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Classification of Activity Patterns in Small Neural Networks in terms of Network Architecture

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Classification of Activity Patterns in Small Neural Networks in terms of Network Architecture

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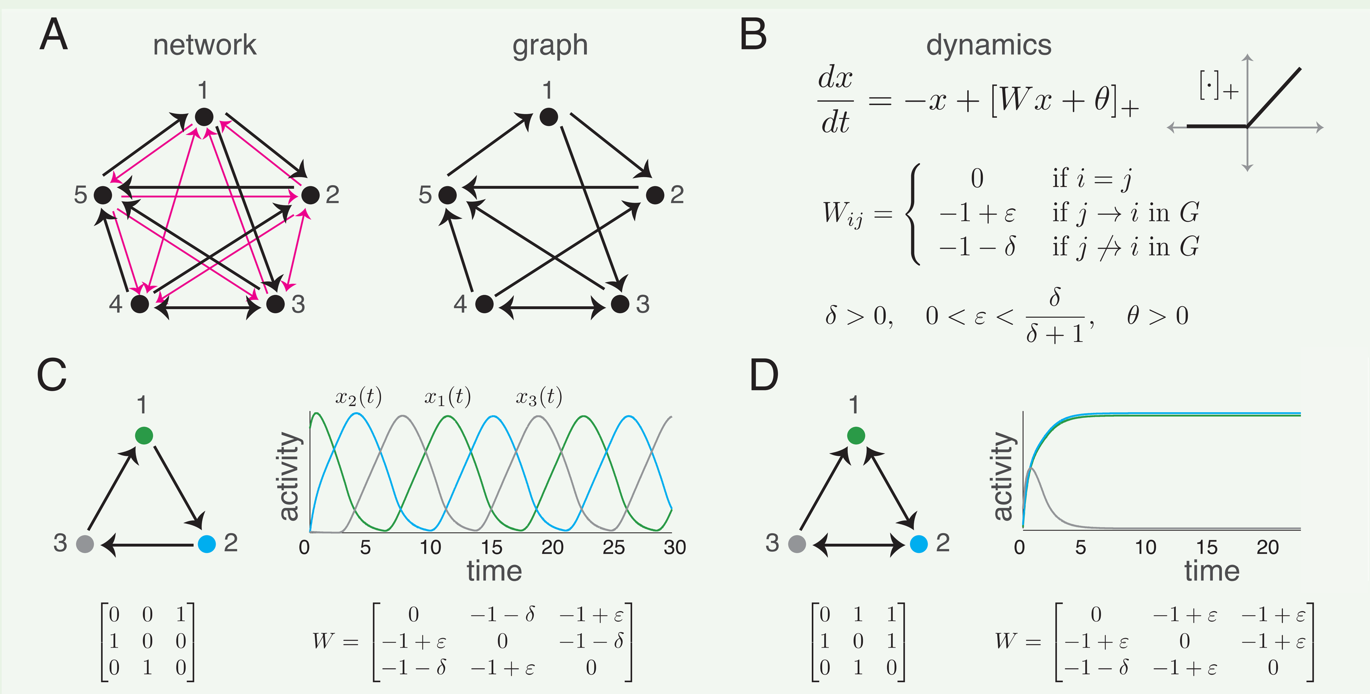
⟷ Combinatorial threshold-linear networks (CTLN) ⟷

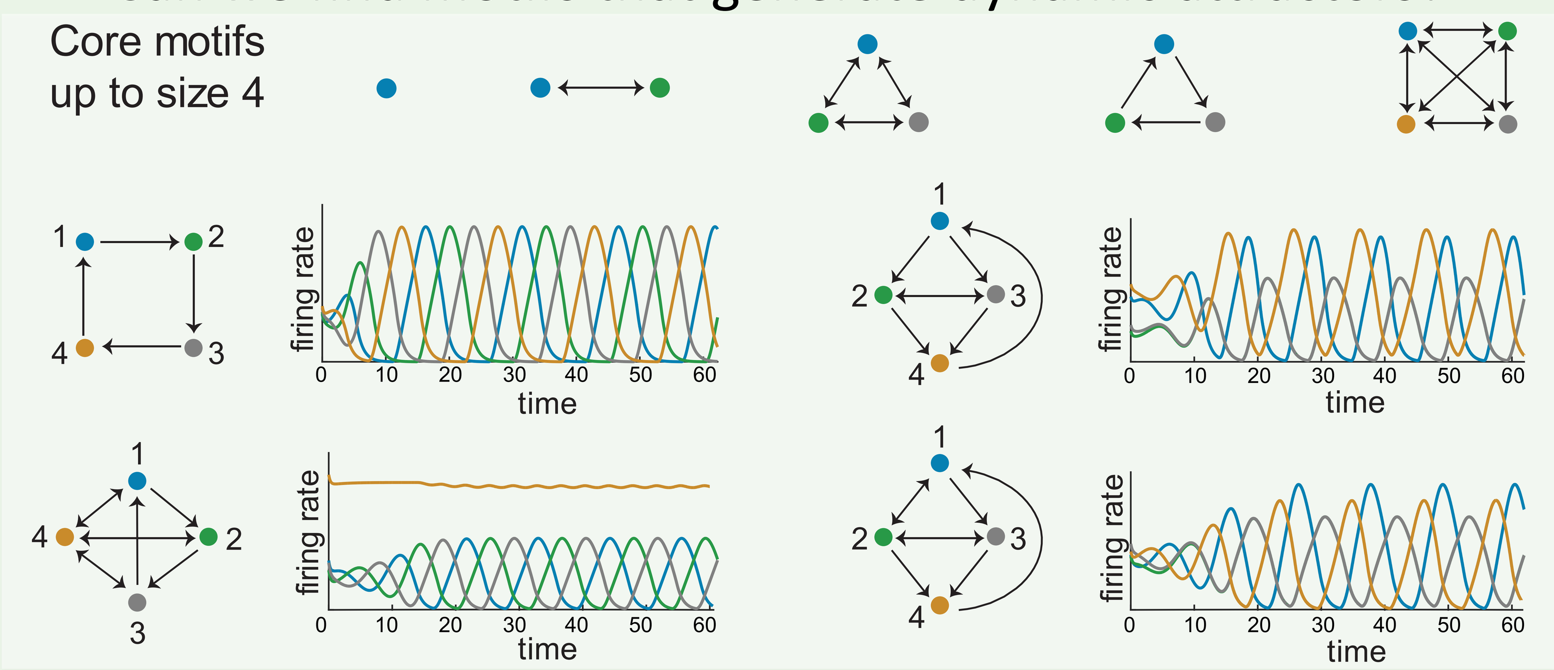


Unless otherwise noted, ϵ=.51, δ=1.76, ϑ=1 in all simulations. Thus, differences in dynamics are due only to differences in the graph G.

Motivating Questions:

How does network connectivity shape emergent dynamics?

Can we find motifs that generate dynamic attractors?



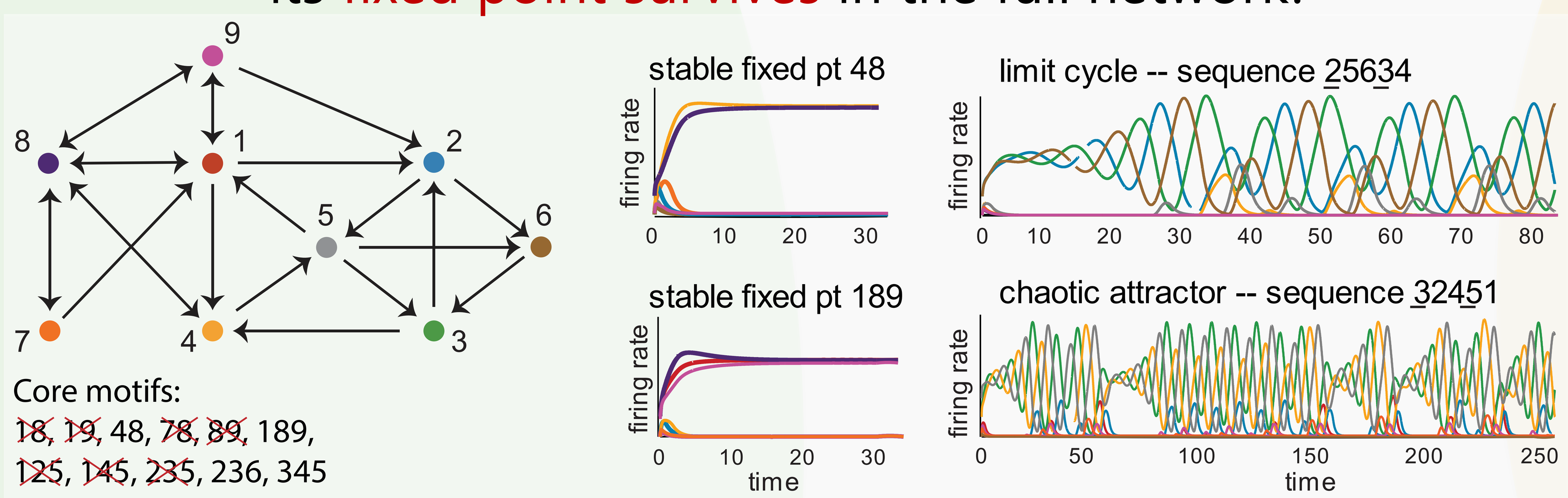
A core motif is a subgraph $G|_\sigma$ with a unique fixed point $\mathbf{x}^*$ that has full support, i.e. $x_i^* > 0$ for all $i \in \sigma$.

Graphs	Survives addition of k	Does not survive addition of k
4-cycle	at most one edge to k	at least two edges to k
4-ufd	at most two edges to k	at least three edges to k
4-clique	at most three edges to k	all four edges to k
fusion 3-cycle	if 4 → k, then at most one edge from the 3-cycle 123 to k; if 4 ↛ k, then all edges from 123 to k allowed	4 → k, and at least two edges from the 3-cycle 123 to k
4-cycu	at most one edge to k; or any pair of edges from {1,2,3} to k; or 2,4 → k; or 3,4 → k	at least three edges to k; or 1,4 → k

A set of graph rules determines whether the fixed point of the core motif survives the addition of a node *k*. The fixed point survives in the full network if it survives the addition of each external node individually.

Rule of thumb:

Every core motif has an associated attractor
if and only if
its fixed point survives in the full network.

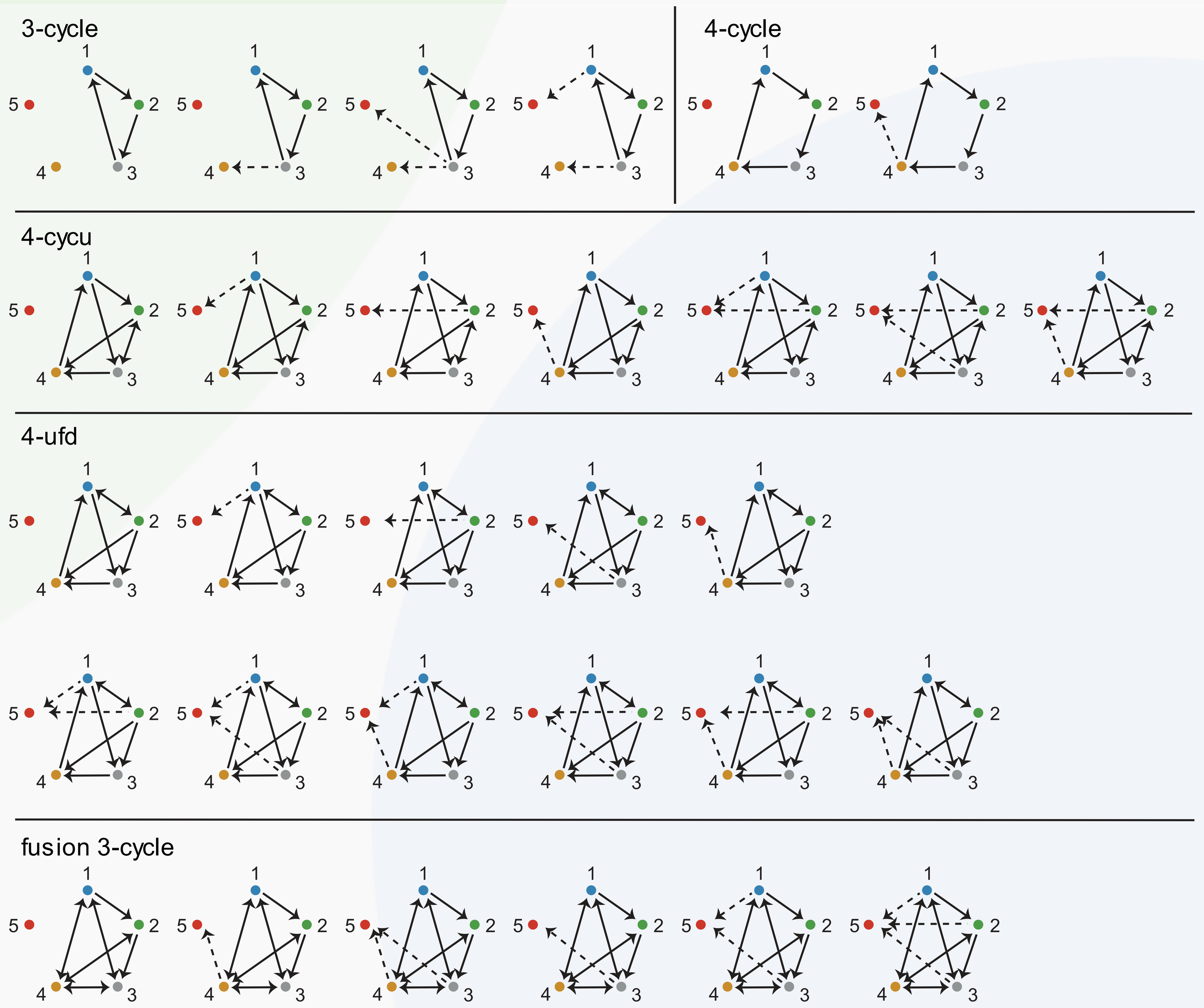


This n=9 graph contains 11 core motifs, but only 4 have a fixed point that survives in the entire network. Thus, 4 attractors are produced.

Testing the rule of thumb

Of the total 9608 graphs of size 5, the Rule of Thumb holds for all but 19 graphs.

We focus on classifying the dynamic attractors of the 1014 graphs that contain a core motif of size up to 4 that produces a dynamic attractor.

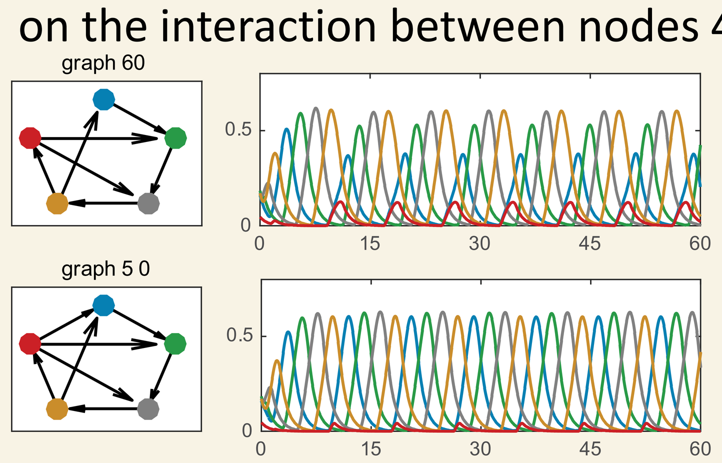


There are 5 core motifs up to size 4 that produce dynamic attractors. The table above shows all the core-periphery classes for graphs of size 5. Each class consists of a core motif and a set of outgoing edges to node 5 such that the fixed point of the core motif survives.

⟷ Summary of n=5 classification of attractors ⟷

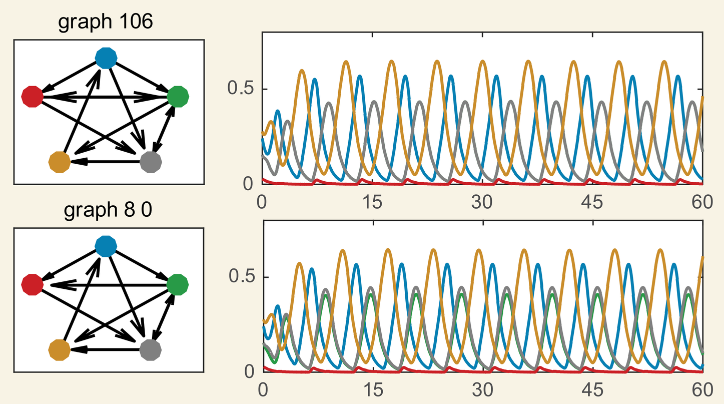
The core-periphery classes largely predict the structure of the dynamic attractors of the graphs of size 5, irrespective of the back edges from node 5 to the core motif.

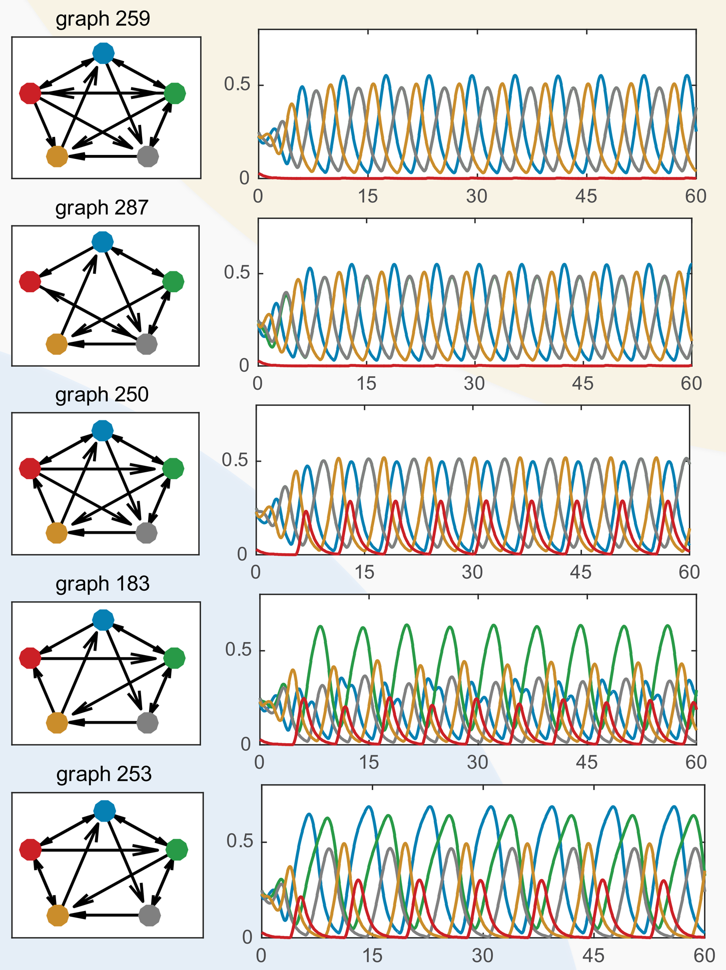
3-cycle: The core-periphery classes perfectly predict the structure of core attractors for embedded 3-cycles, except there is a split in the third class based on the interaction between nodes 4 and 5.



4-cycle: There is slight variation between the attractors in this class based on the height of the peripheral node depending on whether 5 sends an edge to 1.

4-cycu: There are some discrepancies between the attractors of the graphs when the peripheral node sends an edge to node 2 or 3 that breaks the symmetry. Additionally, whenever there is no edge from 1 to 5, the attractor is missing.





4-ufd: This class has significantly more variation between attractors depending on back edges from the peripheral node.

- There is no firing of node 5 when it receives 1 or no edges
- Low firing when node 5 receives from
- 1 & 2, 1 & 3, or 2 & 3
- Higher firing when 5 receives from 1 & 4
- A back edge from 5 to 2 or 3 can break the symmetry
- When there is no edge from 5 to 2, then the attractor is missing

Fusion 3-cycle: This class is perfectly predicted by the core-periphery classes.

Key Takeaways

1. Surviving core motifs give attractors.
2. Core-periphery classes predict general features of attractors, and back edges from peripheral nodes often have little effect.
3. Back edges from the periphery to the core *do* matter when they break symmetry between nodes in the core.

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