Virtual manipulatives: Making effective instructional choices

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Abstract: This article gives three tips for the effective use of virtual manipulatives in mathematics instruction to promote active engagement and student learning: (1) match the virtual manipulative to the learning target; (2) determine the task to pose to students; and (3) encourage discourse.

Keywords: virtual manipulatives, technology, engaged teaching, discourse

Virtual manipulatives (VMs), like all instructional tools, require intentional design to maximize learning. I offer three practical tips to make sound instructional choices to select and implement VMs in mathematics lessons.

A VM is “an interactive, technology-enabled visual representation of a dynamic mathematical object, including all of the programmable features that allow it to be manipulated, that presents opportunities for constructing mathematical knowledge” (Moyer-Packenham & Bolyard, 2016, p. 13). Physical manipulatives (e.g., pattern blocks, base-ten blocks, counters, fraction bars, geoboards, etc.) provide concrete and tactile ways for students to connect with abstract mathematical concepts and explore mathematical relationships. VMs offer similar benefits while leveraging digital technologies to enhance students’ learning experiences. They are available in a wide range of content domains (e.g., addition, multiplication, algebra, fractions, probability, etc.), and they provide a viable option in conjunction with or in place of physical manipulatives.

**TIP #1: MATCH THE VIRTUAL MANIPULATIVE TO THE LEARNING TARGETS**

Besides the need to select a VM that matches the mathematical content that you are teaching, it is also important to select a VM whose features align with the structure and purpose of the lesson.
Different types of VMs provide different types of benefits. In fact, five distinct types of VMs have been identified by mathematics educators (Moyer-Packenham & Bolyard, 2016): multi-representation, single-representation, tutorial, gaming, and simulation. Table 1 gives descriptions, examples, and suggestions for using each VM type.

Table 1. Virtual manipulative types, examples, and suggested instructional uses. (adapted from Moyer-Packenham & Bolyard, 2016). Examples are from illuminations.nctm.org

<table>
<thead>
<tr>
<th>VM Type</th>
<th>Description</th>
<th>Examples</th>
<th>Suggested Usage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multi-representation</td>
<td>Dynamically links at least two types of representations</td>
<td>Coin Box, Factorize</td>
<td>Promote discourse, build conceptual understanding, connect to procedural knowledge</td>
</tr>
<tr>
<td>Single-representation</td>
<td>One type of representation (typically pictorial)</td>
<td>Pan Balance – Shapes, Patch Tool</td>
<td>Promote discourse, enhance the physical version of the manipulative, carefully guided tasks</td>
</tr>
<tr>
<td>Tutorial</td>
<td>Series of self-paced, step-by-step instructions</td>
<td>Grouping and Grazing, Algebra Tiles</td>
<td>Individual instruction focused on learning or for practicing mathematical procedures</td>
</tr>
<tr>
<td>Gaming</td>
<td>Structured as leveled tasks to solve and to earn achievements</td>
<td>Fraction Game</td>
<td>Increase motivation and engagement; requires prior basic understanding of content</td>
</tr>
<tr>
<td>Simulation</td>
<td>Models mathematical situations and results of manipulations</td>
<td>Adjustable Spinner</td>
<td>When a physical demonstration is unrealistic or undesirable (e.g., flipping a coin 1000 times)</td>
</tr>
</tbody>
</table>

The Factorize applet (Figure 1) is a multi-representation VM that connects abstract and pictorial representations of multiplication. Factorize presents students with a target number (abstract) and a grid (pictorial). Students click and drag on the grid to draw rectangles and to find the factors of the target number. The area of the rectangle is...
displayed and changes as the student draws the rectangle (e.g., the red rectangle in Figure 1). If the area is not equal to the target number, the rectangle will disappear. Students are also prompted to type in the factor pairs (e.g., $3 \times 7$). If the student types a factor pair not already on the grid, the rectangle will appear. Factor pairs are color-coded to the corresponding rectangles. Students can build conceptual understanding when they make connections between different representations. This VM presents opportunities for a teacher to point out connections between the factor pairs and the rectangle representations.

Figure 1. Example of multi-representation VM: Factorize. 
https://www.nctm.org/Classroom-Resources/Illuminations/Interactives/Factorize/

TIP #2: DETERMINE THE TASK YOU WILL POSE TO STUDENTS

Some VMs, such as the gaming and tutorial types, contain built-in tasks that are presented to students. Other VMs require the classroom teacher to create tasks to accompany the VM. Some important questions to consider when designing a VM task are:

- Do I want my students to have free exploration time?
- Do I want to present my students with targeted learning activities?
- Are there any activities suggested by the virtual manipulative that I could leverage in my lesson?

Another key element to consider when posing VM tasks is the level of students’ familiarity with the VM. If students are less familiar with a virtual manipulative used in a lesson, the teacher will need to orient the students to its functional features. This orientation will help students to focus their cognitive energies on the mathematical task at hand. For
example, in the Factorize VM, students can click and drag to draw a rectangle and they can type in known factor pairs. The number of white boxes under “Factorizations” indicates how many factor pairs exist for the target number. They can click on the checkmark to check their answers. An orientation to these features at the beginning of a lesson will facilitate students’ use of and learning with the VM.

**TIP #3: ENCOURAGE DISCOURSE**

The use of VMs can enhance students’ explanations and mathematical arguments. For example, one student described his ideas related to factorizations while using the Factorize VM (see Figure 2): “So…if that’s $3 \times 4$ (pointing to the 3 by 4 rectangle) and that represents 12, then a factor pair for 24 will be twice as long as that…twice as many groups of four…so six groups of four. $6 \times 4$!” This student was able to explain and justify his thinking by using the visual representations he had created on the VM.

![Figure 2. Student’s representation of factorizations.](image)

The communication of mathematical ideas through classroom discussion is a critical element of mathematics classrooms. When facilitating classroom mathematics discussions with students, it is important to ask questions that focus the students’ attention on the representation provided by the VM. For example, when working with the Factorize VM, a teacher might ask, “Why do you think the length and the width of the rectangles represent the factors of the target number? If you double a number, what happens to the number of factor pairs? Do you notice a pattern in the factor pairs of your original number and the doubled number?” Asking questions like these can help students connect mathematical concepts with their experiences working with the VMs and develop conceptual understanding.
MAKING THE MOST OF VIRTUAL MANIPULATIVES

VMs have the potential to add considerable value to your mathematics instruction and to your students’ learning. Be strategic in selecting which VMs to use and how to integrate them into your instruction (including mathematical discussions). Before using a VM with your students, become familiar with it yourself. Make note of any parts of the VM that you want to point out as you introduce it to your students and anticipate how they might approach the VM.

Technology is a tool for teaching. It is not a substitute for teaching. The role of the teacher is critical when using VMs for mathematics instruction. The tips offered in this article can help teachers make the most of VMs in mathematics instruction.

REFERENCE


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