IN THE CLASSROOM

Tasks that Promote Problem Solving

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It takes ¾ cup of sugar to make one pie. If Ms. R has four cups of sugar, does she have enough to make 5 pies?

This was the rich task that engaged Ms. Rosenberg's fifth graders for nearly two hours this morning. Initially, the class claimed there would certainly not be enough sugar for the five pies. After diligent work deciphering the meaning of numerators, denominators and multiplication of fractions, drawing and modelling and labelling, discussing and deciding, learners changed their thinking: indeed, there would be enough sugar—in fact 3 ¾ cups was required to make all five pies.

This task is one of dozens Ms. Rosenberg presents to students each year to invoke their mathematical genius. Sometime she grabs tasks right out of her school's math curriculum; at other times she borrows them from online resources. Often she conjures them from her own creativity by modifying existing tasks while keeping an eye on the standards.

So what makes a rich, juicy task that can deeply engage learners in thinking and understanding important mathematical ideas? In my experience working with K–12 math teachers keen to challenge students as thinkers, I have found that a great task:

- offers varied entry points,
- is open-endedness, and
- invites multiple representations.

Varied Entry Points

In order to succeed as mathematicians, everyone needs a foothold. A task with varied entry points is a task that welcomes less confident math learners to get started. We open entry points by putting mathematics into contexts students can understand, using friendly numbers, welcoming thinking, and focusing on ideas rather than simply seeking solutions.

Sometimes the entry point is not in the print version of the task itself but rather the way it is presented to students: building or reviewing background knowledge before launching a work time, modelling varied approaches to beginning the task (without identifying a solitary strategy), or offering learners a chance to think alone, then partner with peers, can each offer options for beginning. Some teachers even allow students to decide how they want to proceed: start independently, or stick with the teacher on the rug. Differentiating by process in these ways can invite all learners to find their own starting place.

Open-Ended

Math education expert Dr. Jo Boaler describes effective tasks as those that have a "low floor and high ceiling." The high ceiling is the open-endedness that invites all learners to demonstrate understanding to the greatest possible extent in their own unique ways. Open-ended problems are those that invite learners not only to solve mathematical situations but also to articulate their thinking and ideas.

In order to make a typical problem more open-ended, we might ask questions that could be interpreted differently, look for how a specific scenario illustrates a common pattern, or present layered inquiries that continue to challenge motivated mathematicians. Open-ended tasks are intrinsically motivating because they get us and keep us thinking; there is no such thing as, "done."

Multiple Representations

Another way to enliven tasks is to open doors to diverse ways of knowing, and to invite learners to express themselves by more than one means. Their work and solutions could come in the form of models, pictures, tables, diagrams, narrative, graphs, arithmetic, cartoons, slides, movies, and more. When we invite learners to share what they know in a variety of formats, this nudges them to go beyond a "right" answer and instead to figure out how to best explain their thinking. Deep discourse with peers can help thinkers hone their understanding and refine their communication skills.

So, how do we actually do this, provide multiple entry points, offer open-ended inquiries, and invite multiple representations in the context of authentic math learning? I recently developed a sample lesson based around the task, Painted Cubes:

Imagine a large cube made up from 27 small red cubes. Imagine dipping the large cube into a pot of yellow paint so the whole outer surface is covered, and then breaking the cube up into its small cubes. How many of the small cubes will have yellow paint on their faces? Will they all look the same? (Source: https://nrich.maths.org/2322)

To create more space in this problem for learners at all levels, I decided to expand the entry points: rather than presenting merely a 3x3x3 cube, I invited participants to consider a series of cubes starting with 2x2x2 on up to 5x5x5. I offered learners a data table to record, for each cube, how many of their component cubes had one, two and three sides painted, then to explore what patterns they could discover and describe algebraically.

To scaffold success, I modelled the search for patterns and how we might describe those by algebraic rules using a different data set. I also offered omnifix cubes with which students could build representations, and allowed them to choose their own partners. The focus of our reflective conversation at the close of the work time was not what everyone "got" in each box of the data table, but rather on what patterns they had discerned and how we could describe those using algebraic equations. Participants recorded their thinking using three-dimensional models, two-dimensional drawings, tables, equations and more.

This is but one example of how multiple entry points, open-endedness, and an invitation to use multiple representations moved an already interesting math task to something juicy and accessible for diverse learners.

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