned today?
- What questions do you have about \_\_\_\_\_? If you don't have a question, write a similar problem and answer it.
- Describe a mistake that you or a classmate had in class today. What did you learn from this mistake?
- How did your group approach today's question? Was your approach successful?

What teachers assess and grade in their classroom demonstrates to students what is valued. For example, if teachers just grade homework, quizzes, and tests, it tells your students you only value formal assessments. I recommend grading students during a productive struggle session with a rubric to demonstrate the value of mathematics conversations happening in the classroom. This rubric gives students explicit guidance about what is expected of them during the productive struggle math group time. A group grade can demonstrate that conversation and understanding of mathematics is more important than the math answer. Teachers may use the rubric below by placing a tally mark each time they observe a group exhibiting one of the behaviors.

Table 2: Productive Struggle Evaluation Rubric

Group A	Group B	Group C	Group D	
				Leaning in and working in the middle of the table
				Equal air time (everyone takes a turn talking)
				Sticking together discussing each problem before going to the next one
				Explaining how they solved a task with justification and/or reasoning
				Listening to each other when someone is talking
				Asking each other questions to clarify and understand
				Providing solutions using multiple strategies
				Students persevere, persist, and don't give up
				Following group roles or jobs
				Students encourage each other

Overall Rating: \_\_\_\_\_

After teachers have all of the tallies recorded, they can give an overall group grade or they may use it as an informal assessment for that day's productive math struggle. I use a 0-3 scale as an overall rating for all of the items on the rubric:

0 = No evidence during the observation

1 = A few isolated instances of evidence being observed (only a few items attempted; 5 out of 10 with at least one tally mark)

2 = Some evidence observed but does not seem frequent (many items attempted; 6 or more with multiple tally marks)

3 = Strong and frequent evidence observed; is regularly present (multiple tally marks in each)

Through questions, discussion stems, reflective questions, and assessment rubrics, teachers can create conditions for a productive math struggle. Through group conversations, students can delve deeper into mathematics. As a result, classroom math discussions can become more vibrant.

## REFERENCES

- Common Core State Standards Initiative. (2010). Standards for mathematical practice. <http://www.corestandards.org/Math/Practice>
- Hiebert, J., & Grouws, D. (2007). The effects of classroom mathematics teaching on students' learning. In F.K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning*. (pp. 371-404). Information Age.



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