Almost-common denominators

Nat Banting
University of Saskatchewan

ARTICLE

Abstract: This article describes a classroom episode where a student's unanticipated reasoning reminds us that foregrounding student sensemaking requires more than listening for strategies from a prepared list. Student knowing can be abrupt and surprising, and the teacher must honor and accentuate mathematical sense as it emerges.

Keywords: Teaching and Learning, comparing fractions, number sense, lesson planning

"I did it a different way."

Ambitious policy documents in mathematics education call teachers to establish a classroom culture that honors learners as active, mathematical knowers (e.g., NCTM, 2014). Inviting student sensemaking means also inviting moments that puzzle, elude, pique curiosity, and even change the course of the lesson entirely.

Providing an environment filled with potential requires anticipation of how students might take up this potential to make mathematical sense. However, as my students have reminded me time and time again (e.g., Banting, 2018), all preparations are susceptible to surprises. Here, I recreate a sequence of prompts that provided a recent, and abrupt, reminder of this.

A LESSON PREPARED

I developed the prompts in preparation for a lesson with preservice elementary school teachers (PSETs) about comparing the sizes of fractions, a topic typically included in standards from the third to fifth grade. Determining which fraction is larger builds toward a foundational conception that fractions have magnitudes (Siegler et al., 2011), which students should explore before working with fraction operations (Bruce et al., n.d.).

Guided by four common strategies for comparing fractions (see Van de Walle et al., 2015), I developed the sequence to, hopefully, elicit a variety of strategies and compare their affordances. I was not interested in establishing a “best” way to compare fractions; rather, I was interested in debating the efficiency and robustness of a variety of solutions. The four prompts were presented one at a time, and the reader is encouraged to spend some time with each prompt as it is presented here.

Which is larger? $\frac{3}{5}$ or $\frac{3}{7}$

Figure 1. Our first prompt.

Many PSETs answered the first prompt (Figure 1) by reasoning that the numerators (the number of pieces) were identical, but the denominators (the size of the pieces) were not. They justified three one-fifths as larger because, although both options resulted in owning three pieces, the one-fifths were larger than one-sevenths. A few PSETs also created a common denominator, claiming that they were once told to do so.

Which is larger? $\frac{3}{5}$ or $\frac{2}{7}$

Figure 2. Our second prompt.

Most students justified their solution to the second prompt (Figure 2) using comparative methods. Some compared both fractions to a benchmark of one-half, determining that three one-fifths must be larger because it is greater than one-half, while two one-sevenths was less than one-half. Another subsection of PSETs referenced the first prompt and claimed that if three one-sevenths was smaller than three one-fifths, then two one-sevenths certainly was as well! In my observations, this novel approach was a sign that they were making sense on their own terms and signaled that more reasoning was to come.

Which is larger? $\frac{3}{5}$ or $\frac{7}{9}$

Figure 3. Our third prompt.

The third prompt (Figure 3) still involved three one-fifths, but now both fractions were greater than the one-half benchmark. This prompted some PSETs to compute a common denominator, still with little justification for why this process worked—it just did. In contrast, others argued that both fractions were only two sections away from a whole. They used this fact to reason that the two one-fifths missing was larger than the two one-ninths missing, therefore seven one-ninths is larger than three one-fifths.

Which is larger? $\frac{3}{5}$ or $\frac{8}{11}$

Figure 4. Our fourth prompt.

The fourth prompt (Figure 4) was designed to evade the common student reasoning strategies I reviewed in my lesson preparation (again, see Van de Walle et al., 2015), and show that the precision of creating a common denominator is valuable in certain situations. Most PSETs did use a common denominator, and I was about to move forward when my lesson trajectory was interrupted:

“I did it a different way.”

She went on to explain that while she did not use a common denominator, she found an *almost-common denominator* by re-writing three one-fifths as six one-tenths. Her new fraction was composed of one-tenths, which are larger pieces than one-elevenths. Six one-tenths is missing four one-tenths from a whole, while eight one-elevenths is only missing three one-elevenths. Therefore, her *almost-common denominator* was enough to convince her that eight one-elevenths was missing fewer pieces of a smaller size—guaranteeing that it was larger.

As a class, we then revisited the previous prompts to see whether this new *almost-common denominator* strategy helped us make sense. It proved useful for the third prompt but was not as efficient for the first two prompts.

Then one student suggested that in order to use *almost-common denominators*, one denominator should be an *almost multiple* of the other. That is why the strategy was useful for the third prompt (with denominators of 5 and 9, where 5•2 is *almost* 9) but not as useful on the second (with denominators of 5 and 7). In response, I created a fifth prompt (Figure 5) and asked the class whether the almost common denominator strategy was relevant.

I now extend the same question to the reader.

$$\text{Which is larger? } \frac{3}{5} \text{ or } \frac{10}{16}$$

Figure 5. A new prompt designed to confound our new reasoning strategy.

A LESSON LEARNED

This episode illustrates the importance of inviting and amplifying student sensemaking. It also suggests a larger connection between planning and surprise.

My mental list of anticipations allowed me to identify, honor, and amplify emerging student sensemaking. However, I was too quick to assume that these anticipations made me surprise-proof.

Instead of allowing me to sidestep surprise, my preparation ensured that I was able to meet surprise head on because I could earnestly pursue my student’s thinking behind the almost common denominator. My anticipations and improvisations informed each other, and my ability to act spontaneously emerged from my pre-lesson scaffolds.

Foregrounding student sensemaking involves more than listening for a strategy from a pre-determined list; it involves listening *with* students as they make sense. This episode reminds us that preparation should make us more sensitive to unorthodoxies and increase our ability to spot leverage points to open new, unexpected, and productive avenues.

REFERENCES

Banting, N. (2018). Survival of the fitness: Mathematical activity as a union of content and process. *Ohio Journal of School Mathematics*, *79*, 32-27.

Bruce, C., Flynn, T., & Yearley, S. (n.d.). *Fractions learning pathways (FLP)*. http://www.fractionslearningpathways.ca/

NCTM (2014). Principles to actions: Ensuring mathematical success for all. Reston, VA: Author. https://www.nctm.org/Store/Products/Principles-to-Actions--Ensuring-Mathematical-Success-for-All/

Siegler, R. S., Thompson, C. A., & Schneider, M. (2011). An integrated theory of whole number and fractions development. *Cognitive Psychology*, *62*, 273-296. https://doi.org/10.1016/j.cogpsych.2011.03.001

Van de Walle, J. A., Karp, K. S., Bay-Williams, J. M., McGarvey, L. M., & Folk, S. (2015). *Elementary and middle school mathematics: Teaching developmentally, 4th Canadian ed.* Toronto: Pearson.

Author: Nat Banting is a mathematics teacher and mathematics education lecturer at the University of Saskatchewan. Nat's interests include the ecological and biological roots of cognition and teacher decision making. Nat is the curator of FractionTalks.com and is the winner of both the 2019 Margaret Sinclair Memorial Award and the 2019 Rosenthal Prize. Nat's website is natbanting.com and Nat is on Twitter at @NatBanting.

Associate Editor: Geoff Krall edited this article.

Suggested Citation:

Banting, N. (2020). Almost-common denominators. *Colorado Mathematics Teacher, 53*(1), Article 2. https://digscholarship.unco.edu/cmt/vol53/iss1/2