8-1-2009

Traditional and nontraditional preservice elementary teachers' perceptions about mathematics and mathematics teaching

Anna Marie Wheeler
University of Northern Colorado

UNIVERSITY OF NORTHERN COLORADO

Greeley, Colorado

The Graduate School

TRADITIONAL AND NONTRADITIONAL PRESERVICE ELEMENTARY TEACHERS' PERCEPTIONS ABOUT MATHEMATICS AND MATHEMATICS TEACHING

A Dissertation Submitted in Partial Fulfillment
of the Requirements for the Degree of
Doctor of Philosophy

Ann Marie Wheeler

College of Natural and Health Sciences
School of Mathematical Sciences
Educational Mathematics

August, 2009

This Dissertation by: Ann Marie Wheeler

Entitled: *Traditional and Nontraditional Preservice Elementary Teachers' Perceptions about Mathematics and Mathematics Teaching*

has been approved as meeting the requirement for the Degree of Doctor of Philosophy in College of Natural and Health Sciences in School of Mathematical Sciences, Program of Educational Mathematics

Accepted by the Doctoral Committee

____
Dr. Hortensia Soto-Johnson Ph. D., Chair

____
Dr. William Blubaugh Ph. D., Committee Member

____
Dr. Steven Leth Ph. D., Committee Member

____
Dr. Jennifer Harding-DeKam Ph. D., Faculty Representative

Date of Dissertation Defense____

Accepted by the Graduate School

____
Robbyn R. Wacker, Ph. D.
Assistance Vice President of Research and Extended Studies
Dean of the Graduate School

ABSTRACT

Wheeler, Ann Marie. *Traditional and Nontraditional Preservice Elementary Teachers' Perceptions about Mathematics and Mathematics Teaching*. Published Doctor of Philosophy dissertation, University of Northern Colorado, 2009.

In this qualitative dissertation, I examined six nontraditional (based on age) and six traditional preservice elementary teachers' beliefs and changes in beliefs about mathematics and mathematics teaching. These case study participants were enrolled in one of three mathematics content courses designed for preservice elementary teachers at a mid-sized doctoral granting university in the western United States. I selected the twelve participants based on the predetermined criteria of gender (female), age (less than 30 for traditional and at least 30 for nontraditional), mathematics instructor, preservice mathematics course, and group dynamics.

Data collection consisted of two approximately 45-minute long interviews per preservice teacher, two approximately 30-minute long interviews per instructor, and classroom observations. After data collection, I coded the data using *NVivo* and searched for themes in the participants' responses. From the coding, I found six themes in the data: *Senses*, *Socio-cultural*, *Standards Aligned Beliefs about Mathematics*, *Nonstandards Aligned Beliefs about Mathematics*, *Standards Aligned Beliefs about Mathematics Teaching*, and *Nonstandards Aligned Beliefs about Mathematics Teaching*. Findings included the fact that nontraditional preservice teachers, on average, ranked themselves higher in their self confidence in teaching mathematics at the K-6 grade levels than the traditional participants. Nontraditional participants also were less likely than traditional

participants to change their belief systems based on preservice mathematics content courses. A common finding among participants included the fact that all participants believed they would teach using all five senses in their future classroom. Implications for teaching of preservice elementary teachers consist of the following: offering activities involving family member participation as classroom practice, providing additional tutoring support and/or a cohort grouping for nontraditional preservice teachers, and giving traditional preservice teachers extra support to decrease possible self efficacy concerns they may have about teaching mathematics.

ACKNOWLEDGEMENTS

During my life, God has blessed me by placing so many people in my life that have provided assistance and encouragement. Throughout all my years of schooling, my late father, Truit, my mother, Martha, and my brother, David, have helped me achieve my goals. This support helped me persevere through the tough times when I was young, as well as when I worked on my more advanced degrees. I will always remember the Sunday afternoon softball sessions and long drives in the back country of Arkansas to relieve life's stresses.

My biggest confidant through my later work has been my husband, Roger. His never-ending encouragement proved to be invaluable to my sanity. He always understood my work ethic, even when coding for 15 hours to South Dakota on a weekend trip may sound a little crazy to most people. Even though I worked hard, Roger knew when I should take a break and go to a Rockies game or skiing.

With my doctoral work, I would like to thank my committee members: Dr. William Blubaugh, Dr. Steven Leth, Dr. Jennifer Harding-DeKam, and especially my research advisor, Dr. Hortensia Soto-Johnson. Dr. Soto has proven to be a significant mentor and friend. I admire her drive for a balance in her academic career and family life. I appreciate her candid honesty in all aspects of life. Even though I was her advisee, she valued my opinions and treated me as an equal in many respects, which I will always appreciate. Thank you for letting me see how you truly live and work; I will always cherish those memories.

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INTRODUCTION

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According to the National Council of Teachers of Mathematics ([NCTM], 2000), mathematics instructors should place an emphasis on conceptual understanding of mathematics in order for students to comprehend the underlying mathematics behind procedures. “Teachers must not only be able to describe the steps for following an algorithm but also discuss the judgments made and the meanings of and reasons for certain relationships or procedures” (Ball, 1990, p. 459). Many who enter preservice teacher programs have not had the opportunity to learn mathematics conceptually; those who learned conceptually continue to focus on memorization of rules and procedures rather than teaching for understanding (Eisenhart et al., 1993). Some teachers embrace the idea of teaching mathematics conceptually by providing students with opportunities to discuss processes and answers theoretically (Boaler, 1998; Kazemi & Stipek, 2001; Lampert, 1990). These activities can lead students to understand mathematics at a deeper level, but some teachers do not provide their students with such learning opportunities.

A reason for this deficit of conceptual teaching could stem from the teachers’ K-12 schooling. Kagan (1992) found that preservice teachers hold certain beliefs about teaching and about themselves as learners that follow them throughout their teacher program and into their classrooms. Many instructors in teacher education programs take this knowledge for granted and believe preservice teachers are “simply lacking particular knowledge and skills” (Ball, 1988, p. 3). For teacher programs to be effective, researchers need to understand more about what preservice teachers believe about mathematics and mathematics teaching. The faculty of teacher programs not only need to understand these beliefs, but they also need to comprehend the differences that may arise among preservice teachers of all ages who have varying opinions about teaching and learning.

There is much research pertaining to preservice and inservice teachers’ beliefs about mathematics and mathematics teaching. Through a careful review of these articles, I discovered that preservice and inservice elementary teachers hold varying opinions about mathematics (Philippou & Christou, 1998; Raymond, 1997; Thompson, 1984). Some feel that mathematics consists of procedural fluency (Ball, 1988), while others possess a conceptual knowledge that goes beyond rules and memorization (Cooney, 1985). Throughout the literature, I found articles that discussed mathematics topics, such as addition, multiplication, division, estimation, probability, algebra, and geometry (Adi, 1978; Ball, 1990; Battista, Wheatley, & Talsma, 1982; Canada 2006; 2008; Canada & Makar, 2006; Dollard, 2006; Dutton, 1951; Eisenhart et al., 1993; Glidden, 2008; Gliner, 1991; Harding-DeKam, 2005; Ma, 1999; Mayberry 1981; Tirosh & Graeber, 1989; 1990; van Dooren, Verschaffel, & Onghena 2002; Yang, 2007; Zazkis & Liljedahel; 2002).

There are also several articles about preservice and inservice elementary teachers’ beliefs about mathematics teaching. Personal experience plays a key role in how preservice teachers think about mathematics teaching (Borko et al., 1992; Mewborn, 1999). Even though prior experience is important, not all preservice teachers think about teaching mathematics similarly. Some preservice and inservice teachers plan to teach mathematics procedurally, some conceptually, and some procedurally and conceptually (Crespo, 2003; Eisenhart et al., 1993; Raymond, 1997; Thompson, 1984; Vacc & Bright, 1999). Preservice teachers often feel mathematics lessons should be fun, no matter the cost or mathematical value in the lessons (Borko et al., 1992; Eisenhart et al., 1993; Gellert, 1998; 2000; Wiegel & Bell, 1996). Some prospective elementary teachers would like to teach using multiple strategies but are not familiar with alternative strategies to the way they learned mathematics (Ball, 1988). Preservice teachers who believe mathematics teaching would be difficult for them commented that they hope to teach lower grades so that their lack of mathematical knowledge would not affect their students' learning. By using manipulatives, some preservice teachers feel they comprehended mathematics better than in the past (Fuson, 1975).

Although there is much research related to the beliefs and attitudes of preservice elementary teachers, none of this research isolates the beliefs and attitudes of nontraditional preservice elementary teachers. Researchers may have studied nontraditional, those 25 years old and older (National Center for Education Statistics [NCES], n.d.), preservice teachers but did not identify them as such. Research related to studies about nontraditional students centers on external factors that influence student attrition. For example, nontraditional students face several obstacles in attending college, including family and financial concerns (Bean & Metzner, 1985; Bundy & Smith, 2004; Richardson, 1994). With these issues, it is not surprising to find that nontraditional students have lower college completion rates than traditional students (Taniguchi & Kaufman, 2005). To help them overcome these hurdles, adult learners often need motivational support systems from friends and family, as well as financial support (Blair, McPake, & Munn, 1995; Chao & Good, 2004). Even though nontraditional students suffer from certain hardships that traditional students might not, they can succeed in mathematics courses (Elliot, 1990) and contribute to classroom discourse (Howard, Short, & Clark, 1996).

As can be seen, several researchers have investigated preservice elementary teacher beliefs about mathematics and mathematics topics but have not examined preservice traditional and nontraditional teachers. In this study, I attempt to distinguish the beliefs related to mathematics and the teaching of mathematics between nontraditional and traditional preservice elementary teachers. Since no researchers have conducted studies about traditional and nontraditional preservice elementary teachers, it will fill this gap in the literature and pave the way for future research in this area.

Purpose of the Study

The purpose of this qualitative case study is to understand the beliefs about mathematics and mathematics teaching of traditional and nontraditional preservice elementary teachers at a midsized doctoral granting university in the western United States. Besides understanding more about the way nontraditional and traditional preservice teachers' describe mathematics, I also investigate how preservice teachers view ideas, such as standards aligned and nonstandards aligned mathematics and mathematics teaching.

Definitions

Before posing my research questions, I will clarify some terms used throughout my investigation.

1. *Preservice elementary teacher*: A preservice elementary teacher is an undergraduate who is enrolled in a teacher education program who plans to attain a teacher license in elementary education.
2. *Nontraditional preservice elementary teacher*: A nontraditional preservice elementary teacher is a preservice elementary teacher who is 25 years of age or older (NCES, n.d.). The institution where I conducted my research defines nontraditional students on its website as students who: audit classes, take only summer classes, and/or return after a 12-month absence. In addition, the university also classifies graduate students who have not been admitted to the graduate program and senior citizens as nontraditional students.
3. *Traditional preservice elementary teacher*: A traditional preservice elementary teacher is a preservice elementary teacher who is less than 25 years old.
4. *Conceptual learning*: Conceptual learning is "comprehension of mathematical concepts, operations, and relations" (Kilpatrick, Swafford, & Findell, 2001, p. 5).
5. *Procedural learning*: Procedural learning is "skill in carrying out procedures flexibly, accurately, efficiently, and appropriately" (Kilpatrick et al., 2001, p. 5).
6. *Conceptual teaching*: Conceptual teaching is teaching students how to comprehend "mathematical concepts, operations, and relations" (Kilpatrick et al., 2001, p.5).

7. *Procedural teaching*: Procedural teaching is teaching students the “skill in carrying out procedures flexibly, accurately, efficiently, and appropriately” (Kilpatrick et al., 2001, p. 5).

8. *Traditional mathematics learning*: Traditional mathematics learning is characterized by the ideas that mathematics is a fixed “unrelated collection of facts, rules, and skills” (Raymond, 1997, p. 556).

9. *Nontraditional mathematics learning*: Nontraditional mathematics learning is characterized by the ideas that mathematics is “dynamic, problem driven, and continually expanding” (Raymond, 1997, p. 557).

10. *Traditional (nonstandards aligned) teaching*: Traditional (nonstandards aligned) teaching is characterized by the ideas that mathematics teaching consists of lecture, “right answers” (Raymond, 1997, p. 558) with no explanation of processes, no group work, no deviation from set lesson plans, and memorization of facts.

11. *Nontraditional (standards aligned) teaching*: Nontraditional (standards aligned) teaching is characterized by the ideas that mathematics teaching consists of difficult questions, an emphasis on process and not the answer, group work, flexible lessons, and a teacher as a facilitator of learning (Raymond, 1997).

12. *Discovery based learning*: Discovery based learning involves students discovering mathematics concepts by investigating and seeking out answers rather than teachers telling them the underlying mathematics concepts and ideas.

13. *Manipulatives*: Manipulatives are "objects designed to represent explicitly and concretely mathematical ideas that are abstract" (Moyer, 2001), such as color tiles, Cuisenaire rods, pattern blocks, and geoboards.
14. *Math 100*: Math 100 is the first of a three course conceptually based mathematics sequence for preservice elementary teachers that addresses topics, as stated in the university catalogue, as "mathematical structures, including numeration systems, natural numbers, integers, rational numbers, relations, functions, and equations." The class is a three credit semester course and meets either on Mondays, Wednesday, and Fridays for 50 minutes each or Tuesdays and Thursdays for 75 minutes each. Instructors use the Beckmann (2007) text, which concentrates on the explanations behind mathematics concepts and procedures, as well as details common mathematics misconceptions.
15. *Math 200*: Math 200 is the second course of a three course conceptually based mathematics sequence for preservice elementary teachers that addresses topics, as stated in the university catalogue, as "representing, analyzing, generalizing, formalizing, and communicating patterns and probabilities." The class is a three credit semester course and meets either on Mondays, Wednesday, and Fridays for 50 minutes each or Tuesdays and Thursdays for 75 minutes each. Instructors use the Beckmann (2007) text.
16. *Math 300*: Math 300 is the third course of a three course conceptually based mathematics sequence for preservice elementary teachers that addresses topics, as stated in the university catalogue, as "two- and three-dimensional

shapes, their properties, measurements, constructions, and transformations". The class is a three credit semester course and meets either on Mondays, Wednesday, and Fridays for 50 minutes each or Tuesdays and Thursdays for 75 minutes each. Instructors use the Aichele and Wolfe (2008) discovery based mathematics content text that does not provide definitions of terms and is activity based.

17. *Social constructivism*: Social constructivism is a learning theory that emphasizes the idea of no absolute truth; learners co-construct knowledge in a social setting, which they then can internalize (Schunk, 2004).

Research Questions

Through case studies, I used qualitative methods, such as the use of classroom observations and interviews, to answer the following guiding research question (Q1) and four sub-questions (Q2-Q5):

Q1 What is the nature of nontraditional and traditional preservice elementary teachers' experiences with and/or perceptions about mathematics and the teaching of mathematics?

Q2 How do nontraditional preservice elementary teachers perceive "mathematics" in terms of standards aligned and nonstandards aligned mathematics in comparison to traditional preservice elementary teachers?

Q3 How do nontraditional and traditional preservice elementary teachers' opinions about "mathematics" evolve (as collective traditional and nontraditional groups and as individuals) throughout a semester long mathematics content course designed to teach preservice elementary teachers in a conceptual format?

Q4 How do nontraditional preservice elementary teachers perceive "mathematics teaching" in terms of standards aligned and nonstandards aligned teaching in comparison to traditional preservice elementary teachers?

Q5 How do nontraditional and traditional preservice elementary teachers' opinions about "mathematics teaching" evolve (as collective traditional

> and nontraditional groups and as individuals) throughout a semester long mathematics content course designed to teach preservice elementary teachers in a conceptual format?

Timeline for Research

The following timeline details my pilot studies and dissertation work.

Table 1

Research Timeline

Time Frame	Work Progress	Completed
Fall 2007	I conducted Pilot Study I. This was a case study of two preservice elementary teachers who enrolled in Math 200 classes under my direction.	Fall 2007
Spring 2008	I conducted Pilot Study II. This was a case study of eight preservice elementary teachers, four enrolled in Math 100 and four enrolled in Math 300. There were two traditional and two nontraditional students from each course. One traditional and one nontraditional participant completed their coursework under my direction.	Spring 2008
Spring 2009	I collected data for my dissertation, a study of 12 preservice elementary teachers, four enrolled in Math 100, four enrolled in Math 200, and four enrolled in Math 300. Half of the participants were traditional and half were nontraditional preservice elementary teachers. Of the four from each Math 100/200/300 course, two were nontraditional and two were traditional participants. None of the participants enrolled in courses under my direction.	

Researcher Stance

In any qualitative study, there are biases that may exist because of the subjectivity in the research. Thus, it is important to state the researcher's background and stance to have a sense of the values and perspectives the researcher is bringing with him/her to the research (Merriam, 1998). For these reasons, I detail in the following paragraphs my educational background and beliefs about mathematics, teaching mathematics, and nontraditional preservice elementary teachers.

Progressing through my elementary and high school, I became increasingly better at algorithms and felt good about my understanding of mathematics. My actual knowledge consisted of rules and memorized facts, which made college mathematics harder for me because of the holes in my mathematics background. If my initial understanding of mathematics was conceptual, then I might not have struggled as much in my college mathematics courses. Conceptual learning helps individuals attain a better sense of how mathematical concepts relate to one another. Conceptual learners understand more than algorithms that often are memorized techniques for arriving at answers.

As I progressed through college, I graduated with a degree in mathematics with an emphasis in education. My certifications include secondary mathematics and English Language Learners (ELL) in the state of Arkansas. I taught three years in the public school system and three years at the collegiate level. Two of my three years of teaching at the collegiate level included instructing preservice elementary teachers. Currently, I work as a teaching assistant/consultant in a local middle school and high school, where my duties include developing lessons/activities, teaching students, assisting with field trips, tutoring students, and grading papers.

These experiences made me an attentive teacher to diverse student populations and learning styles. Not all students learn in the same way, and teachers need to be willing to help all students achieve to the best of their abilities. Teachers can use a standards aligned teaching approach to help their students gain conceptual knowledge of mathematics. In a standards aligned course, instructors act as facilitators in their classrooms, where students work in groups and present their problems and answers to the class for review. I prefer this approach of teaching and feel it is an effective way to instruct students, especially given the holes in mathematics from my own public schooling.

My beliefs about nontraditional students stem from my experiences with teaching preservice elementary teachers. Many of the nontraditional preservice teachers did not learn mathematics conceptually and struggle with the nonprocedural aspects of the class, but I believe they can learn conceptually through practice and perseverance. They are persistent in their work but often lack the mathematical knowledge that several of the younger students possess. Some prefer the procedural methods because of their mathematics background of algorithms. With my data collection, I plan to test these assumptions by observing the preservice teachers learning in their mathematics content course. In addition, my interview questions relate to their concerns or possible struggles with mathematics.

Theoretical Perspective

To understand the lens through which researchers conduct a study, they often explicitly state the theoretical perspective (Creswell, 2007). In the following paragraph, I outline my theoretical perspective of social constructivism.

Constructionism is "the view that all knowledge, and therefore all meaningful reality as such, is contingent upon human practices, being constructed in and out of interaction between human beings and their world, and developed and transmitted within an essentially social context" (Crotty, 1998, p. 42). Social constructivism, as I use in this research, is also concerned with the participants' meaning of what they consider as valuable pedagogical ways to teach mathematics and important ideas to know about mathematical topics (Simon, 1995). I chose this theoretical perspective because the classroom environment in which the students learn is of a social constructivist nature. By interacting with fellow classmates, the preservice teachers will negotiate meanings (Simon, 1994) and co-construct knowledge that they will internalize as individuals. My research question consisted of participants' beliefs about mathematics and mathematics teaching, where the classroom environment of social constructivism influenced these beliefs. These influences materialized in the participants' responses in interviews, which determined the choices of code words. In addition, I interpreted the data based on social constructivist beliefs because of the social constructivist influence in the structure of my research questions and analysis.

Limitations

For the study, there are certain limitations concerning bias, participant sampling, and interview protocol. More specifically, the limitations include the following:

1. I gathered and coded all of the data, which could lead to biases in my interpretations of the data. To combat biases, I implemented member checking, expert checking, triangulation of data, and peer debriefing (Schwandt, 2001).
2. All the study participants volunteered, which might result in certain findings that are characteristic of the type of preservice teacher who would be more likely to volunteer (i.e., high achieving and motivated).
3. I employed a semi-structured interview protocol as described by Merriam (1998). This entailed posing follow-up questions, which might result in unconsciously leading the participants to certain answers.

Delimitations

Two main delimitations in my study involve narrowing the sample and focus of my study. Descriptions of some delimitations of the study are as follows:

1. Since there are few male preservice teachers, I interviewed female preservice elementary teachers who enrolled in Math 100, Math 200, or Math 300 during the spring 2009 semester at a mid-sized doctoral granting university in the western United States.
2. There are also a small number of nontraditional preservice teachers in the elementary education program so I limited my study to 12 participants with an equal number of traditional and nontraditional participants.
3. Since my research involved analyzing participants' views about mathematics and mathematics teaching, I concentrated on preservice teachers' attitudes and beliefs about mathematics and mathematics teaching.

Significance of Study

The findings of the research will be important for mathematics educators who participate in the training of prospective elementary teachers. This research may allow mathematics educators to retain and help procedurally motivated preservice elementary teachers succeed in a conceptually focused elementary education teacher program. In addition, if there are a significant number of traditional and/or nontraditional preservice teachers who feel procedural learning is more important than conceptual learning, mathematics educators can take steps to address these perceptions about mathematics learning in their classes. Through my research, I might also identify factors that influence traditional and/or nontraditional prospective teachers' success in the program, which could inform teacher educators and universities about their teacher education programs. The findings will fill the gap in the literature about comparing and contrasting traditional and nontraditional preservice teachers' thoughts and perceptions about mathematics and mathematics teaching.

Summary

In this chapter, I have detailed the need for research about traditional and nontraditional preservice elementary teachers' views about mathematics and mathematics teaching. To justify my future study, I have explained the significance of the research, which includes having a better understanding of the ways in which nontraditional and traditional preservice elementary teachers comprehend mathematics and view the importance of procedural and conceptual learning of mathematics. Since there are no studies which compare traditional and nontraditional preservice elementary teachers, I have found a gap in the literature about preservice teachers.

My theoretical perspective of social constructivism and my personal stance of a former secondary teacher will play significant roles in the ways in which I conducted and analyzed the data. In regards to limitations of the research, they include the fact that I coded and analyzed all the data, which might result in biases. In addition, all the participants volunteered, which might lend to certain sample characteristics that are not representative of the entire population of preservice elementary teachers. Lastly, I utilized semi-structured interviews, where my follow-up questioning may consist of unconsciously relying on leading questions. For delimitations, I restricted my research to 12 female preservice elementary teachers who took Math 100, Math 200, or Math 300 during the spring 2009 semester at a midsized doctoral granting university in the western United States.

In the following chapter, I summarize the literature related to my research question and describe how it informed my study. The literature centers on adult learners, preservice/inservice teachers' beliefs about mathematics including topics from Math 100/200/300, and preservice/inservice teachers' beliefs about teaching mathematics.

CHAPTER II

LITERATURE REVIEW

Introduction

In this chapter, I summarize the literature related to my guiding research question (Q1) and four sub-questions (Q2-Q5) that include:

Q1 What is the nature of nontraditional and traditional preservice elementary teachers' experiences with and/or perceptions about mathematics and the teaching of mathematics?

Q2 How do nontraditional preservice elementary teachers perceive "mathematics" in terms of standards aligned and nonstandards aligned mathematics in comparison to traditional preservice elementary teachers?

Q3 How do nontraditional and traditional preservice elementary teachers' opinions about "mathematics" evolve (as collective traditional and nontraditional groups and as individuals) throughout a semester long mathematics content course designed to teach preservice elementary teachers in a conceptual format?

Q4 How do nontraditional preservice elementary teachers perceive "mathematics teaching" in terms of standards aligned and nonstandards aligned teaching in comparison to traditional preservice elementary teachers?

Q5 How do nontraditional and traditional preservice elementary teachers' opinions about "mathematics teaching" evolve (as collective traditional and nontraditional groups and as individuals) throughout a semester long mathematics content course designed to teach preservice elementary teachers in a conceptual format?

Within the scope of the sub-questions, I will investigate how preservice teachers view traditional and nontraditional aspects of teaching and learning as detailed in Raymond's (1997) work with inservice elementary teachers. I categorized preservice teachers in my first pilot study based on LaBoskey's (as cited in Griffin, 2003) three categories of thinkers: concrete, alert, or pedagogical. Concrete thinkers tend to think in a procedural manner, where their interests center on algorithms and correct answers. On the other hand, pedagogical thinkers are interested in higher order thinking, multiple strategies for understanding, and conceptual learning. They possess sound content knowledge and want to teach their future students conceptually. Alert thinkers are in a middle ground between concrete and pedagogical thinkers. In my first pilot study, I determined how traditional and nontraditional preservice elementary teachers thought about mathematics. I felt Raymond's work contained a broad scope that encompassed many ideals of nontraditional and traditional preservice teachers, so I switched my concentration to Raymond's methodology for my second pilot study.

Raymond (1997) described traditional mathematics beliefs as a fixed set of "unrelated collection of facts, rules, and skills" (p. 556). Raymond detailed traditional mathematics teachers as individuals who tend to lecture, to strive for right answers with no explanation of processes, to not utilize group work, to maintain inflexible lesson plans, and to want students to memorize facts. On the other hand, nontraditional mathematics beliefs consist of the ideas that mathematics is "dynamic, problem driven, and continually expanding" (p. 557). According to Raymond, nontraditional mathematics teachers ask difficult questions, care more about the process than the answer, utilize group work, maintain flexible lesson plans, and act as facilitators of learning.

Since I am already using the terms "traditional" and "nontraditional" to refer to the age of preservice elementary teachers, I will therefore describe "traditional" beliefs about teaching and learning as "nonstandards aligned" and "nontraditional" beliefs about teaching and learning as “standards aligned.” I chose these terms because of the differences inherent in traditional and nontraditional approaches to teaching and learning that are consistent with NCTM (2000) views of standards and nonstandards aligned teaching and learning.

For the literature review, my focus areas center on adult learners, preservice/inservice teachers’ beliefs about mathematics, and preservice/inservice teachers’ beliefs about teaching mathematics. In the following paragraphs and sections, I discuss my rationale for inclusion of each of these sections in my literature review, as well as detail articles from each category.

I include articles about adult learners because I am comparing traditional and nontraditional prospective teachers. When I discuss nontraditional preservice elementary teachers and/or nontraditional students, I refer to individuals who are at least 25 years old. In my literature search, I found that several researchers had a different definition for nontraditional students. In such cases, I specify the researchers’ definitions of nontraditional. If I do not specify a range of ages, a reader may assume that a nontraditional student is 25 years of age or older.

Since the literature about mathematics teachers and teaching is extensive, I narrowed my literature review to articles about K-8 preservice and inservice mathematics teachers. Although elementary teachers tend to be certified to teach K-6, some of these teachers are employed in middle schools where they may teach 7th and 8th grade mathematics. The direction of my research question includes preservice elementary teachers’ thoughts and perceptions about mathematics and mathematics teaching so it was natural to include such articles.

Although my study concentrates on preservice elementary teachers, there are a few articles about inservice elementary teachers (Raymond, 1997; Thompson, 1984) that influenced my dissertation research. From these articles, I found ideas that affected my research questions and interview protocol. Thus, I felt it necessary to include articles about inservice elementary teachers in my literature review.

Adult Learners

In this section, I detail articles about adult learners in regards to mathematics achievement, affective concerns, burdens for success in college, motivational factors, and classroom discourse. Elliot (1990) and Richardson (1994) discussed aspects and/or struggles nontraditional students have towards mathematics achievement. Bean and Metzner, (1985), Schuetze and Slowey (2002), Bundy and Smith (2004), Taniguichi and Kaufman (2005), Viskic and Petocz (2006), and Trueman and Hartley (1996) detailed burdens that nontraditional students face in attending college. These burdens include such topics as family, finance, and remedial coursework. With the topic of motivation, Chao and Good (2004), Schloglmann (2006), and Blair et al. (1995) investigated nontraditional students' motivational factors that consisted of family and support systems. Howard et al. (1996) examined the idea of classroom discourse of traditional and nontraditional students and their reasoning for participating in class discussions. Most of the articles are from research conducted years ago and none of the authors discusses prospective teachers.

Elliot (1990) found that nontraditional freshman students scored at the same level as traditional freshman students on basic mathematics. Elliot administered an arithmetic pretest and an algebraic posttest to 75 nontraditional and 75 traditional students enrolled in basic algebra classes at seven Michigan universities. The researcher also investigated affective variables of “causal attribution, confidence in learning mathematics, and perceived usefulness of mathematics” (p. 160). In order to measure these variables, Elliot administered three student surveys at the beginning of the course based on each of the three affective variables. Sample survey items included, “Mathematics always has been one of my most difficult courses,” and “Using mathematics will be necessary in earning my living” (p. 161). Elliot used a multiple regression analysis to discover that the more a nontraditional female student felt luck played a role in her mathematics achievement, the lower her grades on mathematics exams were ($p < .05$). The researcher did not find this link between success and luck to be significant with any other participant group (i.e., nontraditional males, traditional females, and traditional males). There is no additional discussion about this result, which is a weakness of this article.

In support of Elliot’s (1990) findings, Richardson (1994) also detailed the sentiments that mature students are able to succeed in higher education. Through a synthesis of research on adult learners, he contended that adult learners do not lack the basic skills to productively study and further their education; rather, adult learners face issues such as personal or financial reasons that may cause them to withdraw from school. Richardson also addressed the inaccurate myth that older students are not capable of learning in higher education because of the aging process. Instead, nontraditional students obtain a type of wisdom that traditional students have not yet acquired.

Nontraditional students can face burdens that make school challenging and even unattainable. Similar to Richardson (1994), Bean and Metzner (1985) conducted a meta-analysis of obstacles that nontraditional students have outside of their schoolwork. They defined a nontraditional student as someone who “is older than 24, or does not live in a campus residence, or is a part-time student, or some combination of these three factors” (p. 489). Through a meta-analysis of the literature, the authors created a conceptual model for undergraduate nontraditional students’ dropout patterns. Bean and Metzner discovered the four main variables of GPA, intent to leave, past educational performance, and environmental factors like family commitments affect dropout rate for nontraditional students. The authors also found “nontraditional students are more affected by the external environment than by the social integration variables affecting traditional student attrition” (p. 485). Some of these demands include employment, commute time (Schuetze & Slowey, 2002), financial aid, childcare, and remedial coursework (Bundy & Smith, 2004). These demands impede nontraditional students from completing college. In fact, nontraditional students have lower completion rates of college than traditional students (Taniguchi & Kaufman, 2005).

Taniguchi and Kaufman (2005) found the results through a quantitative study of 792 males and 911 females that were over the age of 20. The researchers analyzed data from the National Longitudinal Survey of Youth (p. 918) to see how many nontraditional students completed a four-year undergraduate degree program. They discovered that 59% of males and 65% of females who enroll part-time do not complete their degrees. In addition, nontraditional students with young children significantly decreased their successful attainment of college degrees.

As a consequence of student attrition, Viskic and Petocz (2006) found it difficult to conduct research in their preparatory mathematics course. The authors discussed how half of the “mature-age” (p. 7) students, those aged 21 years or older, taking their beginner’s mathematics course dropped the class. Student reasons for leaving the class included needing to move for job purposes, finding it difficult to manage school and family, and believing the mathematics was too difficult.

Initially, Viskic and Petocz (2006) conducted research in Australia on student reflections about classroom projects in their beginner’s mathematics course. The authors investigated nontraditional students’ views about mathematics using class assignments. Viskic and Petocz collected data in the form of written reflections that students wrote individually or as group activities. The reflections addressed material from class projects on various mathematics topics, such as counting systems, infinite series, radiocarbon dating, and women in mathematics.

Viskic and Petocz (2006) examined the reflections using a previous framework they had developed. Three components of the framework included the following:

1. *Components*: Students think of mathematics as compartmentalized.
2. *Models*: Students view mathematics as representations or models that individuals can shape into mathematical relationships.
3. *Life*: Students believe mathematics connects to everyday situations.

To this list, the researchers added *Techniques*, the idea that students believe mathematics consists of mathematical techniques. This component came from research conducted by one of the lead authors.

Viskic and Petocz (2006) found several examples of their framework types, which they outlined in their study. Below is an example of one nontraditional student reflection on strategies and successes in working with different bases that the researchers classified as demonstrating the framework of *Conceptions*.

> We started by working out how to change between bases, particularly 5 and 10, and initially found it so difficult it almost hurt. A big breakthrough occurred when we were working on the Alien question, doing a lot of trial-and-error work and having some bizarre discussions! After that, the notion of base and power fell into place, and the ease of calculation between bases increased considerably (p. 10).

The authors also discovered that students who reflected on the mathematics course felt self-aware and confident. Several expressed positive attitudes about group work and an increased ability to manage their time, as detailed in the following quote.

> Looking back, I actually appreciate what I have picked up with time management. I don't know whether I had made many discoveries, rather than what I have been learning about myself. I seem to have a lot more confidence in myself, knowing that I can do the work that is required. Also learning that it will take time, I won't get it straight away but if I stick to it then it will eventually all unfold. I loved that feeling (p. 13).

Trueman's and Hartley's (1996) work also adds to the research knowledge base about classroom management. They found that older students, at least 26 years old, have better time management skills than do younger students.

Researchers examined other factors, such as the motivation of nontraditional students. Chao and Good (2004) conducted a grounded theory study of 43 nontraditional undergraduate students' beliefs about their college experiences. The authors interviewed participants and transcribed their conversations. Through a grounded theory analysis, Chao and Good created a main category of hopefulness for nontraditional students with five related sub themes: "motivation, financial investment, career development, life transition, and support systems "(p. 7). Several nontraditional participants felt motivated to finish college with assistance from support systems, such as friends, family, and faculty who provided encouragement.

Schloglmann (2006) quantitatively examined the motivation of 419 adult learners enrolled in 1 of 19 varying mathematics courses, such as basic mathematics and upper level classes, at 7 Austrian universities. Each participant completed a lengthy questionnaire that included demographic information, as well as various questions about such topics as attitudes towards mathematics and improvements needed in mathematics education. The participants' teachers also completed similar survey items. Schloglmann found that the greatest motivational factors for adults in basic classes consisted of "improvement of personal education, increased vocational demands, joy of learning new subjects, and to cope with life problems" (p. 10). Through a principle component analysis, the author discovered four factors of motivation, which included "professional and economic advancement," "personal motives," "general professional performance orientation," and "change in job" (p. 10). Adults who were taking vocational classes described "acquisition of latest professional knowledge, increased vocational demands, improvement of personal education, and security in economically unstable times" (p.10) as their highest motivational factors.

Blair et al. (1995) also examined the motivational goals of adult learners, who returned to school. They conducted research in the United Kingdom with 50 nontraditional students, ranging in age from 21-70. Data collection consisted of one semi-structured interview per participant that included discussion topics, such as reasons for reenrolling in school and future plans. The researchers coded the data and found two main themes, *Goals* and *Conditions* (circumstances), which influence nontraditional students' involvement in education. Blair et al. concluded that "gaining qualifications or skills to secure a better job, enjoyment, learning for its own sake, getting out of the house, making new friends, and gaining a place on a more advanced course" (p. 637) were various reasons adults enroll in school. Participants mentioned grants, student loans, and support systems as necessary aids in returning to school. Jane, one of the participants, shared her reasons for returning to school that included bettering her children's future and socializing.

> This is me trying to get back. [I wanted] to provide my youngest son with some type of nursery education and give myself something to take my mind off everything…It gives me the chance to meet other people—otherwise I would be totally isolated…I was given encouragement and support…It helped me get myself sorted out and it's given support for me and my children (p. 644).

With classroom discourse, Howard et al. (1996) found that nontraditional students, students over the age of 24, participated more in class discussions than traditional students did with a rate of 37.5% to 56%. The researchers selected 13 student volunteers to observe 13 different introductory courses of various topics at a university, which resulted in 3,521 observations of 247 different students. The courses included such topics as English, education, music, business, and anthropology (p. 11). Nontraditional students consisted of 37% of the students observed in the classes. Besides classroom observations, 170 students and 13 teachers answered surveys about class participation and personal views about classmates and their instructors. Howard et al. collected additional information about classroom participation through 22 student and five teacher interviews. The researchers found that nontraditional students made two or more comments per class meeting, which was almost twice (40.4% as opposed to 20.7%) the number of comments made by the traditional students. The authors did not remark on the types of student responses but only on why students do not participate in classroom discussions. The four main reasons nontraditional students gave for not contributing in class were the following: "feeling that I don't know enough about the subject (50%), I had not done the assigned reading (35.3%), large size of class (30.9%), and feelings that my ideas are not well enough formulated (29.4%)" (p. 17).

Many of the articles (Blair et al., 1995; Schloglmann, 2006; Trueman & Hartley, 1996; Viskic & Petocz, 2006) included in my literature review detailed adult learners in various countries other than the United States. Through my literature search, I did find some articles about nontraditional students in the United States (Chao & Good, 2004; Elliot, 1990; Howard et al., 1996), but research mainly from other countries were more relevant to my study.

Preservice Elementary Teachers' Beliefs about Mathematics

Since my research will concentrate on the nature of nontraditional and traditional preservice elementary teachers' thoughts and perceptions about mathematics and mathematics teaching, it is only natural to include a section about preservice elementary teacher's beliefs about mathematics. I discuss some overarching values that preservice teachers demonstrate about mathematics and various views on specific topics in Math 100, Math 200, and Math 300 classes, the classes from which I solicited participants for my pilot studies and dissertation. Ball (1988), Cooney (1985), and Philippoiu and Christou (1998) discovered varying preservice teachers' beliefs about mathematics, which centered on ideas such as procedural beliefs, conceptual beliefs, hatred of mathematics, and enjoyment of challenges. Ball (1990), Crespo and Nicol (2006), Dutton (1951), Eisenhart et al. (1993), Glidden (2008), Gliner (1991), Tirosh and Graeber (1989; 1990), Wheeler and Feghali (1983), and Yang (2007) detailed examples of preservice teachers' beliefs about Math 100 topics, such as addition, subtraction, estimation, division, and multiplication. Adi (1978), Canada (2006; 2008), Canada and Makar

(2006), Dollard (2006), van Dooren et al. (2002), and Zazkis and Liljedahel (2002) examined preservice teachers' beliefs about Math 200 concepts such as algebra, probability, and data analysis. Battista et al. (1982), Charles (1980), Martin and Harel (1989), Mayberry (1981), Soto-Johnson, Cribari, and Wheeler (2009) conducted research on Math 300 geometric topics, such as shapes, proofs, symmetry, geometry facts/conceptual understanding, and spatial visualization, respectively. In the following paragraphs, I will describe work conducted by researchers about mathematics in general, number sense, probability/data analysis, algebra, and geometry.

General Mathematics.

Preservice teachers' views about mathematics in general can vary noticeably from person to person. Some may love mathematics, while others may find it their worst subject. Researchers, Philippou and Christou (1998), conducted a mixed methods study through a three-year investigation of preservice elementary teachers' attitudes about mathematics. Their research included the following questions:

1. What are the attitudes towards mathematics of prospective primary teachers entering University Education programs?
2. Can the attitudes of candidate teachers be altered by mathematical experience in their preparatory program?
3. Do changes in beliefs for preservice teachers vary by individual characteristics (p. 194)?

Philippou and Christou (1998) administered participant (the sample size ranged from 128-160) surveys at the beginning, in the middle, and at the end of the participants' courses in mathematics content and methods. In addition, the researchers interviewed participants. Below are sample survey items.

1. I detest mathematics and avoid using it at all times.

2. I have never liked mathematics.
3. I enjoy doing problems, when I know how to do them.
4. I would like to spend more time at school working on mathematics (p. 197).

Philippou and Christou discovered the most common reasons for participants liking mathematics were the following: “it [mathematics] develops mental abilities (47%), it is practical and useful (39%), it is interesting and challenging (35%), and it is necessary for modern life (35%)” (p. 198). The main reasons for not enjoying mathematics were, “I was afraid of it (29%), because of poor teaching (27%), and lack of teacher enthusiasm (25%)” (p. 198).

One preservice teacher in the study (Philippou & Christou, 1998) discussed how her beliefs formed from the ideas that mathematics is only about right answers. Another participant expressed the view that the “proper way to learn mathematics” (p. 202) was to memorize rules and facts, where answers were right or wrong. One preservice participant believed that “mathematics will hunt me forever” (p. 202). The only positive response came from a preservice teacher who said that mathematics was like a journey of interesting experiences, where “I felt more confident when I realized that even great mathematicians did mistakes as frequently as I did” (p. 202).

Teachers may view certain ideas about mathematics as either procedural or conceptual in nature and these viewpoints may influence the way they teach mathematics. In Ball’s (1988) study, she investigated preservice teachers’ beliefs and how beliefs influence mathematics teaching. Through analysis of preservice teachers’ responses to a project about permutations, she found several prospective elementary teachers believed knowing mathematics involved calculating right answers without any knowledge of why the procedures worked. Cindy, a preservice teacher, echoed these sentiments when she talked of high school and how performing procedures made her a good mathematics student, though she did not understand the formulas. Two other preservice teachers mentioned mathematics knowledge as time-consuming, frightening, and unnerving.

Unlike Ball's (1988) work, Cooney (1985) investigated a preservice teacher who loved conceptual mathematics. Cooney conducted a case study of a preservice teacher named Fred to examine his beliefs about mathematics and mathematics teaching. Through a series of seven interviews that lasted approximately 45 minutes each, Fred described aspects about mathematics and mathematics teaching. The researcher varied the content of the interviews. With the initial two interviews, Fred answered questions that came from a prescribed interview guide. The next two interviews consisted of content that addressed Fred's responses from the earlier meetings. In the fifth interview, Fred analyzed his previous responses and synthesized his ideas from all four past sessions. The sixth and seventh interview also addressed Fred's overall beliefs that he detailed throughout the research. Cooney discovered that Fred believed mathematics to be "useful, logical, axiomatic, fun, and hard" (p. 327). Fred felt that mathematics consisted of problem solving activities like puzzles.

Number Sense and Operations.

Besides understanding about preservice teachers' beliefs about mathematics in general, I also feel it is important to know about preservice teachers' beliefs on number sense and geometry since this is the focus of the Math 100 and Math 300 courses, respectively. Yang (2007) examined preservice teacher approaches to solving questions about number sense. The 15 participants consisted of five mathematics majors, five language education majors, and five elementary education majors. Yang collected data in the form of interviews that included 12 questions relating to number sense ideas, such as estimation, benchmarks, and fraction size. The examples below represent some of Yang's interview questions.

1. Without calculating, circle the best estimate for 103 x 48?
   (1) 100 x 50 (2) 103 x 50 (3) 100 x 48

2. Without calculating, order the following numbers from smallest to largest:
   13/38, 0.966, 7/29, 0.4828, 17/16, 8/15

3. What's the reasonable estimate of 61027 ÷ 33.275 (p. 295)?

The author scored the responses based on accuracy and classified the answers into one of the following three categories:

1. *Number sense-based (NS-based)*: The participant's strategies could be identified as one of the following groupings: "understanding the meaning of numbers, operations and their relationships," "recognizing relative number size," developing and using benchmarks appropriately," or "judging the reasonableness of a computational result by using the strategies of estimation" (p. 295).

2. *Rule-based*: The participant solved problems procedurally with no conceptual knowledge of the algorithms.

3. *Wrong explanation*: The participant was unable to describe their reasoning process.

Through his research, Yang (2007) discovered that preservice teachers preferred procedural work when trying to decide the size of numbers. Ten out of the 15 participants resorted to common denominators or decimal equivalents to answer these questions. When Yang asked the participants to solve the problem in a different way, almost none of the participants could think of another method. Similarly with estimation questions, the participants utilized computations instead of estimation strategies to find the solutions.

Twelve of the 15 preservice teachers used the division algorithm to answer an estimation question about division with decimals.

Tirosh and Graeber (1989) also investigated preservice elementary teachers' number sense beliefs, specifically about multiplication and division. The authors conducted a study of 136 preservice elementary teachers enrolled in either a mathematics content or mathematics methods course designed specifically for undergraduate elementary education majors. The participants answered the following six multiplication and division statements as true or false with explanations of their reasoning.

1. In a multiplication problem, the product is greater than either factor.
2. The product of .45 x 90 is less than 90.
3. In a division problem, the quotient must be less than the dividend.
4. In a division problem, the divisor must be less than the dividend.
5. The quotient for the problem 60/.65 is greater than 60.
6. The quotient for the problem 70/ ½ is less than 70 (p. 81).

The preservice teachers also completed either 16 or 21 problems that consisted of mainly division and multiplication problems. The researchers interviewed approximately one half of the participants about division and multiplication questions.

Tirosh and Graeber (1989) discovered that a "majority" of preservice teachers held the belief that the "quotient must be less than the dividend," while some also believed that "multiplication always makes bigger" (p. 91). The authors claimed that preservice teacher mistakes with multiplication and division problems are due to the lack of conceptual knowledge the preservice teachers have about the two operations.

Tirosh and Graeber (1990) also conducted a study of preservice elementary teachers' thinking about division. They discovered that 12 of 21 participants described division in terms of the partitive model, where sharing or equally distributing among individuals was the focus of dividing. Other preservice elementary teacher opinions about division included the partitive and measurement model, the inverse operation of multiplication, and the written procedure. The authors cited other reasons for preservice teachers' misconceptions about division, such as, "thought only of whole numbers, assumed that with decimals it [division] works in the same way as with natural numbers, found the decimals confusing and misleading, and conclusions from the standard algorithm" (p. 103).

On the number sense topic of order of operations, Glidden (2008) conducted a quantitative study of 381 preservice elementary, early childhood, and special education teachers who enrolled in a mathematics content course. He studied how participants solved five multiple choice problems that involved order of operations. The five problems included the following: "-3^2, 2 x 3 + 5, 3 + 4 x 2, 9 – 4 + 3, and 24/2 x 3" (p. 132). Glidden administered the five problem (no calculator) exam on the first day of class. The author found that less than one-tenth of the participants completed the four order of operations problems successfully. More than 50% of the preservice teachers correctly answered only two or fewer problems. In addition, approximately one-fifth of the participants could not solve the problem involving multiplication and addition correctly.

Within the scope of order of operations, a researcher could investigate preservice teachers' beliefs about division; Ball (1990) is an example of such a researcher. In Ball's study, she investigated 217 preservice elementary teachers' and 35 preservice secondary mathematics teachers' beliefs about mathematics/writing, mathematics teaching/writing, and students' learning. All the preservice participants completed questionnaires, while the researcher interviewed a sample of the preservice teachers. Topics addressed in the research consisted of "rectangles and squares, perimeter and area, place value, subtraction with regrouping, multiplication, division, fractions, zero and infinity, proportion, variables and solving equations, theory and proof, slope and graphing" (p. 451).

Through her work, Ball (1990) discovered that half of preservice elementary teachers in her study liked mathematics and believed they were good at mathematics. More than one third of them thought they were bad at mathematics and avoided it as much as possible. Preservice elementary teachers believed division was a difficult topic to comprehend. No elementary preservice teacher in her study appropriately created a representation for 1 ¾ divided by ½. Most of the preservice teachers did not understand that dividing in half was a different mathematical idea than dividing by ½. Besides struggling with division, most of the participants could only discuss division in terms of round food items like pizzas and pies, the most common representations one might generate with division.

Researchers, such as Dutton (1951), examined number sense in regards to the operation of arithmetic. Dutton asked preservice elementary teachers to write about their attitudes (both good and bad) about arithmetic. Two hundred and eleven preservice teachers responded to the prompt. By grouping responses as favorable or unfavorable (p. 85), the author found that the most common negative responses about arithmetic included the following: "never taught the reason why, disassociated from life and social usage, pages of word problems, boring drill, and poor teaching" (p. 86). The most common positive responses towards arithmetic consisted of "always enjoyed arithmetic/very good in it, enjoyed arithmetic because of good teachers, vital subject in the elementary-school curriculum, and enjoyed advanced mathematics after having some difficulty with arithmetic" (p. 88).

As another example of research that involves arithmetic, Eisenhart et al. (1993) conducted a two-year case study of eight preservice K-8 teachers with Ms. Daniels, a preservice teacher, at the center of their work on preservice teacher beliefs about procedural and conceptual knowledge of mathematics. The researchers collected lesson plans, student handouts, classroom observation data, and three sets of interview data about such topics as beliefs about mathematics, pedagogy, and teaching. Ms. Daniels felt that arithmetic, "basic skills like addition and subtraction, multiplication and division..." (p. 14) is comprised of memorization of rules, where a student does not have to comprehend the procedures. Ms. Daniels' own understanding of decimal multiplication was procedural, and she complained about the confusing conceptual instruction she had received on the topic in college. On the other hand, Ms. Daniels considered "doing mathematics" (p. 14) to involve conceptually understanding mathematics, though she never defined what doing mathematics entailed.

Other topics in Math 100 classes consist of concepts, such as estimation and understanding mathematics vocabulary such as zero. Gliner (1991) investigated the estimation capabilities of 141 preservice elementary teachers and found that generally the participants struggle with the topic. Preservice teachers scored a 90% or above on only 1 out of 25 estimation problems. An additional finding was that preservice teachers performed significantly better on application problems than computational problems. In Wheeler's and Feghali's (1983) work about preservice elementary teachers' beliefs about the number zero, they discovered that many preservice teachers do not understand the concept of zero. The researchers conducted a qualitative study of 52 preservice elementary teachers, where 47 were female and 5 were male. The participants enrolled in a mathematics methods course designed for elementary education majors. Wheeler and Feghali administered an 18-item test on division, as well as conducted interviews with the participants. In the findings, the preservice teachers often did not recognize zero as a number, did not divide with zero correctly, or did not classify using zero. The following statements detail some of the participants'' responses about zero.

1. Zero is the number found between -1 and +1 on the number line.
2. Zero is a number that indicates nothing.
3. Zeros is nothing; no objects.
4. Zero is the dividing point between positive numbers and negative numbers.
5. Zero keeps us from getting confused (p. 151).

Another study regarding a concept about zero included Crespo's and Nicol's (2006) research about division by zero. The researchers' research questions were the following:

1. How do prospective elementary teachers respond to the questions of division by 0 before they have opportunities to discuss their ideas or investigate the topic?
2. How do prospective teachers participating in two different instructional interventions respond to the question of division by 0 after their explorations (p. 86).

The participants consisted of 32 preservice teachers with 18 enrolled in a post baccalaureate teacher program and 14 enrolled in an undergraduate elementary teacher education program. All participants enrolled in a mathematics education course taught by one of the authors and completed an assignment about division of zero. The authors gave the first task to the post baccalaureate preservice teachers, which consisted of a video of a child answering questions about dividing by zero. The post baccalaureate students then answered questions about the students' responses and how they might teach the child to understand the concept. The undergraduate participant group watched and answered similar questions to the first group but also investigated the topic outside of class to arrive at their answers.

Crespo and Nicol (2006) found that only 5 of the 32 participants could initially provide a correct explanation for why division by zero does not work. Fifteen of the 32 preservice teachers answered the question correctly but could not explain their reasoning. Below is a sample response that illustrates incorrect thinking about division by zero.

> When I think about it, I'm not really sure what the answer is. I should know what 5 ÷ 0 is. It seems that it would be 5 because when you divide 5 into nothing, the answer should stay the same…I then think the answer is 0. I'm a little confused now and I'm not sure what I would say to teach this to my students (p. 88).

Algebra.

Algebra, along with probability/data analysis, is one of the main topics discussed in Math 200. In the following paragraphs, I describe word conducted by Adi (1978), van Dooren et al. (2002), and Zazkis and Liljedahl (2002) about preservice elementary teachers and the topic of algebra. Adi (1978) conducted quantitative research with 75 preservice elementary teachers using the following research hypotheses:

1. On reversal equation solving, subjects at the early formal operational stage will perform at least as well as those at the late concrete operational stage who in turn will perform at least as well as those subjects at the early concrete operational stage.

2. On formal equation solving, subjects at the early formal operational stage will perform at least as well as those at the late concrete operational stage who in turn will perform better than those subjects at the early concrete operational stage.

3. Differences in mean performance scores between reversal and formal equation solving are greater for the early concrete group than for the early formal group (p. 206).

Adi (1978) collected data in the form of a written 15 multiple choice test about balancing a beam, a 5 question pre-test over solving equations, and a 12 question post-test over solving equations. Between the pre-test and post-test, the researcher provided the treatment, which included five classes focused on solving equations. The specific objectives for the course consisted of the following two goals:

1. Given an equation with one unknown, and the unknown occurring only once, the subject will find the solution set by applying inversions through the cover-up method.

2. Given an equation of one unknown, the subject will find the solution set by applying compensations to both members of the equation, and the subject will also check whether the numbers defining the solution set satisfy the given equation (p. 208).

Through the results of the 15 question test over the balance beam, Adi (1978) classified 37 participants at the “early concrete operational stage,” 26 participants at the “late concrete operational stage,” and 12 participants at the “early formal operational stage” (p. 208). Adi performed a one-way ANOVA with the three classifications of participants and discovered a significant F-ratio of 7.1 with $p < .01$ (p. 211). Results showed with formal solving of equations that the mean scores of preservice teachers grouped under late concrete operational thought and early formal operational thought were significantly higher than the mean scores of early concrete operational thought participants (p. 211).

Besides equation solving, the researchers, Zazkis and Liljedahl (2002), studied another aspect of algebra, namely patterns. They examined how 36 preservice elementary teachers made sense of a pattern of numbers. The researchers gave the participants two weeks to explore a pattern and write about their conjectures on the following questions:

1. How can you continue this pattern?
2. Suppose you continue it indefinitely. Are there numbers that you know ‘for sure’ where they will be placed? How do you decide?
3. Can you predict where the number 50 will be? 150? And how about 86? 87? 187? 392? 7386? 546?
4. In general, given any whole number, how can one predict where it will appear in this pattern? Explain the strategy that you propose (p. 383).

The researchers selected 4 of the 36 participants to interview further about their thought processes provided in their journal writings.

Zazkis and Liljedahl (2002) found that participants’ attempt to utilize algebraic styles of symbolism seemed unproductive. Participants more easily could discuss the generalizations than write these generalizations in algebraic terms, which made the participants uneasy about their “incomplete” solutions (p. 400).

Van Dooren et al. (2002) further studied preservice elementary teachers’ ideas about algebraic thinking in a comparison mixed methods study with preservice secondary teachers. They compared the thought processes of these two groups with respect to solving problems involving addition and algebra. Ninety-seven preservice teachers participated in this Flandish study with 62 classified as elementary and 35 classified as secondary. During the first part of data collection, participants answered 12 word problems (6 arithmetic- and 6 algebra-based). Below is an example of an algebraic word problem.

> A furniture factory uses large and small trucks to transport 632 beds from England to Germany. A large truck can carry 26 beds. A small truck can carry 20 beds. In the truck convoy that transports the beds, there were 4 more small trucks than large trucks. How many trucks of each type were in the convoy (p. 327)?

Findings related to algebra showed that preservice teachers struggled with solving the algebra problems. Additionally, the secondary preservice teachers performed much better on difficult algebra problems than the preservice elementary teachers.

The second data collection sequence consisted of six word problems from the part one material along with three correct solutions per problem. The preservice teachers ranked the solution strategies based on how well they liked the techniques utilized, as well as provided reasoning for their selections. Through use of a multivariate ANOVA, van Dooren et al. (2002) found that preservice elementary did not support one global technique for solving all problems; they preferred arithmetic techniques for arithmetic problems and algebraic techniques for algebra problems.

Probability/Data Analysis.

The other major concepts taught in Math 200 are probability and data analysis. In the following paragraphs, I detail the research of Canada (2006; 2008), Canada and Makar (2006), and Dollard (2006) with preservice elementary teachers and probability/data analysis. Dollard qualitatively investigated 24 preservice elementary teachers' ideas about probability. In particular, he examined the following research questions.

Q1A How do preservice elementary teachers think about situations involving fundamental concepts of probability?

More specifically,

Q1B How do preservice elementary teachers think about situations involving simple probability, the law of large numbers, compound events, and conditional probability? (p. 8).

Each of the participants in Dollard's (2006) research were enrolled in a number sense course designed for preservice elementary teachers. None of the preservice teachers had completed the probability course designed for preservice elementary teachers. Dollard interviewed each participant for about an hour, where preservice teachers answered probability questions about "traditional paper-based mathematics problems involving probability," as well as game based activities, such as "cards, dice, spinners, or bags of colored beads" (p. 9). During the game scenarios, participants made predictions and then carried out the scenarios.

Dollard (2006) found that one third of the preservice elementary teachers could not correctly define probability or do simple probability problems. I listed two incorrect responses about the definition of probability in the following quote.

> Amber said that, "I think it means the different outcomes that can happen .. from an event or a specific happening. All the different answers you can get." The remaining participant, Jessica, said that probability was "whether or not it's going to happen." (p. 148)

In addition, Dollard (2006) commented "more than three fourths of the participants did not have an adequate understanding of theoretical probability, experimental probabililty, and/or the law of large numbers" (p. iii). Jessica again provided an example of an incorrect answer. The dialogue shown below consists of a discussion between Jessica and Dollard about the probability of rolling a two on a six-sided die.

> Jessica: "Six? cause there's 6 sides. So, .. or maybe, five? Six or five? Cause, I .. so …
>
> Interviewer: Think about it, just think out loud, whatever you think about it.
>
> Jessica: Then I would say that the probability of rolling a two would be . mm you have … six shots;.. five shots at it, or six shots at it. Six cause there's six sides, but, if you're counting the two, .."
>
> Interviewer: Yeah, there's six sides.
>
> Jessica: So, yeah" (p. 151).

An additional researcher who studied the beliefs about probability with preserivce elementary teachers was Canada. Canada (2006) qualitatively examined 29 preservice elementary teachers' probability answers to survey instruments. Canada gave the preservice teachers enrolled in a course designed for preservice teachers a pre-survey about flipping 6 groups of fair coins 50 times, an instructional probability intervention, and a post-survey about spinning 6 groups of partial black and partial white spinners 50 times (p. 2). After each data collection, the researcher interviewed 10 preservice teachers about their logic in responding to the surveys.

The instructional interventions consisted of class activities over data collection, graphing, and probability scenarios. A sample activity, "the Known Mixture Activity" is listed below.

> The band at Johnson Middle School has 100 members, 70 females and 30 males. To plan this year's field trip, the band wants to put together a committee of 10 band members. To be fair, they decide to choose the committee members by putting the names of all the bad members in a hat and then they randomly draw out 10 names (Canada, 2006, p. 2).

Canada (2006) coded the responses in one of four ways, ranging from incorrect answer and explanation (Level 0) to correct answer and explanation (Level 3). The results showed that more participants gave correct responses and justification in the post-survey than in the pre-survey (11 versus 2, respectively). Over half of the participants justified their responses by activities they completed in class.

In a later study, Canada and Makar (2006) compared their qualitative dissertation findings about preservice elementary (Canada) and preservice secondary teachers' (Makar) ideas about variation, respectively. Both researchers utilized pre-tests, post-tests, and interviews. Canada's study comprised of 29 preservice elementary teachers; Makar's work consisted of 17 preservice secondary participants. In Canada's work, participants analyzed boxplots and histograms to discuss variability, while the participants in Makar's research analyzed dotplots. Findings suggested that preservice teacher groups possessed an "intuitive" feel for variation and utilized similar terminology in describing given data sets, such as "clustered" and "scattered" (p.7).

In 2008, Canada again conducted qualitative research with preservice elementary teachers. He investigated the thought processes of 58 preservice elementary teachers and 50 middle school students on the concept of data distribution. The researcher gave the participants a scenario about two trains that travel between two specific cities. "For 15 days (and at different times of the day), data are gathered for time the trip takes on each of the trains" (p. 2). Canada wanted to see whether or not the participants believed the hypothesis that there was "no real difference between the two trains because the data have the same means." (p. 2).

Canada (2008) collected data in the form of the participants written answers and classroom discussion data. The researcher found that 20 of the 58 preservice teachers initially believed the hypothesis, while 16 of 50 middle school students also agreed. Some responses included justifications, such as "Because the averages are the same," and

"Each train had the same average time" (p. 3). The second main finding centered on the fact that the most number of participants who disagreed with the hypothesis utilized reasoning about distributions.

Geometry.

Besides Math 100 and 200 concepts involving number sense, algebra, and probability, I incorporated some research on preservice teachers' beliefs about geometry, which is the focus of Math 300. Mayberry (1981) investigated the geometry knowledge of 19 preservice elementary teachers through two interviews. During the interviews, the researcher gave each participant questions of varying difficulty based on the van Hiele levels. Mayberry found that 70% of the preservice teachers' response patterns who completed high school geometry scored below the third van Hiele level. At level III learning, individuals should be able to construct proofs with logical reasons for each step of their proof. Many preservice teachers struggled with properties of geometric shapes. For example, 12 of the 19 participants did not believe that a right triangle had a largest side, and 7 out of the 19 preservice teachers did not believe that a right triangle had a largest angle.

Battista et al. (1982) also conducted research on geometry. At the beginning and end of a geometry course for preservice teachers, the authors administered the Purdue Spatial Visualization Test: Rotations to preservice elementary teachers. The test consisted of 30 questions about visually rotating figures. In addition, the preservice teachers took a modified version of the Longeot test of cognitive development with questions that address proportional and combinatorial reasoning. Battista et al. found that the 82 preservice elementary participants increased their scores on spatial visualization tasks after completing the geometry course, where spatial visualization was a component of several of the class activities.

With the concept of symmetry, Charles (1980) conducted a four-day study with 18 preservice elementary teachers enrolled in a mathematics/methods course and 72 second graders. The research focused on “the issues of whether teachers can be trained to use “E or C moves” in a teaching situation and whether the use of these moves by teachers facilitates concept acquisition” (p. 11). “E moves” refer to exemplification moves, where teachers may present examples or counterexamples about a concept. “C moves” are characterization moves, where teachers may state related or unrelated aspects of a concept. The researcher randomly chose preservice teachers to be in a control or experimental group. In addition, Charles randomly selected four second graders to work with each preservice teacher. Each preservice teacher participated in training sessions on the previous Monday and Friday before the study. All teachers learned about the concepts they would be teaching, bilateral and rotational symmetry, as well as the experimental group learned how to teach students to use the E and C moves.

Students in Charles’s (1980) work took pretests and posttests over the symmetry material on the first and fourth day of the study. During the second and third days, the preservice teachers each taught rotational and bilateral symmetry to the second graders the researcher had assigned to them. Charles found that preservice teachers used significantly more exemplification moves classified as examples than counterexamples. In addition, the author discovered that “for unidimensional concepts teachers naturally use C moves related to the relevant attribute” (p. 18).

One of the most recognizable concepts with geometry is the idea of proofs, which Martin and Harel (1989) studied at the preservice elementary level. Martin and Harel quantitatively examined 101 preservice elementary teachers' conceptions about proofs. The authors' research questions consist of the following:

1. Do preservice elementary teachers accept inductive arguments as proof of mathematical statements? Are their evaluations of inductive arguments dependent on their familiarity with the statement?

2. Are preservice elementary teachers more convinced by some types of inductive arguments than others?

3. Do preservice elementary school teachers accept that a deductive argument constitutes a mathematical proof? Are their evaluations of deductive arguments dependent on their familiarity with the statement?

4. Are students' judgments of an argument influenced by its appearance in the form of a mathematical proof—the ritualistic aspects of proof—rather than the correctness of the argument?

5. How do students view deductive arguments presented in the particular case, that is, mathematical proofs in which the parameters are changed to specific numbers?

6. Is the acceptance of inductive arguments and deductive arguments as mathematical proofs mutually exclusive" (p. 42).

All participants enrolled in a sophomore-level mathematics course. Preservice teachers examined proofs to rate whether they considered the proofs valid or not. Martin and Harel (1989) found that several preservice teachers rated inductive and deductive arguments as correct proofs, no matter the context. In addition, the authors stated, "students who correctly accepted a general-proof verification also showed high levels of acceptance of a particular proof (using specific numbers)" (p. 49).

Others investigated the types of geometric knowledge preservice teachers reflect on in written assignments. Soto-Johnson et al. (2009) conducted a mixed methods study of 55 preservice teachers who enrolled in a mathematics geometry content course for elementary education majors. Their research questions included the following:

1. Do written reflections in a geometry course designed for prospective elementary teachers affect their performance on content related to the reflections? i.e. the null hypotheses are:

   a. Preservice elementary teachers who reflect on activities will perform as well on related warm-up exercises as participants who do not reflect on activities.

   b. Preservice elementary teachers who write strong reflections will perform as well on related warm-up exercises as participants who do not write strong reflections.

   c. Preservice elementary teachers who reflect on activities later in the semester will write comparable reflections to those written by participants who reflect earlier in the semester.

2. What experiences, intentions, and perceptions do preservice elementary teachers' share through written reflections to guided questions pertaining to geometry lessons (p. 1)?

Data collection consisted of "a pre-test, 7 written reflections, 14 warm-up exercises, 4 quizzes, 2 tests, and a final comprehensive exam" (p. 3). Results that are pertinent to my study include qualitative findings about how preservice elementary teachers reflect about geometry. Most (54 out of 55) preservice teachers sometime reflected about their discovery based mathematical learning in a procedural fashion, using algorithms or facts. Forty percent of the participants expressed their ability to see connections between the class material and the real world.

Inservice Elementary Teachers'
Beliefs about Mathematics

From my literature search, I found two articles about inservice elementary teachers, Thompson (1984) and Raymond (1997), which play key roles in my dissertation work. These two articles are case studies, similar to mine, that investigate teachers' perceptions about mathematics. I also detail work about inservice elementary teachers from Ambrose (2004), Collopy (2003), Sztajn (2003), and Ma (1999). Ambrose, Collopy, and Szatjn conducted case studies of inservice elementary teachers in the United States with varying opinions about mathematics that ranged from procedural to conceptual. Ma conducted research with inservice teachers from the United States and China and felt that United States' elementary teachers often lack the mathematics knowledge they need in order to understand mathematics conceptually. Chinese teachers, on the other hand, possess a deep understanding of mathematics that includes conceptual knowledge with multiple teaching strategies.

Thompson (1984) conducted a four-week comparative case study of three seventh and eighth grade teachers named Kay, Lynn, and Jeanne and documented discrepancies between their thoughts and beliefs about mathematics and mathematics teaching. Participants taught at least three years at their current grade level and volunteered for the study. Thompson observed the three participants' teaching each day for two weeks. The second two weeks of her study consisted of the same daily observations but with subsequent teacher interviews. As a form of triangulation of her research data, Thompson asked the teachers to answer six written prompts about their beliefs about mathematics and mathematics teaching throughout her work.

From her research, Thompson (1984) found that Kay believed mathematics is a difficult, thought-provoking discipline, where one can reason logically to arrive at answers and use mathematics as a science tool. Mathematics to Kay is ever expanding with new discoveries affecting the scope of the discipline. Lynn felt mathematics was full of procedures and methods that individuals can use to arrive at right answers.

Mathematics to Lynn cannot change and is free from interpretations or creativity. Similar to Lynn, Jeanne believed mathematics is fixed and certain with no inconsistencies. She also talked of mathematics as interrelated, logical, mysterious, and impossible to fully understand.

In a comparable study to Thompson's (1984) work, Raymond (1997) conducted a 10-month case study of six first and second year elementary teachers and their beliefs about mathematics, mathematics learning, and mathematics teaching. The author gathered teacher belief information for each participant through seven interviews, five classroom observations, a survey questionnaire about mathematics beliefs with respect to teaching, and documents. The documents consisted of items, such as lesson plans and a concept map about the participants' connections between mathematics beliefs and teaching. Through a review of literature, Raymond developed a visual mapping of how inservice teacher beliefs about mathematics and mathematics teaching influence one another, which provided a theoretical framework for her research. In her study, Raymond took an in-depth look at one of the six teachers, a fourth grade teacher named Joanna, and her beliefs about mathematics and how they related to her mathematics teaching.

Joanna (Raymond, 1997), a second year teacher, decided to teach mathematics because she felt there was not much preparation needed on her part to teach the subject. As a child and college student, Joanna loathed mathematics so she felt she had to put on a fake persona to cover her dislike for the subject for her own students. In her analysis, Raymond found that Joanna believed mathematics was a fixed discipline that was predictable and full of rules, rote memorization, and problem solving, which matched well with a nonstandards aligned view of mathematics. Joanna attributed her beliefs about mathematics to her experiences as a student.

Not all inservice teachers share Joanna's nonstandards aligned views. Ambrose (2004) conducted a longitudinal study of four preservice Danish teachers who obtained their first teaching positions. Christopher, one of those four participants, felt that students who are good at mathematics should be able to "systematize, plan, delimit open problems, reflect on their own learning, have the ability to cooperate, independently find solution strategies and models, and relate critically [to information and problems]" (p. 9).

Collopy (2003) detailed another example of teachers with standards and nonstandards aligned beliefs of mathematics using a case study of two inservice teachers, Ms. Clark and Ms. Ross. Ms. Clark was a fifth-grade teacher with 26 years of experience, while Ms. Ross was a fourth-grade teacher with 11 years of experience. Collopy collected data in the form of 28 interviews and 41 classroom observations about Ms. Clark's and Ms. Ross's experiences with a new reform mathematics curriculum. From observations, the researcher found that Ms. Clark stressed to her students her mathematical beliefs of speed and accuracy of rules, algorithms, and computations. Ms. Clark shared her opinions about mathematics during an interview.

> Math is like a game. If you listen carefully, listen to the instructions, you'll learn how to play the game, and it is a game. It's learning the patterns to it. There are certain methods, techniques. Once you learn those, you know how to do it (p. 295).

On the other hand, Ms. Ross (Collopy, 2003) felt mathematics did not have an obvious structure. The new reform curriculum helped Ms. Ross change from teaching in a nonstandards aligned fashion that emphasized procedures and correct answers to a standard based routine that utilized conceptual understanding of mathematics and reasoning skills. As with the traditional text in previous years, Ms. Ross followed the reform curriculum carefully. The reform program provided several activities involving manipulatives, one of the key reasons Ms. Ross agreed to adopt the material. Thus, Ms. Ross easily adapted her lessons to fit in with a reform minded classroom.

Sztajn (2003) also conducted a case study of two inservice teachers, Teresa and Julie, who held opposing views about mathematics. Teresa, a third grade teacher, defined mathematics as a set of rules that students must remember and practice. "Problem solving, critical thinking, and other higher-order thinking skills" (p. 62) also characterize mathematics, but she focused her teaching on the former list. The other inservice teacher in Sztajn's work was Julie, a fourth grade teacher. Julie emphasized mathematics as problem driven and a set of memorized rules.

As an international example of number sense research, Ma (1999) found inservice teachers of different nationalities varied in their mathematics knowledge. Ma discovered that Chinese teachers held conceptual understandings of subtraction, multiplication, division, and geometry ideas unlike the United States teachers. Most United States teachers felt that learning procedural tasks, such as "borrowing" for subtraction, "lining up digits" for multiplication, and "invert and multiply" (p. 108) for division was sufficient knowledge for these topics. On the other hand, Chinese teachers explained the rationale for each mathematical topic and often implemented multiple novel strategies to solve problems.

Preservice Elementary Teachers' Beliefs about Teaching Mathematics

Since my research question also consists of the nature of nontraditional and traditional preservice elementary teachers' beliefs about mathematics teaching, I incorporate a literature review section on research related to preservice elementary teachers' views about teaching mathematics. The three main areas consist of standards/nonstandards aligned mathematics beliefs, the role of the teacher, and senses. Ball (1988), Beswick (2006), Borko et al. (1992), Crespo (2003), Eastman and Barnett (1979), Eisenhart et al. (1993), Fuson (1975), Harding-DeKam (2005), and Mewborn (1999) conducted research about preservice teachers' standards and nonstandard based beliefs. Ambrose (2004), Ball (1988; 1990), Cooney (1992), and Vacc and Bright (1999) investigated preservice teachers' beliefs about the role of the teacher in the classroom. Gellert (1998; 2000) discovered findings about preservice teachers' attitudes towards entertaining mathematics and their necessity in the classroom.

Standards and Nonstandards Aligned Mathematics Beliefs.

Teachers often possess certain beliefs about mathematics that are standards and nonstandards aligned, which can change as time progresses. Beswick (2006) conducted a quantitative study of 94 preservice elementary teachers enrolled in their first and second mathematics components of their education program. The following are Beswick's two research questions:

1. What is the net impact of the first and second mathematic education units on the attitudes and beliefs of preservice teachers?

2. Which aspects of the units are most effective in positively influencing preservice teachers' beliefs" (p. 38)?

Data collection consisted of participants' answers to pre-tests and post-test results to a nine-item belief questionnaire about mathematics myths. Two myths consisted of, "Some people have a maths mind and some don't. Maths requires a good memory" (p. 40).

Beswick also collected data through a 21-item survey about perceptions about mathematics and mathematics teaching. Below are three sample items.

1. I am interested and willing to use mathematics in everyday life.
2. A teacher's energy and enthusiasm for mathematics can positively influence students' attitudes to mathematics.
3. An important aspect of mathematics teaching is engaging children in interesting mathematical investigations (p 43).

During the second course, the participants took the two mentioned tasks again, as well as a survey about the usefulness of different topics addressed in their mathematics course. The author administered all three as pre-tests and post-tests. Beswick analyzed the data through paired samples *t*-tests.

Over time, the results (Beswick, 2006) showed that preservice teachers increased their agreement to the survey item "telling students the answer is an effective way of facilitating their mathematics learning" (p. 41). This may seem to conflict with another result that found the participants decreased agreement with the survey item "mathematics is such a precise subject that there can only be right and wrong answers" (p. 41). Other results included the following:

> the need for sequential planning of mathematics teaching focused on establishing connections between mathematical topics, the value of using strategies other than teacher demonstration followed by practice, the role of concrete materials in the development of students' conceptual understanding,…the effectiveness of group work in learning mathematics (p. 42).

Harding-DeKam (2005) conducted a mixed methods study of 289 undergraduate preservice elementary teachers that addressed an aspect of my research on preservice elementary teachers' beliefs about teaching mathematics. Her main goal was to construct and validate the Prospective Elementary Teacher's Mathematics and Attitudes and

Beliefs Survey instrument, but she also discovered findings related to my work. Specifically, her second research question, "What is the impact of the prospective teachers' attitudes and beliefs measure by the Prospective Elementary Teachers' Mathematics and Attitudes and Beliefs Survey instrument" (p. 215), resulted in findings about nonstandards and standards aligned mathematics beliefs that I detail below. Harding-DeKam constructed the Prospective Elementary Teachers' Mathematics and Attitudes and Beliefs Survey instrument, which consisted of the following four subscales:

1. The prospective teachers' personal confidence about mathematics
2. Usefulness of mathematics content
3. Perception of former teachers' attitudes and beliefs about mathematics ability
4. The prospective teachers' attitudes and beliefs on teaching mathematics to elementary students (p. 3)

Participants took the survey three times, at the beginning of their mathematics education class, after five weeks in the mathematics education course, and during their first year of employment as a teacher. Through an ANOVA, Harding-DeKam (2005) found the following results:

> Teachers believe they can teach mathematics concepts to elementary students, no longer view mathematics teaching as challenging to them, believe they can teach low achieving students in mathematics, and believe mathematics should be taught through hands-on manipulatives (p. 217).

In Mewborn's (1999) study, four preservice elementary teachers observed a 4th grade classroom for a semester. The researcher interviewed each of the participants, as well as collected group interview information, journals, and observational teaching data. After a semester in the elementary classroom, the preservice teachers began to create ideas about teaching beyond their personal knowledge. Mewborn included dialogue from

Ashleigh and Hanna, two preservice teachers, who had differing views about teaching mathematics. Ashleigh believed there is a point where you tell students algorithms to solve problems. Hanna felt that students understand mathematics better when you have them struggle to learn the concepts and do not tell the students the answers.

In Borko et al. (1992), the authors analyzed a preservice teacher's lesson about fractions and discussed her beliefs about teaching. The initial study consisted of eight preservice teachers, but the researchers' concentrated on one preservice teacher named Ms. Daniels. Ms. Daniels, a senior elementary education major, utilized her background experiences as a mathematics student, methods student, and student teacher to shape her views about mathematics teaching. During a lesson about division of fractions, she could not decide how to show her students a concrete example of division so she finally had her students use the invert and multiply algorithm. After the lesson, Ms. Daniels explained to the researcher that she was trying a concept from her methods class, but she drew a multiplication problem instead of a division problem. Borko et al. expressed concern about Ms. Daniels' mathematics background knowledge and her lack of desire to understand why the division algorithm for fractions works.

As time passed and she progressed in her student teaching, Ms. Daniels (Borko et al., 1992) incorporated her own teaching experiences into her teaching philosophy about mathematics. Ms. Daniels believed good mathematics teaching included real-world application problems, fun lessons, and straightforward explanations of the logic behind procedures. She also expressed how mathematics needs to be visual for students so they can "see or touch" the mathematics (p. 206).

In a later study conducted by Eisenhart et al. (1993), Ms. Daniels expressed her views about teaching procedurally and conceptually, “I consider myself pretty excellent in arithmetic, because I know how to manipulate the numbers and I use the processes a lot. I’ve had a lot of practice” (p. 17). Ms. Daniels felt that teachers need to assist students by teaching them procedures in detail to perform arithmetic operations and then allowing them to practice until “they [the processes] were engraved in their brains” (p. 15). With conceptual learning, Ms. Daniels did not articulate how teachers might assist students in discovering mathematics. She felt students could learn conceptually but was unclear on how to go about helping students who might struggle with non-routine mathematics.

Teachers could help students gain this conceptual understanding of material by presenting mathematics in various ways. Ball (1988) discovered that preservice teachers believed that they as prospective elementary teachers should know multiple ways to solve problems because “different people understand different examples” (p. 16). Preservice teachers also felt that it was one thing to comprehend mathematics topics for yourself and a completely different thing to teach someone mathematics. Ball found that preservice teachers who were successful in mathematics tended to be less likely to care about alternative strategies in teaching mathematics than those prospective teachers who struggled. The preservice teachers who did not have good experiences with mathematics intended to teach differently yet did not have any optional lessons or ideas.

Even though preservice teachers might struggle with teaching alternate strategies, they can learn and mature in their teaching techniques. Crespo (2003) found preservice teachers’ values about teaching change over time. She investigated the types of mathematics word problems that preservice elementary teachers felt were important to give to students. As her research evolved, the preservice teachers moved from simple, procedural problems to open-ended problems with multiple solution paths. Even though preservice teachers differed in their mathematics capabilities, all of the preservice elementary teachers posed easy, computational problems at the beginning of the study.

With the concept of manipulatives, Fuson (1975) conducted research with 16 preservice elementary Master's level teachers enrolled in a mathematics/mathematics methods course. She examined the effects of manipulative use on preservice elementary teachers. Fuson analyzed responses to such data collection techniques as survey items, interviews, teaching experiences, and written reflections about mathematics concepts. She found that preservice elementary teachers in mathematics/mathematics methods courses wanted to use manipulatives in their future teaching of mathematics. The participants also explained mathematics topics via manipulatives. One participant stated, "I think I truly understood borrowing and carrying as exchanges for the first time while using the Dienes blocks" (p. 62).

Even though the actual use of manipulatives increased interest in the use of manipulatives in the classroom (Fuson, 1975), Eastman and Barnett (1979) examined how preservice elementary teachers could learn vicariously through demonstrations with manipulatives instead of physically moving them. The preservice teachers who only watched others utilizing manipulatives did as well as participants who actually used manipulatives and completed their work in a shorter period.

Senses.

Teachers oftentimes employ multiple teaching methods that utilize students' different senses, such as seeing, touching, and hearing, to make mathematics entertaining for their students. These beliefs can influence how they instruct a classroom. Gellert (1998) found preservice teachers, like Ms. Daniels (Borko et al., 1992; Eisenhart et al., 1993), feel fun mathematics classes are important for students. Gellert conducted a qualitative study of 42 prospective elementary teachers in Berlin, Germany. He investigated the participants' views about mathematics, pedagogy, student needs, and mathematics teaching. All the preservice teachers in the study were enrolled in Gellert's seminar entitled, "Why teach mathematics?-Conceptions for mathematics education in primary school" (p. 29).

As a part of the course, Gellert (1998) had the prospective elementary teachers journal about nine topics that included such ideas as "their [preservice elementary teachers'] beliefs about mathematics and mathematics education," "their conceptions for future teaching," and "the participants' mathematical biographies" (p. 30). Gellert reviewed the participants' writings several times to generate two major themes, "having fun in mathematics class," and "conceptions of a child-centered learning atmosphere" (p.33). Ariane, one of the prospective teachers, explained her concern for teaching mathematics in the following way:

> From personal experience as a pupil, I already know how I do not want to teach mathematics. What I am lacking is only the idea of how to teach mathematics to students in primary schools in a nice and amusing way (p. 33).

In a later article about the same study but with additional findings, Gellert (2000) also found preservice elementary teachers seek teaching materials and problems that are “funny and motivating rather than mathematically substantial” (p. 266). Others expressed the need to make mathematics “invisible” (p. 259) to the students. Thorsten, one of the 42 preservice elementary teachers commented, “In mathematics classes, mathematics should be wrapped up in a way that students do not become aware of the fact that mathematics is taught” (p. 259). Stephanie demonstrated the importance of mathematics. She wrote about how unnecessary certain topics in mathematics like graphs and calculus (p. 260) are so teachers should skip those areas of mathematics. Additional themes Gellert included in his work consisted of the idea that mathematics should be applicable for students, mathematics is important in daily activities, and mathematical knowledge is essential for survival in the world. Wiegel and Bell (1996) also found that preservice teachers look for fun in mathematics. Tina, one of their participants, stated that computer activities involving mathematics can be fun. She commented, “We are having fun playing. I don’t believe I said that in math class! (p. 1).”

Role of the Teacher.

Preservice elementary teachers view mathematics in different ways. Fred, from Cooney’s (1992) work, believed mathematics teaching involved “30% concepts, 20% to 30% problem solving, and whatever remains among the other things—discovering generalizations, developing skills, and applications.” (p. 328). Some preservice teachers, as in Vacc’s and Bright’s study (1999), changed their beliefs about mathematics teaching throughout their coursework and student teaching. For example, Helen believed the role of a teacher was to assist students in learning mathematics from a supportive role. As her experience in the program grew, she began to think that a teacher should show students how to solve problems. During her student teaching, Helen’s beliefs about teaching changed once more; she felt that worksheets might not be the best way to help students learn mathematics. Ultimately, Helen believed the teacher was the motivator and facilitator of mathematics learning, where teachers listen to students' problem solving strategies and modify their lessons based on student feedback. Like Helen, Andrea, another preservice teacher, felt that children develop problem solving strategies that are useful, and teachers should listen to student strategies to gain valuable information for planning lessons. The most important concept about teaching in Andrea's opinion is the role of questioning to understand students' thought processes.

Other ideas about preservice teachers' beliefs about teaching involve the idea of questioning students and answering student questions. In Ball's (1988) study, Maureen, a preservice teacher, felt that how and when she asked questions made a significant difference in student responses. Others worried about answering student questions, which might involve responding to questions about why certain procedures work. Thus, many hoped to teach lower grades since they felt their lack of mathematics knowledge would not affect their teaching in lower elementary. Cathy, a preservice teacher in Ball's (1990) study, also expressed concern about teaching mathematics, such as long division. She could complete the mathematics but was unsure whether or not she could teach it because she felt unsure about her conceptual understanding of the topic and how she would approach teaching the subject.

Even though preserve teachers may feel they have a plan for teaching concepts to students, their plan may not fit in with the students' mathematical abilities. Ambrose (2004) found similar answers with her research of 15 prospective elementary teachers who took a common mathematics course and mathematics methods course. Ambrose collected data through interviews, participants' printed work, surveys, and field notes. Kathy, one of Ambrose's participants, shared her sentiments about teaching mathematics to children.

> I went into class that day thinking, "I'm so excited. I'm going to teach him this. By the end of the hour, he's going to know it and he'll be able to do it forever." And it didn't happen that way, so I guess to just keep that in mind and to know that it's not going to only take an hour for a child to understand a concept (p. 108).

Nina, another preservice teacher in Ambrose's study, expressed what she learned from working with elementary children in the mathematics methods course. Nina commented, "Teaching is not me giving the information, and then them absorbing it, but rather giving them the tools that they need to learn on their own. I think that's probably the most important thing that I learned" (p. 109).

Other preservice teachers in Ambrose's (2004) work felt that the mathematics methods course taught them that preservice elementary teachers at any grade level need sound mathematical understanding to be successful. Cindy, one of her participants, shared these feelings.

> I want to teach young children, so I didn't think I needed to know a whole lot of actual mathematical skills and I really disagree with that now. In order to come up with a creative way to teach it, you need to understand what you're talking about and you need to have the math skills to do that (p.114).

At the end of the course, Ambrose (2004) found that all the preservice teachers believed that teachers should know multiple strategies to teach children mathematics. Some even related how teaching through multiple strategies helped students gain a conceptual understanding of mathematics.

Inservice Elementary Teachers' Beliefs about Teaching Mathematics

Even though my research consists of preservice elementary teachers, inservice elementary teachers' beliefs about teaching mathematics that consist of standards and nonstandards aligned beliefs provide useful ideas that can be adapted for interview questions. Clarke (1997), Collopy (2003), Raymond (1997), Skott (2001), Sztajn (2003), and Thompson (1984) conducted case studies of inservice elementary teachers and their beliefs about teaching mathematics. The participants in these studies consisted of one to three individuals with teaching styles from algorithmic to discovery based. Lampert (1990) detailed her own teaching experiment that involved a social constructivist teaching atmosphere. Puchner, Taylor, O'Donnel, and Fick (2008) and Moyer (2001) investigated how inservice elementary teachers utilized manipulatives in the classroom.

A teacher's philosophy about instruction influences various facets of their classroom etiquette. Thompson's (1984) comparative case study of three seventh and eighth grade teachers is one such article that details teacher beliefs about mathematics teaching. The author discovered that Kay, Lynn, and Jeanne, the three participants, had three different views of mathematics teaching. Kay believed that teachers should create an inviting and appealing classroom atmosphere, where students feel free to question, conjecture, and hypothesize. From Kay's view, teachers should be supportive of student interaction in the classroom, where students can openly ask questions and discuss their opinions about mathematics topics. When students spoke incorrectly, Kay asked probing questions to help the students understand their errors. On the other hand, Lynn felt that mathematics instruction consisted of students observing their teacher perform mathematics procedures with students methodically working problems involving the rules. Lynn's ultimate goal in teaching consisted of her students solving mathematics problems using procedures. In her view of mathematics teaching, Jeanne stressed an orderly classroom, where the teacher is in control of classroom discourse and presents material in a precise method. Jeanne had an inflexible lesson plan, which she implemented on a daily basis. Thus, she believed students should listen to her explanations and questions instead of creating their own beliefs. Another aspect of mathematics teaching Jeanne emphasized was the idea that students need to understand the logic behind the mathematics procedures they use in class, which is different from Lynn's rote memorization she proscribed in her class.

Certain teachers, such as Lampert (1990), took a standards aligned approach to teaching that mirrored Kay's approach to instruction in Thompson's (1984) work. Lampert taught exponents in a guided discovery orientated atmosphere, where fifth grade students freely asked questions, conjectured, and defended their solutions to the class. Lampert investigated the art of teaching mathematics through a teaching experiment using a standards aligned approach. Lampert never told her students the answers; rather, she and her class discovered strategies and helped each other succeed. From her work, Lampert found that fifth grade students are able to reason with mathematics and justify their conclusions without having the teacher explain each step. Students freely learned exponents in a nonstandards aligned format, where the author eliminated the need for procedures and memorization.

Similar to Lampert's (1990) ideas about teaching, Collopy (2003) found a teacher named Ms. Ross who valued standards aligned beliefs of teaching. Collopy conducted a case study of two inservice teachers, Ms. Ross and Ms. Clark, who valued standards aligned and nonstandards aligned beliefs of mathematics, respectively. Ms. Ross, like Lampert, focused her class discussions on students' solutions and reasoning skills. Ms. Ross' students worked to "collaborate, diagram, write, and discuss problem-solving strategies, observations, and solutions" (p. 300). During an interview at the end of the school year, Ms. Ross expressed the value of multiple strategies for student learning.

> I think that what we were really focused on this year was helping them understand that there are many different ways to solve a problem…I mean there were strategies for problem solving that we taught in the other math curriculum, but I don't think we were open to having them explore and come up with different ways of solving problems" (p. 304).

Ms. Ross also expressed the need for mathematics to be fun for students.

> I think this is silly, but I want them [the students] to say math is fun. Because if I have a philosophy at all it's to help them relax with math because I think when any of us, adults or children, are uptight about a subject it's very hard to penetrate and understand. But, if you are relaxed, then you're more open to learning (p. 303).

On the other hand, Ms. Clark felt she should "walk the students through" (Collopy, 2003, p. 296) the mathematics algorithms and rules. Ms. Clark sometimes drew pictures or provided manipulative demonstrations to help students understand the concepts. Oftentimes, she just hurriedly repeated the procedure until the student found the error in their thinking. Ms. Clark then gave students time to work individually on homework. When questioned about the use of multiple strategies and conceptual explanations, Ms. Clark explained that these additions to a lesson confused students and thus excluded them from her lessons.

Sztajn (2003) also conducted research with two inservice teachers, Teresa and Julie, who held varying views about mathematics teaching. Teresa concentrated mathematics lessons on drill and memorization of facts and procedures. She commented on the value of an orderly classroom over problem solving.

> As a teacher, the things that I get very frustrated about when we try to work on the higher thinking skills [is that] (…) sometimes you have such a large range of levels that the kids are dealing with, that when you go to do an activity…It's like a third of them are with you, a third of them maybe has an idea of what you are doing, and another third has no idea of what you are doing. And you are lost" (p. 62).

Julie believed that the NCTM (2000) standards helped her focus her teaching on a problem driven classroom, which she commented about in an interview.

> The *Standards* is really helping me re-focus my teaching. I think that I always knew that I wanted to teach in a problem-solving, creative-type way. But in math I've always been more tied to the textbook than in any other subject. Just because of that belief in getting the basic facts down, which I still think is important. But I think that I really like the de-emphasis on, oh, doing thirty-five long-division problems and things like that" (p. 65).

In addition, Julie expressed her need for making mathematics fun for her students. She believed that activities and manipulatives help students to see the enjoyment in mathematics. Projects are another way that Julie challenges students to think and generate discussion in her classroom.

With Raymond's (1997) 10-month case study of six first and second year elementary teachers and their beliefs about mathematics teaching, the author discovered findings about inservice teachers' beliefs structures that characterize standards and nonstandards aligned ideologies. In her study, Raymond took an in-depth look at one of the six teachers, a fourth grade teacher named Joanna, and her beliefs about mathematics and how they related to her mathematics teaching. For Joanna, her beliefs about teaching mathematics incorporated a hands-on approach using manipulatives and several different types of activities with varying solution strategies. Joanna, similar to Lynn in

Thompson’s (1984) work, felt she had to be the one to explain mathematics to her students. She attributed her beliefs about teaching mathematics to her teaching experiences.

Puchner et al. (2008) qualitatively examined 23 K-8 inservice teachers’ use of manipulatives in lesson development and implementation that originated from a summer mathematics institute. Data collection consisted of each participant’s written report about the institute and detailed lesson on mathematics. The mathematics lesson included such items as a lesson plan, teacher notes, and student work. The researchers developed categories and selected excerpts about manipulatives. Sixteen of the 23 participants discussed manipulatives in their reports. A common statement included comments like, “Manipulatives can bring a whole new understanding to a mathematics concept. Simply using paper representations of brownie pans can give the students a better understanding of fractions and dividing portions” (p. 316).

Puchner et al. (2008) also found teacher reflections that described how some of the inservice teachers utilized manipulatives in their classrooms, but the students did not use them as learning tools. The students calculated their answers using procedures and then tried to make the manipulative answer match their solution. Other teachers discovered that the manipulatives became more of a hindrance to the students’ learning. Students became confused about their use in activities and often wasted class time trying to figure out the manipulatives’ usefulness.

Moyer (2001) conducted a qualitative study of 10 middle grade teachers enrolled in a mathematics institute for middle school teachers. The researcher examined to what extent inservice teachers utilized manipulatives in the classroom. Data collection consisted of “teachers’ interviews, teachers’ and students’ audio-taped verbalizations during classroom observations, fieldnotes of teachers’ and students verbal and nonverbal behaviors during classroom observations and the self-report postcard responses of teachers’ lessons” (p. 180).

Moyer (2001) found teachers utilized manipulatives in 53 of 67 class lessons with hundreds boards, color tiles, and snap cubes as the three most often used manipulatives. Teachers employed manipulative use for lessons on various topics, such as solid geometry, area, percents, prime numbers, place value, and equivalent fractions. When questioned about the use of manipulatives in class, the teachers often commented that manipulatives made mathematics fun for students. Denise, one of the participants, shared how she had to teach “real math” everyday and could not teach “fun math” (p. 187) with manipulatives on a daily basis. She stated, “I can’t do manipulatives every day… The kids sometimes need that kind of structure where I’m in the front of the class and where they’re sitting there working” (p. 188). Joan, another participant, commented how she structured manipulative use in her classroom. The following quote includes information on how Joan utilizes manipulatives on Fridays or at the end of class.

> Friday is free time…that’s the time they can just explore whatever they can do with the blocks. I make it available for them to use…when we have free time on Fridays, or the last 15 minutes of class, if they’ve kept up with their work. It’s their incentive to work (p. 188).

Even though teachers may feel that manipulative use is important for understanding, they may adapt their teaching delivery to fit a particular class of students, which may be in conflict with their personal beliefs about teaching. In Skott’s (2001) work, he found that Christopher, a novice teacher, approached teaching diverse classes of students differently. From a surface analysis of Christopher’s teaching, Skott felt that a researcher might believe that Christopher's views about teaching and his actual teaching styles conflicted. Skott saw Christopher's teaching differently. He felt that Christopher adapted his teaching delivery to meet the needs of the students in each class. Sometimes, the students needed more of a funneling approach, while other class's mathematical ability levels allowed them to be able to discover the material in a conceptual manner. Skott commented on these differences in detail.

> To be more specific, when students' mathematical learning was Christopher's primary interest, he struggled to establish one type of interaction characterized by support of their individual construction of mathematical concepts and skills, and he tried to create a conception of what counted as mathematics that included the process of developing independent solution strategies to given tasks. On the other hand, when his activity was primarily directed at other and more general educational goals, e.g. building student confidence, his contribution to the interaction was dominated by these other goals (p.24).

Group work is yet an additional concept that preservice teachers expressed opinions. In Clarke's (1997) case study work, he investigated two Grade 6 teachers and their evolving roles as mathematics teachers in a standards aligned classroom. Specifically, he addressed the following two research questions:

1. In what ways does the role of the teacher change when a unit of instruction based largely on nonroutine problems is used?
2. What factors influence the process of change, and what is the nature of these influences" (p. 278)?

Primary data collection consisted of classroom observations and teacher interviews.

Clarke (1997) coded the data and created themes based on his findings. He found that Bartlett, one of the two teachers in the study, enjoyed student group work because of her own experiences in mathematics classes. As a student, Bartlett lacked confidence about her mathematics abilities but was willing to listen to other student strategies, which helped her become a better student. This finding seemed out of place, when taking into account the following statement she made before teaching:

> I find myself more intrigued with the idea that I don't necessarily have to have the answer—that I can work with the kids and that that's an okay thing for them to see, too, that I'm struggling with the problem too (p. 287).

She also felt that the strong mathematics students liked helping the other students understand the concepts.

Importance of Literature to Dissertation

The literature informed my dissertation in multiple ways, including research questions, interviews, coding, and analysis. Initially, I utilized LaBoskey's (as cited in Griffin, 2003) work to frame one of my research questions. LaBoskey classified preservice teachers based on how they reflected. For my research, I examine other aspects of preservice teachers' attitudes and beliefs, such as standards and nonstandards aligned mathematics, so Raymond's (1997) study fit well. From Raymond's study, I utilized some of her interview questions about mathematics such as, "What do you think mathematics is all about," and "What most influences your mathematics beliefs?" (p. 555). In addition to interview questions about beliefs about mathematics, Raymond had survey items that pertain to teacher beliefs about mathematics that I incorporated into my interviews. For example, Raymond asked inservice teachers to rate the degree mathematics was "dynamic/static, absolute/relative, and predictable/surprising" (p. 561). From survey items like these, I can ask preservice elementary teachers about their feelings towards mathematics. Raymond's questions and survey items match well with the types of questions I addressed in my own research. In addition, I found certain topics in the literature, which mirror my code words and themes, such as standards aligned, nonstandards aligned, and socio-cultural influences.

With my research, I based several of my interview questions about teaching on Raymond's (1997) interview guides, such as, "What do you think is the most effective way to teach mathematics" and "What are the three most important characteristics of good mathematics teaching," (p. 555). In addition to interview questions about beliefs about mathematics, Raymond has survey items about teacher beliefs of mathematics teaching that I incorporated into my interviews. For example, Raymond asked whether "good mathematics teaching entails, or depends on a good textbook/use of manipulatives, teacher direction/student participation, explicit planning/flexible lessons, and helping students to like mathematics/helping students to see mathematics as useful" (p. 563). I formulated these ideas into interview questions for my dissertation. The author also addressed the use of manipulatives with students, which is a subject I addressed in my interview questions. With my analysis of the data, I created models similar to Raymond's that illustrate the impact of certain "influences" (p. 551) on preservice teachers' beliefs about mathematics and mathematics teaching.

Even with studies on specific needs of nontraditional students, I have not found any research about how nontraditional students view specific subjects or teaching. I have also not seen any research related to nontraditional and traditional preservice teachers. Through my research, I will address this gap in the literature.

Summary

In my literature review, I focused on the key issues of adult learners, preservice/inservice elementary teachers' beliefs about mathematics, and preservice/inservice elementary teachers' beliefs about mathematics teaching. Through these areas of the literature, I molded my research question, sub-questions, interview protocol, and data collection.

Nontraditional students face several obstacles in attending college, including family and financial concerns. To help them overcome these hurdles, adult learners often need motivational support systems from friends and family, as well as financial support. Even though nontraditional students suffer from certain hardships that traditional students might not, they can succeed in mathematics courses and contribute to classroom discussions.

With articles about preservice elementary teacher's beliefs about mathematics, I discovered that preservice elementary teachers hold varying opinions about their feelings towards mathematics. Some felt that mathematics consisted of procedural fluency, while others held to a conceptual knowledge that went beyond rules and memorization. Throughout the literature, I found articles that addressed Math 100, Math 200, and Math 300 topics, such as multiplication, division, estimation, probability, algebra, and geometry. By analyzing these articles, I have a sense of the types of responses preservice teachers may give to my interview questions. I can pose questions to the participants based on the literature, such as the questions in Raymond's (1997) work about inservice teachers' beliefs about mathematics and mathematics teaching.

My last major area of the literature included articles about preservice/inservice beliefs about mathematics teaching, a component of my research question. With articles about preservice elementary teacher's beliefs about teaching mathematics, I discovered that personal experience plays a key role in how preservice teachers think about mathematics teaching. Even though prior experience is important, preservice teachers can change their beliefs about how they plan to teach. Not all preservice teachers think about mathematics teaching similarly. In the literature, I found that some teachers plan to teach mathematics procedurally, some conceptually, and some procedurally and conceptually. Preservice teachers often feel mathematics lessons should be fun, no matter the cost or mathematical value in the lessons. Some prospective elementary teachers would like to teach using multiple strategies but are not familiar with alternative strategies to the way they learned mathematics. Preservice teachers who believed mathematics teaching would be difficult for them commented that they hoped to teach lower grades so that their lack of mathematical knowledge would not affect their students' learning. By using manipulatives, some preservice teachers felt they comprehended mathematics better than they ever had in the past.

For inservice teachers, articles about their beliefs about mathematics teaching included varying opinions and styles of teaching from procedural to conceptual. Inservice teachers have experimented with teaching experiments, where they act as facilitators in the classroom to aid students understanding of mathematics. Besides facilitating class discussion, inservice teachers undertake class activities that involve hands-on approaches to teaching, such as manipulative work. As a similar concept to teachers as facilitators, inservice teachers have utilized group work in the classroom in ways that promoted group learning.

In the following chapter, I detail my two pilot studies with comparisons and contrasts between them discussed in-depth. I also describe how the pilot studies influenced my dissertation plans, including an extensive discussion about my methodology.

CHAPTER III

METHODOLOGY

Introduction

In this chapter, I summarize my two pilot studies from fall 2007 and spring 2008 semesters, as well as my dissertation methodology. One significant change in my dissertation work is the exclusion of LaBoskey's (as cited in Griffin, 2003) categories of thinkers. Though these findings are informative, I found Raymond's (1997) article of value to my dissertation work because of its broad scope of mathematics and teacher classifications. Thus, I modeled my dissertation work after her work. In the following paragraphs, I document my pilots' research questions, research design, rationale for study, document collection, quality of research, analysis, and findings. I also discuss how my pilot studies influenced my dissertation work with the primary purpose of this chapter devoted to delineating the methodology of my dissertation.

Research Questions

For my first pilot study, I conducted a comparative case study (Merriam, 1998) during the fall 2007 semester with two participants enrolled in Math 200. I answered the following qualitative research question and sub-questions:

Q1 What is the nature of traditional and nontraditional preservice elementary teachers' experiences and perceptions about mathematics and the teaching of mathematics?

Q2 What is the nature of nontraditional and traditional preservice elementary teachers' reflections on algebra, data analysis, and probability?

Q3 How do nontraditional and traditional preservice elementary teachers perceive mathematics and mathematics teaching as described by LaBoskey's (as cited in Griffin, 2003) categories of alert, pedagogical, and concrete thinkers?

For my second pilot study, I conducted research during the spring 2008 semester through a comparative case study (Merriam, 1998) of eight participants enrolled in either Math 100 or Math 300. Since I did not teach all the preservice teachers, I decided not to use reflections as a means of data collection. In addition, I implemented ideas from Raymond's (1997) work, which contained an expansive foundation to discuss preservice teachers. Therefore, my research question and sub questions were:

Q1 What is the nature of traditional and nontraditional preservice elementary teachers' experiences and perceptions about mathematics and the teaching of mathematics?

Q2 How do nontraditional and traditional preservice elementary teachers perceive mathematics as described by Raymond's (1997) work with standards aligned and nonstandards aligned mathematics?

Q3 How do nontraditional and traditional preservice elementary teachers perceive mathematics teaching as described by Raymond's (1997) work with standards aligned and nonstandards aligned mathematics teaching?

Overview of Research Design

With both pilot studies, I implemented comparative case studies as my methodological framework (Merriam, 1998). Creswell (2007) defines case study research as "a qualitative approach in which the investigator explores a bounded system or multiple bounded systems over time, through detailed, in-depth data collection involving multiple sources of information, and reports a case description and case-based themes" (p. 73). Each of my pilot studies consisted of two bounded systems (traditional and nontraditional preservice elementary teachers) over a semester long investigation at a doctoral granting university in the western United States. Since I wanted to understand the differences between traditional and nontraditional preservice teachers' ideas about mathematics and mathematics teaching, the use of qualitative case studies appropriately fit my research. Below is a table that summarizes my two pilot studies with some of their similarities and differences. In the following paragraphs, I will provide a description of each pilot study and their distinguishing characteristics.

Table 2

Summary of Pilot Studies

Pilot study	Number of participants	Course	Data collected	Dates of interviews
I	2 (1 traditional and 1 nontraditional)	Math 200	3 interviews, field notes, written reflections, and researcher journal	#1--Sept. 2007 #2--Oct. 2007 #3—Nov. 2007 (via email)
II	8 (4 traditional and 4 nontraditional)	Math 100 or Math 300	2 interviews, participants' final grades, and instructors' teaching philosophies	#1-- Feb. 2008 #2--April 2008

In Pilot Study I, I investigated responses of one nontraditional (34-year-old) prospective elementary teacher and one traditional (21-year-old) prospective elementary teacher. I interviewed participants on three occasions and observed them in their Math 200 classes. I also collected the participants' reflective essays related to mathematics and mathematics teaching. I transcribed the interviews and coded the data to find themes. After my initial coding, I recoded the data in Microsoft Word and created additional codes.

In Pilot Study II, my participants included 4 nontraditional (ages 25 to 42) prospective elementary teachers and 4 traditional (ages 19 to 21) prospective elementary teachers. I interviewed each two times and obtained final grades and teaching philosophies from their respective Math 100 or Math 300 teachers. I collected all the data and transcribed interviews for both pilot studies. I created tables in *Microsoft Word* that contained several of the existing code words from my first pilot study, as well as new code words found in the second pilot study transcriptions.

In Pilot Study I, I conducted two of the three interviews in a mathematics department conference room at the university during the semester. The third interview consisted of email conversations between the preservice elementary teachers and myself. When questions arose about participants' answers for the third interview, I emailed the preservice teachers and received timely responses. I conducted this third interview via email instead of in person because the third interview was a reflective piece similar to the participant's reflective essay assignments. The participants also confirmed information from previous interviews so I felt an email interview would be sufficient. With my second pilot study, I carried out the two participant interviews in the same mathematics department conference room as in the first study.

Participants.

For Pilot Study I, I conducted a comparative case study that consisted of two preservice elementary teachers enrolled in Math 200 under my direction during the fall 2007 semester at a mid-sized doctoral granting university in the western United States. In my second pilot study, I conducted a comparative case study that consisted of eight preservice elementary teachers enrolled in Math 100 or Math 300 during the spring 2008 semester at the same university. Both sets of participants self-identified themselves as either traditional or nontraditional on the same form, which I created that included information about name, instructor name, age, and contact information (see Appendix A). I solicited participants on an individual basis (see Appendix B for informed consent form) based on predetermined criterion (Creswell, 2007) that consisted of age, gender, and Math 100, 200, or 300 instructor. According to the registrar's office at the university, females comprised approximately 94% of the population of elementary education majors in 2007. Thus, I only selected women as participants.

For my first pilot study, I also chose prospective teachers who were at slightly different ability levels based on my classroom observations. Both participants worked hard in class and utilized help from other prospective teachers in the class or myself in order to understand the concepts. I chose the pseudonyms Tina and Naomi for the traditional and nontraditional preservice teachers, respectively. Only one preservice teacher, Naomi, completed a class from me in the past. Even though Naomi and Tina took my class, they enrolled in two different Math 200 classes that I taught. All these requirements allowed me to have a varied sample and "describe multiple perspectives about the cases" (Creswell, 2007, p. 129).

My second pilot study consisted of a pair of traditional and nontraditional prospective teachers from two different Math 300 instructors, since there were not enough traditional and nontraditional preservice teachers from the same class. For the Math 100 participants, I chose all four participants from the same Math 100 instructor, which helped to control for any teacher effect influencing the knowledge of the preservice teachers. I chose the pseudonyms Dr. Ramirez and Ms. Hernandez for the

Math 300 and Math 100 teachers, respectively. I also selected the pseudonyms Nadia, Nicole, Natece, and Natalie for the nontraditional participants and Tracey, Tabitha, Tamera, and Tara for the traditional participants. The table below summarizes demographic information about all the participants as well as class information.

Table 3

Pilot Study Participant Information

Preservice teacher	Age	Pilot study (I or II)	Mathematics course	Mathematics instructor
Naomi	34	I	Math 200	Wheeler
Nadia	42	II	Math 300	Ramirez
Natece	25	II	Math 300	Wheeler
Nicole	25	II	Math 100	Hernandez
Natalie	27	II	Math 100	Hernandez
Tina	21	I	Math 200	Wheeler
Tracey	21	II	Math 300	Ramirez
Tara	19	II	Math 300	Wheeler
Tabitha	19	II	Math 100	Hernandez
Tamera	21	II	Math 100	Hernandez

Setting.

In my Math 200 classes, I used Beckmann's (2007) text that provides a conceptual understanding of mathematics concepts and procedures, as well as illustrates common mathematics misconceptions (see Appendix C for sample questions from my final). I also utilized activities that a previous Math 200 instructor used in teaching the course. This was the first time I had taught the course so I relied on the Math 200 course coordinator and experienced teachers for advice. My Math 200 teaching consisted of an overview of the day's material with activities for preservice teachers to work on in groups. Preservice teachers sought help from each other, while I acted as a facilitator instead of lecturer in the classroom. As a part of my class, preservice elementary teachers completed six to seven reflections (see Appendix D for a sample reflection) about mathematics and mathematics teaching of Math 200 concepts. The topics ranged from reviewing mathematics websites and sample lessons to reflecting on class lessons and activities. Preservice teachers also presented a lesson (see Appendix E for guidelines) that included activities and homework on probability to the class.

In Math 100 and Math 300 classes, preservice teachers learn mathematics in a guided discovery based learning environment. Ms. Hernandez, the full-time lecturer of all the Math 100 participants, described her teaching philosophy for Math 100 in the following way.

> For Math 100, I generally start out the semester lecturing mostly with a few group activities since most students are more comfortable with that scenario. As the semester progresses, I move toward using more and more group work with less lecture. Over the entire semester, I try to phase in discovery learning activities so students have time to adjust to new expectations. Almost all the activities I use in class involve the use of various manipulatives, and these activities are usually geared at encouraging conceptual understanding. For new topics, I tend to focus more on conceptual understanding before I introduce the basic algorithms. Then I move on to discussing different strategies that are available. My main goal is for students to move away from the idea there is only one way to find the correct answer, and have them move toward the idea there is only one correct answer but many different correct ways to get it.

Ms. Hernandez used Beckmann's (2007) second edition text, as well as handouts, similar to the book content, which a former Math 100 instructor created as supplementary material (see Appendix F for sample questions from her Math 100 exams).

Dr. Ramirez, an associate professor, also shared her teaching philosophy for Math 300.

> Although our preservice elementary teachers are exposed to standards aligned mathematics courses in their first two semesters, it is not until Math 300 that our prospective teachers are fully immersed into standards aligned learning. This along with the fact that geometry is new for many of our preservice elementary teachers makes the course challenging. In order to alleviate the frustration that some students experience, I incorporate advanced organizers, reflections/synthesis questions, and warm-up activities into the classroom. Through advanced organizers, I inform students of the goals of the activities, which entail both mathematics and pedagogical concepts. Through reflections/synthesis questions students express the mathematics and the pedagogical ideas that they gleaned by completing the activities. Warm-up activities serve as a quick assessment for both me and for my students, allow for discussions about misconceptions, and lead to wrapping-up the activities. Although, this course follows a Socratic and constructivist viewpoint, I am comfortable straying from this philosophy. For example, I know which activities require a lecture, which activities require extra practice, and which activities require more attention to pedagogical issues.

My approach to teaching Math 300 included the same inquiry based text (Aichele & Wolfe, 2008) as Dr. Ramirez, but I did not provide preservice teachers warm-ups or reflection activities. On occasion, preservice teachers completed quizzes over three to four concepts and projects that included reflection questions and manipulatives, such as compass/straightedge, paper folding, and Geometer's Sketchpad. I occasionally lectured in class but mainly gave the preservice teachers an overview of the day's topic before they worked in groups to learn the material for themselves. If preservice teachers struggled, I looked for another prospective teacher at their table or in the class to help explain the concept to them. If none of the preservice teachers at a table understood a concept, I would scaffold the material so that the preservice teachers learned in a guided discovery fashion. The semester of my second pilot study was the first time I taught Math 300 or any geometry related concepts. Dr. Ramirez coordinated Math 300 so I relied on her expertise in teaching the course by discussing lessons with her and using modified versions of her old exams (see Appendix G for sample questions from both of our final exams).

Theoretical Perspective.

To understand the lens through which researchers conduct a study, they often explicitly state the theoretical perspective (Creswell, 2007). For this reason, I outlined my theoretical perspective of social constructivism in Chapter 1, which provided information about the influences of social constructivism on my research questions, analysis, and interpretations of findings.

Researcher Stance

In any qualitative study, there are biases that may exist because of the subjectivity in the research. Thus, it is important to state the researcher's background and stance to have a sense of the values and perspectives the researcher is bringing with him/her to the research (Merriam, 1998). For these reasons, I also detailed in Chapter 1 my researcher stance that included information about my mathematics background, teacher certifications, and belief systems about nontraditional preservice elementary teachers.

Rationale for Study

The findings of the research will be important for mathematics educators who participate in the training of prospective elementary teachers. This research may allow mathematics educators to retain and help procedurally motivated preservice elementary teachers succeed in the elementary education teacher program. In addition, if there are a significant number of traditional and/or nontraditional preservice teachers who feel procedural learning is more important than conceptual learning, mathematics educators can take steps to address these perceptions about mathematics learning in their classes.

Through my research, I might also find factors that influence traditional and/or nontraditional prospective teachers' success in the program, which could inform teacher educators and universities about their teacher education programs. The findings will fill the gap in the literature about comparing and contrasting traditional and nontraditional preservice teachers' thoughts and perceptions about mathematics and mathematics teaching.

Data Collection

During my first pilot study, I collected data using Creswell's (2007) four basic types of information, which consisted of interviews, audiovisual materials, documents, and observations. For my second pilot study, I utilized interviews, audiovisual materials, and final grades. Through the usual research-based consent, I gained permission to collect information about the preservice elementary teachers who participated in the study. In the following paragraphs, I detail the different types of data collection I used with examples to provide clarity.

Interviews and Audiovisual Materials.

For Pilot Study I, I conducted three semi-structured interviews (Merriam, 1998, p. 74) with each preservice elementary teacher. These interviews took place towards the beginning, middle, and end of the semester. I used the initial 5-7 minute interview (see Appendix H) to build relationships and establish questioning techniques. Some questions involved ideas about group work, discovery learning, and central tendency. Sample questions included the following:

1. What do you dislike about discovery learning?

2. Would you foresee yourself using activity based learning in your future classroom?
3. Based on your responses, is there a different way you would have approached teaching mean, median, and mode?

During the 30-34 minute second round of interviews (see Appendix I), I posed questions about the participants' prior mathematical learning experiences in K-12 and college. I also asked about the participants' views on procedural and conceptual teaching and learning. Students responded to some new questions, such as the following:

1. Are there any past mathematics teachers at the college level that stand out to you as poor teachers? Explain why you would categorize them in this way.
2. If you had an elementary student who refused to learn conceptually, how would you respond?
3. What do you believe are the main goals, or objectives, you should get across to your elementary students during a mathematics lesson?

For the third interview (see Appendix J), preservice teachers answered questions via an email questionnaire instead of face to face interviews because the third interview was a reflective piece similar to the participant's reflective essay assignments. The participants also confirmed information from previous interviews so I felt an email interview would be sufficient. The preservice teachers responded to questions about their teaching experience in my class and about their responses from the previous interviews. Naomi's third interview included questions similar to the following:

1. From previous interviews, you said that you like group work, to teach 50% conceptual and 50% procedural, real world problems, have students put

answers on the board, have fun lessons, and have students to not be afraid to ask questions. Would you still agree with these statements?

2. If you did not include some of the above remarks in your answer, why did you not?

With my second pilot study, I conducted two semi-structured interviews (Merriam, 1998, p. 74) with each preservice elementary teacher. I did not interview the participants a third time because the third interview of Pilot Study I focused on participants' teaching experiences, which was not a component of the current mathematics preservice courses. These interviews took place towards the beginning and end of the semester. I asked questions during the initial 30 minute interview (see Appendix I) that were similar to the questions in the second interview of the first pilot study. From my first pilot study, I felt that the participants were more comfortable answering these questions than the first interview questions so I switched the order of questioning. The only differences in Pilot Study I's second set of interview questions and Pilot Study II's first set of interview questions included questions about mathematics concepts designed specifically for Math 100 and Math 300 content, such as the following:

1. If an elementary student just could not understand the concept of equivalent fractions, how would you further help him/her comprehend the concept?
2. Describe how you would teach an elementary student about shapes.

During the approximately 30 minute second round of interviews (see Appendix K), I posed similar questions to the first round of interviews of Pilot Study I. I included more detail in the second pilot study questions as seen in this example.

1. Do you envision using group learning in your future teaching of mathematics?

    a. In what ways?

    b. Would you group students in same or different ability groupings?

In addition to detailed questions, I based the questions on responses that various preservice teachers gave, such as multiple teaching strategies, scaffolding, and teacher organization as described in the sample questions.

1. Do you believe in using multiple ways to teach a concept?
2. How much should a teacher help students in solving problems?
3. Is teacher organization important to you? In what ways?

Document Collection.

In this section, I discuss the multiple forms of document collection in my two pilot studies. I kept a research journal and administered preservice teacher reflections for my first pilot study. Since I had a dual role as the researcher and teacher of the two participants, I also had a researcher journal where I reflected on my research progress and my role in the research process. I reflected early in my research about my feelings and concerns with my case study research and interviews.

> I am most interested with the case study design because I am thinking about using a case study with my dissertation work that I will begin next year…I have never conducted a case study or in-depth interviews so I think I will learn a great deal from class and the research…I am interested about reflections and feel confident about my research question. I was not sure about my sample size, but the instructor helped me decide on using only two individuals which lends to better results that are more detailed.

Later in the semester, I reflected on my dual role as a researcher and teacher.

> I will always try to keep in mind that I am the teacher and researcher. I do not want to bias my results by acting differently to the pre-service teachers that are conducting interviews with me. If I spend time after every class reflecting on the

> class experiences, I should become more aware of my actions and curtail any issues.

My second round of interviews confirmed to me my early assumptions about the participants, which I discussed in one of my last journal writings.

> The second interviews went a lot better than my first set of interviews. I felt more confident about what I was doing and had more engaging questions that led to interesting conversations. From the interview data, it is obvious that my nontraditional pre-service elementary teacher thinks procedurally and is worried about answering “correctly” all the time. She hesitates to answer and then feels she has given me wrong answers. After each interview, she has emailed me about how the interviews went and about her answers. My traditional pre-service elementary teacher does not hesitate to answer any question I give her. She speaks confidently about her answers and does not second guess herself. She has a sense of how she wants to teach and knows how she does not want to teach.

Besides my reflection documents, preservice elementary teachers completed six to seven reflections (see Appendix D) about mathematics and mathematics teaching of Math 200 concepts. The topics ranged from reviewing mathematics websites and sample lessons to reflecting on class lessons and activities. I used the reflections to triangulate (Mertens, 2005) or validate what the participants said during interviews, what they expressed in their writings, and how they conducted themselves during their teaching project. Tina completed her reflections during the first half of the course, while Naomi finished her reflections over the last half of the semester. I gave the preservice teachers a week in advance to complete each reflection.

For my second pilot study, I did not have participants complete reflections because reflections were not a mandatory assignment in the preservice elementary classes. In addition, I did not keep a researcher journal because I did not interact with all the participants as their instructor. My data collection, however, did include a record of preservice teachers’ final grades in the Math 100 or Math 300 course to see how well the participants performed in the class. I also asked via email Dr. Ramirez and Ms. Hernandez to describe their teaching philosophies for Math 300 and Math 100, respectively.

Observations.

During the fall 2007 semester, I kept field notes, which included observational information from daily class interactions. Towards the middle of the semester, Naomi had a rough day in class, which I documented.

> Naomi was frustrated today with finding the function values. She asked Mandy (pseudonym for classmate) about one and then was silent the rest of class. I asked her if she needed help and walked her through one example. She finished 3 out of 10 problems in a 50 minute class.

I also utilized observational data from the participants' teaching projects. As part of the teaching project, each group taught a probability lesson to their fellow classmates. I recorded any transformations in the classroom culture that may have taken place between the interviewee, other prospective elementary teachers, and me (Creswell, 2007). After Tina taught, she commented to me about her experience, which I later journaled.

> Tina was frustrated because she thought students were not paying attention. She felt teaching was hard. Some preservice teachers did not listen to her when she was teaching and she addressed their inattentiveness. Tina was a leader of the group and told the others in her group what to do.

For my second pilot study, I did not keep field notes because I did not observe Dr. Ramirez's or Ms. Hernandez's classes. I observed and took field notes of the participants' classes for my dissertation work.

Since I am the lead researcher, it is impossible for this research to be anonymous, but I took measures in both studies to respect the privacy of the participants and to secure the data. I have sole access to the data files related to the individuals. In order to maintain confidentiality, I created computer files of interviews and replaced participants' names with pseudonyms. I maintained all audio recordings on my home computer. The names of participants will not appear in any professional report of this research. It will not be easy to identify participants from the final report since there are several sections of the course. I will destroy all data once published.

Quality Research

To establish rigor in my research methods, I followed Mertens' (2005) criteria for creditability, transferability, and dependability. In the following paragraphs, I document the steps I took to produce quality research.

Creditability.

To establish creditability, I used processes outlined by Mertens (2005), including member checking, triangulation, peer checking, and expert checking. As a way of member checking (Mertens, 2005) in both studies, I transcribed the interviews and sent them to the preservice teachers who added supplementary information if they wished. The quote below came from an email that Naomi sent me after one of our face-to-face interviews. She provided in depth answers to several questions relating to group work and discovery learning.

> Hi Ann, I think to answer the questions better I need to know more about activity based, group based and discovery based learning. In our classroom you said we use a constructivist approach which is Vygotsky right? So the discovery based would be more Piaget is that right? We use activity and group based in this math class. I just like the way you teach and I know I probably will have to teach group and activity work in math so I am trying to get used to doing it. As far as liking it, the answer is sometimes I do, but not always. It just depends on if the other people in your group want to work with you or if they prefer to work alone you are kind of stuck. I think it is much different for elementary students. They like to work in groups and will go out of their way to help each other. (at least that is what I have observed in the past working as a Special Ed. para and doing my observations) I like the constructivist theory but it doesn't work for every student.

> I like working with manipulatives because I am a bodily/kinesthetic learner and I also like discovery based learning.

I also asked some of the preservice teachers to comment on questions that I had during the transcription process about their answers. To “check the integrity” or validity of my inferences, I employed the use of triangulation of the data in my first pilot study, where I examined my themes through interviews, reflections, and field notes (Schwandt, 2001, p.257). My second pilot study also included triangulation in the form of interviews, instructors’ teaching philosophies, and participants’ final grades. After I created code words based on interviews, I checked the other forms of data collection to see if they verified my initial coding. A graduate student peer checked and a mathematics education professor expert checked my findings to create a “consensual validation” of initial themes (Schwandt, 2001, p. 188).

Transferability.

Through a rich, thick description of the participants and setting, I allow my readers to “transfer information to other settings and to determine whether the findings can be transferred to comparable situations” (Creswell, 2007, p. 209). An additional component for case study research involves the use of multiple cases. With each pilot study, I increased my number of participants for each case study to improve the transferability of my findings; I increased the number of my participants in my dissertation to include 12 preservice elementary teachers.

Dependability.

Using Merriam’s (1998) techniques to ensure dependability, I utilized an audit trail, a detailed account of how I collected the data (Schwandt, 2001). With both pilot studies, I made changes in my research processes, which I documented and provided examples for clarity.

Data analysis

During the analysis phase of both pilot studies, I utilized open coding (Corbin & Strauss, 2008) to create categories that emerged from the data through a content analysis as described by Merriam (1998). For my first pilot study, I coded by hand after I had transcribed the interviews. I then recoded in *Microsoft Word* to create tables with additional code words. With my second pilot study, I coded in *Microsoft Word* using tables after I transcribed my interviews. From these tables and transcriptions, I created code words and themes, which I will detail in the next section.

Findings

In this section, I discuss the findings of both pilot studies that include code words and themes. With my first pilot study, I found 14 code words about mathematics and/or mathematics teaching. The codes informed my second pilot study, since a majority of the codes also surfaced in my second pilot study. Table 4 contains a summary of the code words for both pilot studies (see Appendix L for definitions of code words and sample coding).

Table 4

Summary of Pilot Study Code Words

Pilot study I	Pilot study II
Group work	Group work
Entertaining mathematics	Entertaining mathematics
Teacher personality	Teacher personality
Family	Family
Future teaching	Future teaching
Discovery learning	Discovery learning
Real world	Real world
Conceptual	Conceptual
Procedural	Procedural
Visual	Visual
Multiple strategies	Multiple strategies
Lecture	Lecture
Repetition	Repetition
Manipulatives	Hands-on (Manipulatives)
	Research
	Connections
	Mental Mathematics
	Technology
	Auditory

For my second pilot study, I discovered four main themes: standards aligned beliefs, nonstandards aligned beliefs, senses, and socio-cultural. I created a model (See Figure 1) that depicts the relationships between the four themes and my code words of my second pilot study, where the four themes corresponded with certain code words grouped under each theme. The arrowheads represent the flow of how the four themes influence the other themes and preservice teachers' beliefs about mathematics and mathematics teaching. In the following paragraphs, I detail these themes, related quotes from the preservice elementary teachers, and modified models for participants in Pilot Study I and II. I analyzed both pilot studies to create my findings, which allowed for a richer description of the participants in my studies.

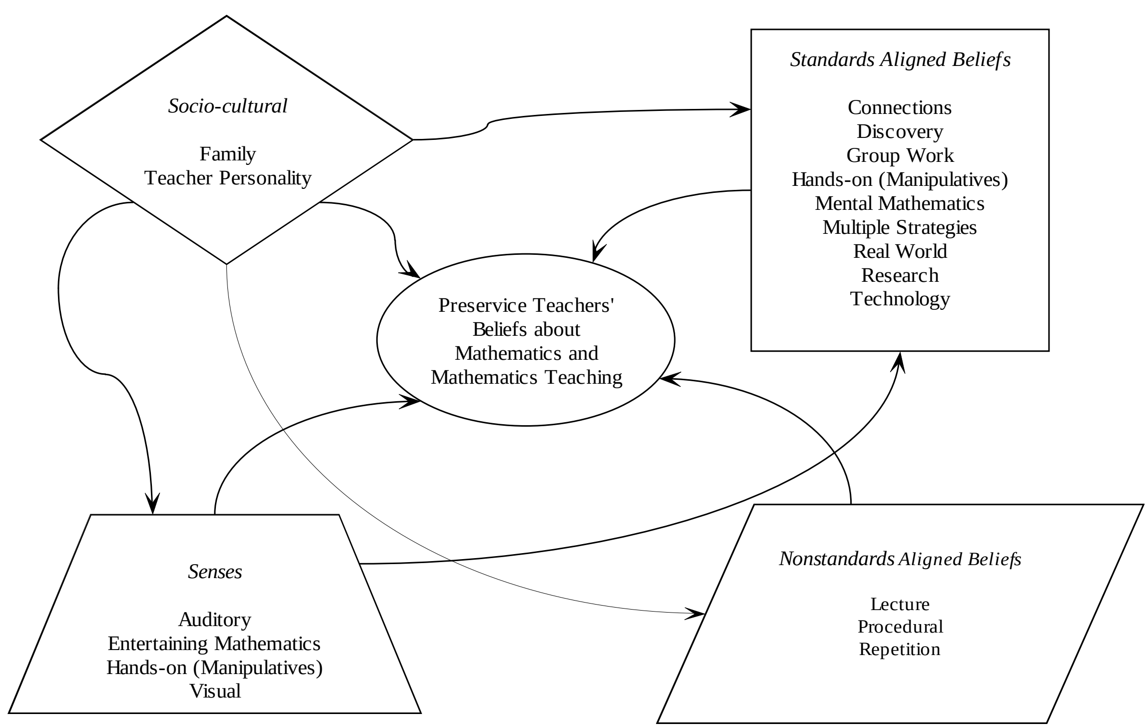

Figure 1. Model for participants' beliefs about math and math teaching.

Standards Aligned Beliefs.

I incorporated Raymond's (1997) work and her classifications of preservice elementary teachers to condense some of my initial codes under standards aligned and nonstandards aligned mathematics and mathematics teaching. Raymond utilized the terms nontraditional (standards aligned) and traditional (nonstandards aligned) to refer to types of mathematics and mathematics teaching. Raymond classified preservice teachers with nontraditional beliefs of mathematics who think of mathematics as "dynamic, problem driven, continually expanding, surprising, relative, doubtful, and aesthetic" (p. 557). With nontraditional preservice teachers, Raymond believed, "The teacher's role is to guide learning, pose challenging questions, and promote knowledge sharing" (p. 559). In addition, nontraditional beliefs about teaching mathematics include the ideas that "the teacher clearly values process over product, provides only problem-solving, manipulative-driven activities, has students work in cooperative groups at all times, and helps students to like and value mathematics" (p. 559).

All preservice teachers mentioned various aspects of both standards and nonstandards aligned mathematics and mathematics teaching that they liked. For example, Math 100 preservice teachers discussed standards aligned beliefs, such as the implementation of mental mathematics to solve addition problems. Some participants also described their standards aligned beliefs about future teaching of mathematics to connect mathematics to other subject areas like science or to ideas in research articles. To categorize participants into a particular group, I examined themes in their interviews that illustrated tendencies to lean more one way than the other. Three preservice teachers, Natece, Naomi, and Tara, spoke admittedly about both belief systems so I categorized them as combinations. From my interview questions, it was difficult to tease the ideas of standards aligned and nonstandards aligned mathematics and mathematics teaching apart from one another. As a result, I grouped a participant under the standards aligned beliefs category if they held standards aligned beliefs for mathematics and/or mathematics teaching.

I categorized one nontraditional (Nadia) and two traditional (Tracey and Tabitha) preservice teachers as having standards aligned beliefs about mathematics and mathematics teaching. Nadia possessed clear views about standards aligned beliefs. She expressed how guided discovery learning helped students to think conceptually and to create meaning of mathematics. Nadia also valued the use of manipulatives and cooperative learning.

> I like discovery learning. I had never done it before college. It is so abstract and makes you think. Students are challenged, and the material is not dumbed down…If you do not discover, mathematics has no meaning…It would have been helpful to use manipulatives and cooperative learning in secondary school.

Tracey also expressed the pros of discovery learning and group work. She included ideas about how group work provided an opportunity for her to teach others.

> I like the trial and error of discovery because I think you can learn from your mistakes. If you see that something doesn't work out you can try it again to see if it will work out…I would use discovery learning in my classroom because it helps kids know their own learning. You know, metacognition… With group work, it helps to get others' inputs when you work together. I also taught my neighbor who was behind, which helped me too. What they don't understand, I can try to figure out and put another way and explain it differently.

Tabitha shared the pitfalls of procedural, nonstandards aligned learning, which she felt hindered student creativity and conceptual thinking.

> I think memorization is misleading because then you think this is the way it is and that's it. It didn't touch on that aspect of critical thinking where you come up with the solution or answer yourself…I don't think that's good, especially in

> elementary school because then you are told something and that’s the way it is because your teacher tells you. You’re not really thinking for yourself. It’s good to start thinking for yourself and developing your own ideas for yourself, not just retaining ideas…It is really neat to explore that and really understand it and give reasons to why something works the way it does because I think that is the big absence in my elementary mathematics, the absence of why.

Nonstandards Aligned Beliefs.

On the other side of the spectrum, some preservice teachers held nonstandards aligned views about mathematics and mathematics teaching. According to Raymond, preservice teachers who believe mathematics is traditional advocate mathematics as “an unrelated collection of facts, rules, and skills” and “is fixed, predictable, absolute, certain, and applicable” (p. 556). Raymond (1997) categorized preservice elementary teachers’ beliefs as traditional if they believe in the following ideals: “the teacher’s role is to lecture and assign individual seatwork, the teacher seeks right answers and is not concerned with explanations, the teacher emphasizes mastery and memorization, and the teacher instructs solely from the textbook” (p. 559). Tina’s, Nicole’s, Natalie’s, and Tamera’s views about mathematics and mathematics teaching fit within the nonstandards aligned definitions for mathematics and mathematics teaching. I provide excerpts from interviews to illustrate my classifications. Tina believed that procedural learning was more important than conceptual learning. If students understood the procedures, Tina felt satisfied. The below quote consists of Tina’s conversation about the importance she places on procedural teaching and learning instead of manipulatives.

> They (future students) would have just need to at least understand the formula… I think it’s (algorithms) important...because I mean if they (students) only know how to do it (problems) the block way (with manipulatives) when they’re 24 years old and they’re trying to average something out I mean they can’t be going back to using blocks and stuff. They’ll need it (procedures) for future use, and I’ll try to explain that to them that it will be so much easier when you’re older.

Nicole remarked candidly on her dislikes for group work and discovery learning, which she found annoying.

> I am a little annoyed by group members. I wish they would just do it! I don't like pairing. I would rather do the work on my own. In my classroom, I would have rows of desks and students could come together for group work…I am not a huge fan of manipulatives and sometimes see how they (manipulatives) are annoying…

Nicole's beliefs seem to mirror Ms. Daniels (Eisenhart et al., 1993) who believed that conceptual mathematics could confuse students. She felt that her main goal as a teacher consisted of procedural teaching.

Natalie shared opinions about how students ultimately will always turn to formulas.

> Everybody will go to the procedure on the test. The main goal is to use the procedure in the right way. You can't teach the conceptual learning because you cannot see how students think. I would stress the importance of procedures more.

This belief in mathematics teaching is similar to Lynn in Thompson's (1984) work, who felt the central aim in teaching was to help students solve procedures.

Tamera's views mirrored Natalie's in that she also preferred nonstandards aligned mathematics, such as lectures and prescribed methods to solve problems instead of discovery learning. She believed students could not learn by discovery because of their limited mathematics background.

> Procedural learning is I guess the way I would rather learn. I like the lecture and writing everything out with lots of examples and lots of homework. I like learning through formulas which break down into smaller pieces…I like the step by step part. I don't think I would teach this way (discovery learning) because the students don't have a complete understanding of math like we do. We know how to problem solve and do the problems different ways. It would be hard for students to do it.

From Figure 1, a reader can see how nonstandards aligned beliefs influence preservice teachers' beliefs about mathematics and mathematics teaching. There is also a link between socio-cultural, specifically the teacher personality aspect of socio-cultural, and nonstandards aligned beliefs. Thus, teacher personality influences the way he/she teaches, which could include nonstandards aligned methods. Nonstandards aligned beliefs only influence socio-cultural with the teacher personality aspect. Nonstandards aligned beliefs do not influence any other theme. From this disconnect with the other data, a researcher might conjecture that preservice teachers with nonstandards aligned beliefs might struggle in a standards aligned atmosphere. This might occur because they are unable to make connections with the other aspects of teachers' lives.

Combination Standards and Nonstandards Aligned Beliefs.

Natece's, Naomi's, and Tara's beliefs about mathematics and mathematics teaching did not fit exactly in standards or nonstandards aligned beliefs. I classified them as a combination of the two because they possessed certain values from both genres of thought. In the following paragraphs, I detail their categorizations with quotes to clarify their grouping.

Natace expressed her dislike for discovery based learning. She felt that students who struggle in mathematics would not be successful in discovery learning environments.

> Discovery learning is a way for students to figure out the concepts they are trying to learn, but I feel it kind of throws them out on a boat with no paddle. It is a disconnect between the teacher and student. I am not a big fan of discovery based learning. You need to have some kind of instruction to get the ball rolling and then leave it up to the students to try to figure it out. It is okay for kids to have failures, but when they constantly fail, this is not good for their self esteem. Kids that are bad in math can only fail so many times before they give up. If you use just discovery learning, you will lead students to have bad attitudes.

Even though Natace disliked discovery learning, she shared about other aspects of standards aligned curriculum, such as multiple strategies and group work that she enjoyed. She believed that teachers should provide students with multiple ways to solve problems because of the different abilities of students.

> I know a classroom of students are all different. I would approach each lesson from different vantage points, such as with an overhead and talking using different styles of learning. I would provide lesson plans for multiple ways of teaching and have tables and group work for all students.

Naomi loved group work and detailed how her group helped her understand the concepts, but she often kept them from completing their work because she asked them several questions.

> I didn't do that (group learning) when I was in school but I like it. I like group work because you get other people's feedback and people in your group they help you if you need help…I am a little bit slower so if I get too far behind then the people in my group proceed without me…I ask too many questions, and sometimes they get frustrated with me like because they are trying to do their own work…I would definitely use group work with my future teaching because you can pair students one that is having trouble with another one that is not having as much trouble to help the student that is behind.

Even though Naomi enjoyed standards aligned beliefs like group learning, she did not feel that students should learn conceptually, as long as they knew the procedure.

Similarly to Natace, Tara believed in alternative methods to discovery learning. She enjoyed learning procedurally and believed algorithms made mathematics easy for students.

> I would probably use procedural more at the elementary level because I feel it is more helpful than just here is the basic stuff and learn the rest on your own. Step by step procedures will be easier for littler kids. I would be open to using conceptual if the students have some sort of prior knowledge. I don't want to go into too much knowledge and bore them.

Though Tara liked procedural learning, I classified her as a combination of procedural and conceptual beliefs because she commented on the importance of manipulatives.

> As a teacher, I think hands-on learning is important because there are some students who only learn by hands-on and I don't want them to not learn ever. For

> example, take the idea of volume. I think it would be cool to have each student have a glass of water (soda) and see how much volume they (students) are putting into themselves. A good way (teaching strategy) would be how we (in class) moved a volume between two different shapes and saw how it (the amounts of volume) related…to the equation.

Summary of Standards and Nonstandards Preservice Teachers' Beliefs.

From my work, I found that preservice teachers vary in their beliefs about mathematics and mathematics teaching, which supports the work of Crespo (2003), Eisenhart et al. (1993), Raymond (1997), Thompson (1984), and Vacc and Bright (1999). Table 5 summarizes the participants' demographic and class information for all preservice elementary teachers.

Table 5

Preservice Teachers' Summary Data

Preservice teacher	Classifications	Age	Mathematics course (with instructor)	Final grade
Naomi	Standards/Nonstandards	34	Math 200 (Wheeler)	C
Nadia	Standards	42	Math 300 (Ramirez)	D
Natece	Standards/Nonstandards	25	Math 300 (Wheeler)	B
Nicole	Nonstandards	25	Math 100 (Hernandez)	B
Natalie	Nonstandards	27	Math 100 (Hernandez)	A
Tina	Nonstandards	21	Math 200 (Wheeler)	A
Tracey	Standards	21	Math 300 (Ramirez)	B
Tara	Standards/Nonstandards	19	Math 300 (Wheeler)	A
Tabitha	Standards	19	Math 100 (Hernandez)	A
Tamera	Nonstandards	21	Math 100 (Hernandez)	B

From the findings, I discovered that two traditional and one nontraditional participant possessed standards aligned beliefs of mathematics and mathematics teaching. The one nontraditional preservice teacher who espoused to standards aligned beliefs was the only participant who failed the preservice mathematics course. On the other hand, I classified two traditional and two nontraditional preservice teacher as nonstandards aligned believers of mathematics and mathematics teaching. The remaining three participants, one traditional and two nontraditional, held combination views of standards and nonstandards aligned beliefs.

It surprised me that the participant who performed the worst in the course was Nadia, the nontraditional preservice teacher who supported all aspects of standards aligned mathematics and mathematics teaching. She enjoyed the group work and every part of the course but struggled with her mathematics background knowledge, which she admitted lacked substance. Possible reasons for her poor performance could be the fact that she was 15 years older than any other participant except Naomi, and her ability to recall her previous mathematics work was difficult. In addition, her Math 300 instructor, Dr. Ramirez, taught from a more inquiry-based approach than Ms. Hernandez and I did. Due to my lack of experience teaching discovery based courses, I feel that my Math 300 classes could have been easier than Dr. Rameriz's classes, which could lead to discrepancies in grades from Nadia (D)/Tracey (B) (not in my Math 300 class) and Natece (B)/Tara (A) (in my Math 300 class).

Naomi, the second oldest participant, only made a C, which was the second lowest grade of all the preservice teachers. Her low grade might be due to her mathematics background and/or length of time away from school.

I also discovered the two preservice teachers from Dr. Ramirez's class held standards aligned beliefs about mathematics and mathematics teaching, while three of the four preservice teachers from my classes held combination beliefs about mathematics and mathematics teaching. I feel these belief systems match well with Dr. Ramirez's and my own personal beliefs about mathematics and mathematics teaching. Thus, instructor beliefs may play a factor in preservice teacher attitudes about mathematics, which supports work by Philippou and Christou (1998).

The Math 100 preservice teachers had the same instructor, which helped to control for teacher effect. Similar to my above argument about the Math 300 classes, Ms. Hernandez's classes might be easier than Dr. Ramirez's so that the grades might be an indicator of these differences. Through analyzing their old exams, I found that Ms. Hernandez's never asked preservice teachers questions about student mathematics misconceptions, while Dr. Ramirez routinely did. Some preservice teachers in my experience enjoy those types of problems, while others find them challenging because there is not a procedural process to follow. Ms. Hernandez also mentioned that she lectured a significant amount at the beginning of the Math 100 course, which might be an easier type of instruction to learn than inquiry based methods. Ms. Hernandez's emphasis on lecture might also be a reason why three of the four preservice teachers in Math 100 espoused to nonstandards aligned beliefs, similar to Ms. Hernandez's teaching philosophy.

Tabitha's standards aligned beliefs may stem from her previous teaching experiences. She remarked during her first interview about these experiences, "I did do teacher cadet (a preparatory program for teachers) my junior and senior levels in high school." This finding would support Kirtman's (2008) work that teaching increases preservice teachers' mathematical understanding. One of Kirtman's participants stated, "It (teaching) has helped me to see that students understand math when we bring in many types of resources" (p. 98). Thus, Tabitha might have obtained her standards aligned beliefs through her teaching practices.

Senses.

The category of *Senses* addresses all the preservice elementary teacher's comments about mathematics and mathematics teaching that involve seeing, hearing, or touching. I asked every preservice teacher about entertaining mathematics, visual mathematics, and hands-on mathematics so it is natural that each participant commented on these topics.

When questioned about her use of manipulatives in the class, Tracey expressed her reasoning for liking hands-on learning. She believed manipulatives aided kinesthetic learners and added entertainment to mathematics classes.

> Yes, I think hands-on learning, (or manipulatives use), is important because some kids are kinesthetic learners. If kids are just sitting there, doing word problem after word problem, they are going to get bored and they are going to lose interest.

Tabitha also discussed her need for manipulatives and visual models in mathematics, which she felt created meaningful visual and kinesthetic learning.

> The second time I learned multiplication (at my new school) was so much easier, having that second way with all the pictures and visual aids because I am a very visual learner, kinesthetic, hands-on. I really need those examples.

All preservice teachers felt mathematics should be entertaining, which support Collopy's (2003) and Gellert's (1998; 2000) research about preservice elementary teachers and their belief that mathematics needs to be fun. Tara shared her feelings of how teacher personalities influence the fun in mathematics and mathematics teaching.

> You, as a teacher, have to be having fun. You can't put one of those teachers who is bored with it. You have to make it seem like it is not horrible. It's not horrible, but students still think it is so you have to trick them into think it is not. Maybe not trick them…

With one of Tina's written reflections, she visited a website and rated how well she thought her future students would benefit from the website. Tina mentioned in her reflective essay how fun a website about scatterplots was and how the children could amuse themselves with the interactive feature of the site.

> The web site is a very fun active web site with many activities conducive to learning how to make and interpret the scatter plots…I learned that this method of teaching is a very fun way for kids to be able to interact with the computer and with each other…Children are able to play with the placement of the plots to be able to see how the line of best fit will fit.

For Naomi, using plastic toys and candy to make her mathematics lessons more entertaining was important to her. She detailed these beliefs in a reflective essay.

> One of the main objectives you should be thinking about in a lesson is fun…Because if it's not fun, then the students will not learn as much…A fun thing to add to a lesson is the materials and the activities, and you could present it in a fun way to…scenarios or…Instead of just blocks you could use…for percents like fake bugs or anything…like M & M's.

In addition to the codes that I addressed specifically in the interviews, Nadia and Tara commented on using auditory resources to aid in mathematics learning. Nadia mentioned the idea of using CD's in the classroom to teach adding, while Tara remembered how rhymes helped her memorize different concepts in mathematics.

From these findings, I discovered that all preservice teachers value different forms of teaching as it relates to seeing, touching, and hearing. I did not think there would be a need to differentiate between visual (two-dimensional) and hands-on (three-dimensional) mathematics, but some of the preservice teachers enjoyed teaching utilizing one of those strategies more than the other did. For example, Nicole did not like using manipulatives or kinesthetic objects but shared her love for visual aids that consisted of two dimensional pictures.

> I am more of a visual learner. For example, geometry I like because you can see (visual) and then you have to find the reason…Teachers always drew graphs on the board, which I liked… I am a very visual person.

Socio-cultural.

The idea of *Socio-cultural* summarizes the last theme. Under *Socio-cultural*, I categorized participants' remarks about family members and teacher personalities. All comments relate to how the participants view people in their lives, which relates to their upbringings. Three nontraditional (Nadia, Nicole, and Natace) and three traditional (Tara, Tina, and Tabitha) preservice teachers commented on how their family, specifically children with nontraditional participants, affected their beliefs about mathematics and mathematics teaching. As a response about using manipulatives in a class, Nadia expressed unease in the below quote.

> As a mother, I'm concerned that my son is still counting with his hands.

On the other hand, Nicole saw the usefulness in manipulatives when she made the following comment

> I totally see how blocks help my four-year-old son.

Traditional preservice teachers described family through their siblings and parents. Tabitha described how her sister connected her knowledge of measurement to everyday situations.

> What is cool is when they start applying that outside of school with their family because I have a 7-year-old sister and she will come home and be like it's

> probably snowed 7 inches right now and that is this many centimeters. She applies everything she is learning and it is really neat to see that. And she is like, “I learned the metric system.”

Tara shared a conversation between her father and herself and the role of real world problems in mathematics teaching, which can involve sports like football.

> I was actually talking to my dad about this the other day. We were talking about how boys usually weren’t interested in math, and he was saying that you could use football stats to figure out how fast the pass is and to convert it into different units. I used to hate word problems, but that is really the only way to incorporate real world problems.

I asked all preservice elementary teachers about past mathematics teachers and what characteristics about their teachers they liked and disliked. Tabitha stated her feelings about one of her teachers who helped her understand mathematics, while another teacher damaged her confidence in mathematics.

> I really understood the information because of the teacher. It was vitally important because of the teacher, which was really neat to see that coming from like freshman year when I had a horrible teacher and everyone failed the final and everyone didn’t care…Other than that, a teacher makes all the world of difference. The material can all be the same but it is really the importance of the teacher being there and connecting with the students and making sure they understand.

Natalie detailed how Ms. Hernandez changed her negative views of mathematics, which consisted of feelings of inadequacy. She also expressed her belief that a teacher’s personality influences students’ perceptions about subjects like mathematics.

> For my whole life, math was hard…If you’re not good at it, then you’re just not good at it, but she (Ms. Hernandez) doesn’t have that attitude…I don’t want to give kids the idea of math being hard because I believe if you hate math or down in the dumps person, you are going to convey that to your students…Students may not like math because of the teacher.

Tina detailed how one of her previous mathematics instructors did not fit well with the dynamics of her school, which caused problems in Tina’s progress in the course.

> She was just well, I think it was just bad personality for our school. She was little tiny, tiny girl from the east coast. She comes to our school in the middle of nowhere with a bunch of big farm kids. She was all quiet. She would just do the work on the board and not explain it to you anything. And so you are like, “So why did you do that? Why did you do that?” She would…She knew how to do it and she was a great math person but she didn’t know how to relate it back to the student.

From these findings, preservice teachers of all ages value family and teacher personalities. I found that three of the four nontraditional participants discussed their children, one of the nontraditional participants also discussed her sister, one traditional participant talked about her father, and one traditional participant mentioned her sister. (One of the two nontraditional participants who did not mention children has children, while I am unsure whether the other nontraditional participant has children.) Family and past teachers influence the way traditional and nontraditional participants think about mathematics and mathematics teaching. Even though I discovered a weak connection between family and senses, none of the research I found examined this link.

Nontraditional and Traditional
Preservice Teachers’ Beliefs

From my analysis, I discovered that some of the links between the themes and preservice teachers’ beliefs are of varying intensities, which I represented through thickness and consistency of lines. I constructed Figure 2 for nontraditional participants and Figure 3 for traditional participants, which are modified forms of my original model.

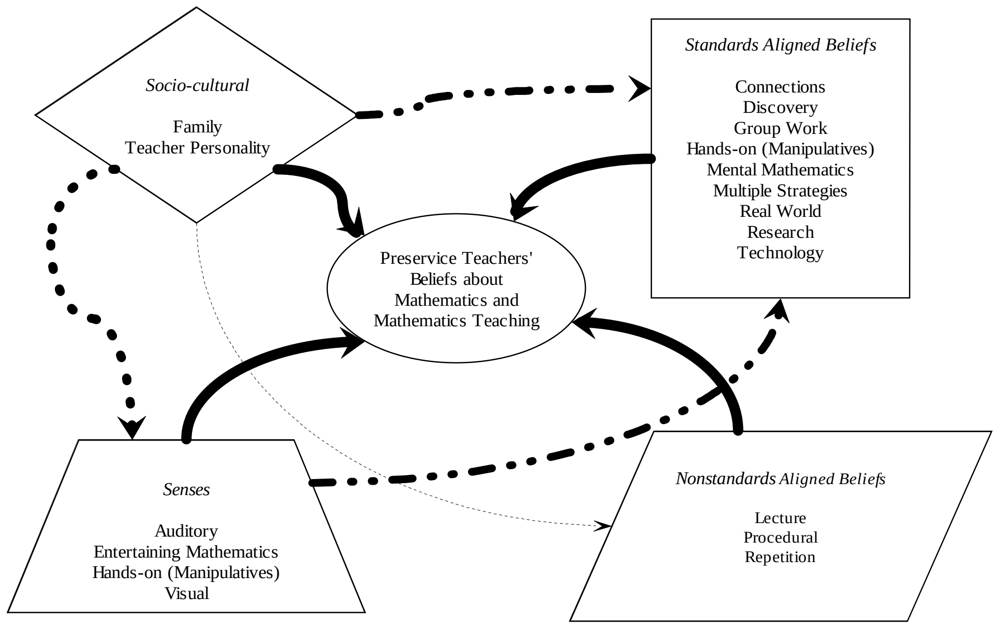

Figure 2: Model for nontraditional participants' beliefs.

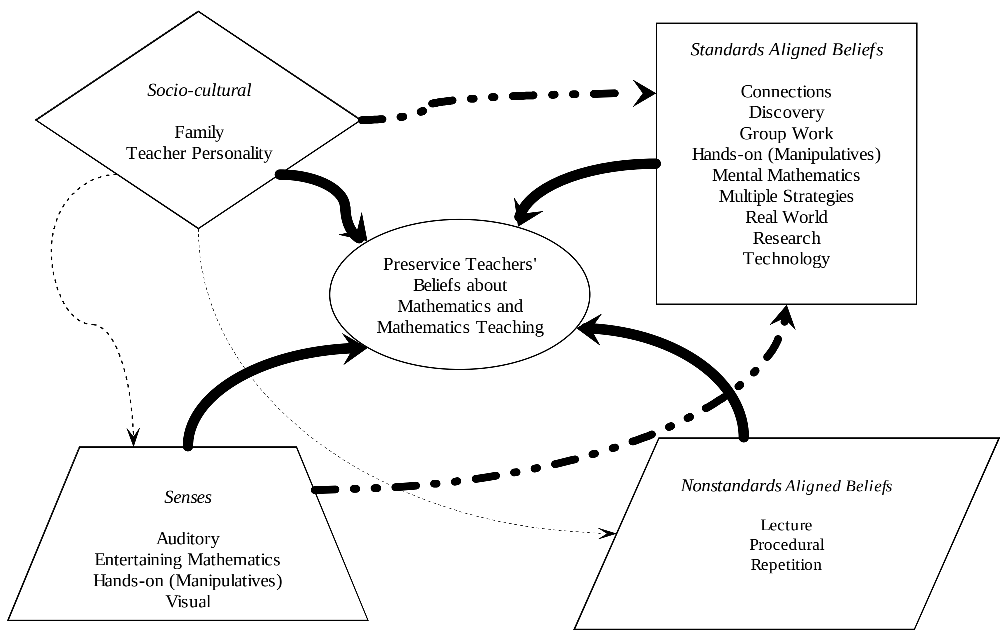

Figure 3. Model for traditional participants' beliefs.

The two models consist of many similarities. All four themes for nontraditional and traditional participants have a bold, solid line going from the theme to preservice teachers' beliefs. This is natural because I selected these themes as the main categories that affected participants' belief systems, which came from transcriptions. In addition, nontraditional and traditional preservice teachers consistently discuss *Socio-cultural* and *Senses* concepts in relation to certain standards aligned beliefs, which are codes underneath *Standards aligned Beliefs*. These connections were strong so I utilized a thick dotted line to represent these findings. I found a weak link between teacher personality and nonstandards aligned beliefs so I used a thin dotted line between these two parts of both of my models. Even though I found many similarities between nontraditional and traditional participants, I discovered one difference. Nontraditional preservice teachers often discussed their family members as reasons for why they hold certain beliefs included under *Senses*. For example, Nadia and Nicole described how their children influenced their perceptions about mathematics learning with manipulatives. Nicole also mentioned her sister during one interview conversation. Some of the traditional preservice teachers mentioned their families but not in relation to *Senses*. I still included a line for both groups of participants because participants talked about how past teachers and their use of *Senses* materials affected their beliefs about mathematics and mathematics teaching. Thus, I bolded the line for the nontraditional participants and only lightly dotted the traditional participants' connection line between *Socio-cultural* and *Senses.*

Summary

In this chapter, I summarized my two pilot studies through research questions, research design, rationale for study, document collection, analysis, and findings. Through the aide of pilot studies, I refined my research questions to fit Raymond's (1997) work instead of LaBoskey's (as cited in Griffin, 2003) research. My work centers on the beliefs about mathematics and mathematics teaching that nontraditional and traditional preservice elementary teachers exhibit.

I interviewed participants from Math 100, Math 200, and Math 300 at the same university to arrive at various opinions from nontraditional and traditional preservice teachers. Utilizing different forms of data collections, I analyzed my data to find themes.

For both pilot studies, I created codes to arrive at four themes that incorporate the values and views of 10 preservice traditional and nontraditional teachers. I found ways that will make my questioning clearer and provide an interview protocol with depth. My findings show that socio-cultural aspects and the senses influence preservice teachers of all ages. I analyzed the participants from both pilot studies to arrive at a rich sample. Two nontraditional and two traditional preservice teachers viewed mathematics and mathematics teaching as nonstandards aligned. Two traditional participants valued mathematics and mathematics teaching as standards aligned, while one nontraditional preservice teacher held the same belief. One traditional and two nontraditional preservice elementary teacher expressed mixed viewpoints. With these findings, I feel a dissertation study could shed light on these differences.

My dissertation work will help to distinguish differences between nontraditional and traditional preservice teachers' beliefs about mathematics and mathematics teaching.

With my pilot studies, I did not ask questions specifically about mathematics. Thus, I could not tease out difference between the participants' views about mathematics and mathematics teaching from my previous work. In addition, too many nontraditional preservice participants were in their twenties, which made it difficult to see differences in their opinions from the traditional participants who were similar in age. I conducted classroom observations, which was a key feature to add triangulation of data (Mertens, 2005) to my research work. With classroom observations, I saw some evolution of participant thought about mathematics and mathematics teaching, which I did not see through my pilot studies.

Importance of Pilot Studies to Dissertation

The pilot studies informed my dissertation in multiple ways, including theoretical perspective, participant sampling, data collection, data analysis, summarization of findings, and quality of research. In the following paragraphs, I detail these aspects of my dissertation.

For my theoretical perspective and participant sampling, I used the same social constructivist stance (Schunk, 2004) and similar sampling procedures as defined by Mertens' (2005, p. 320) description of criterion sampling. Mertens detailed how researchers define specific criteria. My criterion included gender, mathematics course, instructor, age, and group dynamics, which I detail in the following paragraphs.

Participant Selection.

For my research, the participants came from Math 100, Math 200, and Math 300 classes, where they engaged in social constructivict's activities such as group work and co-constructing knowledge. Preservice teachers self-identified themselves on a similar form to the one I used in my pilot studies, which included a question pertaining to their age (see Appendix M). I selected one Math 100, Math 200, and Math 300 class to conduct my research. These classes had the most nontraditional participants. From those interested in participating, I originally selected 14 nontraditional and traditional preservice teachers, which later became 12, with the same number of nontraditional and traditional participants from each instructor (See Appendices N and O for informed consents for preservice participants and instructors, respectively). Two of the participants, a nontraditional and a traditional preservice teacher, dropped out of my study due to family issues. The nontraditional participant had health concerns as well as family health issues that kept her from over a week of classes. The traditional participant had a 4 month-old child. She missed scheduled interview times because of childcare issues so I decided to not include her in my study.

Selection of participants also consisted of preservice teachers who varied in age, especially with the nontraditional participants. I selected nontraditional participants who were at least in their thirties. Since there is not a wide age difference for traditional preservice teachers (ages 18-24), I based my selection of traditional participants on classes where I found nontraditional participants. I also chose traditional participants who interacted regularly in groups with the nontraditional participants. In Table 6, I summarize the participant data. I selected the pseudonyms of Nancy, Nicolette, Nadine, Norah, Nita, and Natalya for nontraditional participants and Tasha, Theresa, Taylor, Tonya, Terri, and Taya for traditional participants. The three instructor pseudonyms included Dr. Flores for Math 100, Ms. Hernandez for Math 200, and Ms. Garcia for Math 300.

Table 6

Participant Information

Preservice teacher	Age	Mathematics course	Mathematics instructor
Nancy	53	Math 100	Flores
Nicolette	36	Math 200	Hernandez
Nadine	34	Math 300	Garcia
Norah	34	Math 100	Flores
Nita	32	Math 300	Garcia
Natalya	31	Math 200	Hernandez
Tasha	20	Math 200	Hernandez
Theresa	20	Math 300	Garcia
Taylor	19	Math 200	Hernandez
Tonya	19	Math 300	Garcia
Terri	18	Math 100	Flores
Taya	18	Math 100	Flores

Unlike previous studies, I did not instruct any of the participants and did not choose any participants who I had previously included in my pilot studies. By choosing new participants, my range of participants' opinions to analyze against my second pilot study grew.

Setting.

In the Math 100 course, preservice teachers learn from the same Beckmann (2007) used for Math 200 that I previously mentioned in my pilot study research. Preservice teachers in Math 100 all came from the same class with Dr. Flores. Dr. Flores is an assistant professor and coordinator of all Math 100 and Math 200 classes (See Appendix P for sample final questions from her Math 100 class). She shared her teaching philosophy for Math 100 in her first interview, which included her personal experiences to enrich the curriculum.

> My goals for the course are to have preservice teachers deepen their understanding of the mathematics that is taught in the K-6 curriculum. I want everyone in that class to have experiences of thinking about the material differently. I believe in doing lots of group work and lots of manipulatives as ways of helping them think about the material differently. In addition, I want them to work with alternative algorithms because many of them come in thinking there is one way to add and to do math problems. They are unaware of other ways of thinking. I want to deepen ideas, such as place value. Place value is a big theme carrying throughout the class. With fractions, I want them to have deeper ideas of how to think about fractions and to develop flexible strategies. I try to bring in my own experiences. My own experiences are somewhat limited, but I do have some that are worth sharing. I try to make it relevant, especially in Math 100, because I think so many students think they are going to teach first grade, and they are unaware of what that means. They are just thinking of their first grade experience and what they learned. They are unaware of the range of student abilities so I try to bring in experiences about that. They are also unaware of current curricula for teaching mathematics and so I think enlightening them a little bit about what is going on out there is important.

Ms. Hernandez, who taught Math 100 in my pilot work, taught all the preservice participants in Math 200 (See Appendix Q for sample questions from her Math 200 final). She utilized a notebook of handouts she collected throughout her years of teaching the course instead of using Beckmann’s (2007) book, which most instructors use to teach Math 200. In the following quote during an interview, Ms. Hernandez expressed her teaching philosophy for the course as one of confidence building in mathematics.

> I really want to engage the preservice teachers in Math 200 at a very basic level of understanding. They have had bad experiences with algebra and statistics, are very afraid of them, or they feel they are complicated. I really want to engage the preservice teachers at a basic level and build up to the things they have seen. In general, it’s about building their confidence and their skills of being able to do those problems. For example, probability has fractions in it. Most of them have very little confidence with fractions. They can do it, but they just second guess

> themselves a lot. I try to build some of their confidence. I try to set it out to be, "See you can do this in an elementary class. See here's an activity that would even work for first graders." I try to have them recognize that it's not high level, super complex ideas that they have to work towards doing. It is something that little kids get. It's something they can do instead of something to be afraid of.

Ms. Garcia, a graduate student, taught the Math 300 preservice teacher course I utilized in my study. She taught the course the previous semester as well and utilized the same Aichele and Wolf (2008) text that Dr. Ramirez and I used when we taught the course during my second pilot study (See Appendix R for sample questions from her Math 300 final). Ms. Garcia detailed, during her first interview, her teaching philosophy for Math 300, which included group discussions and synthesis questions.

> Math 300 is definitely collaboratively based. I try to make sure that the majority of every single class is spent working on group work, whether it's comparing answers to the homework that I assigned the previous class or working on new activities together. I really think that students learn well together socially, talking out the problems. It is very important. We also have a portion of class where we come together as a whole group and we talk about synthesis ideas. We have synthesis questions that I give them for every single assignment, which are questions that I think they may not necessarily recognize they are answering when they work on these activities. These are main ideas that I want to draw out from the activities and questions that I anticipate them having trouble answering. Some are questions that I want to make sure we talk about as a class. I wouldn't say that when they answer these questions, I'm lecturing necessarily. I'm asking them to kind of share with the class as a group what types of answers they come up with and why they work. I'm kind of a facilitator of a class discussion, or that is how I see it.

Data Collection.

Unlike my theoretical perspective and sampling procedures, my data collection methods changed somewhat from my previous work. I conducted two approximately 45 minute face-to-face interviews with each participant in a conference room in the mathematics department of the university where I completed my study. During the first round of interviews, I used an interview guide (See Appendix S) that included interview questions from Pilot Study II and questions stemmed from code words from both pilot studies. I also added questions about how participants view mathematics because I felt I did not obtain a clear answer to this question with my earlier studies. Some of these additional questions came from Raymond's (1997) work. I examined Raymond's work in which she grouped inservice teachers' beliefs about mathematics. I utilized some of her same questions in my interviews, which allowed me to categorize preservice teachers into one of the three classifications, *Standards Aligned Beliefs about Mathematics*, *Nonstandards Aligned Beliefs about Mathematics*, and *Combination of Standards and Nonstandards Aligned Beliefs about Mathematics*. Thus, I sorted preservice teachers as having standards aligned beliefs about mathematics if they regarded mathematics with such notions as changing and unpredictable, two ideals that follow Raymond's research measures. Lastly, I included questions about senses and socio-cultural factors, since these were two themes from my second pilot study.

With both pilot studies, participants seemed most comfortable with the questions about their past schooling so I started with those questions for my first round of interviews. I then progressed to questions about procedural and conceptual learning, as well as other relevant topics and questions stemmed from code words mentioned in my pilot studies and Raymond's (1997) research. In addition, I asked participants who are parents about how their children influence their beliefs about mathematics and mathematics teaching. The following questions are examples of the types of items that I used for the first interview.

1. Do you foresee using group learning in your future teaching of mathematics?
    a. In what ways?

2. What does discovery learning mean to you?

    a. Have you experienced discovery learning? How do you know?

    b. If so, in what ways?

3. "What do you think mathematics is all about" (Raymond, 1997, p. 555)?

4. What roles, if any, do a society's and/or teacher's culture play in the teaching of mathematics?

My second set of interview questions consisted of questions about responses the participants gave during the first set of interviews, as well as similar questions from the first interview (See Appendix T). Since my research question involved the idea of participant's evolution of beliefs, I also asked participants about these issues.

The following questions are representative of the second round of interviews.

1. On a scale of 1 to 10 with 1 being not confident with teaching mathematics to 10 being very confident with teaching mathematics, how confident are you in teaching mathematics at the 4-6 grade level? Explain.

2. If an elementary student who could not understand the concept of (insert Math 100/200/300 topic) through conceptual learning, how would you further help him/her comprehend the concept?

3. How, if any, has your attitudes about mathematics evolved from

    a. The beginning to the end of the semester?

    b. From Math 100 to Math 200/300?

4. How, if any, has your attitudes about mathematics teaching evolved from

    a. The beginning to the end of the semester?

    b. From Math 100 to Math 200/300?

In addition to interviews, I conducted classroom observations on a bimonthly basis of the preservice teachers' Math 100, Math 200, or Math 300 classes. These observations allowed me to examine how the participants interacted with their group and their teacher, as well as see evolutions in their beliefs about mathematics and mathematics teaching. During the classroom observations, I was an observer who did not participate with preservice teacher activities. I did not want my inclusion in the class to influence how the participants interacted with their groups and their teacher. While I observed the class, I followed an observation form (see Appendix U) of certain categories of behavior and class routines.

I also interviewed the instructors (see Appendices V and W) of the participants to confirm my findings from classroom observations and interview data. These interviews consisted of two approximately 30 minute interviews with each instructor. I summarized my classroom observations and preservice teacher interview findings and asked the instructors to comment on their perceptions of the participants and classroom situations. Some sample questions are listed below.

1. How do you think the following preservice teachers utilize group work?
2. How would you respond to these quotes from preservice teachers' interviews and classroom observations?
3. How well do you think the following participants do in your class academically?

I obtained final grades for the participants, similar to my second pilot study, to see if the findings about final grades from my pilot studies match my dissertation work.

My analysis and summarization of the data included transcribing (See Appendices X, Y, and Z for sample transcriptions from a traditional preservice teacher, a nontraditional preservice teacher, and an instructor, respectively.) I then coded using *NVivo* (See Appendix AA for new code words, definitions, and examples of coding). I utilized my preexisting code words and created new ones as they arose. After I finished coding, I searched for themes, similar to my second pilot study. I then summarized my findings with a model of my themes, also comparable to the one I created for my second pilot study. To determine the strength of links in my model, I utilized *NVivo* to see how many times a code appeared in transcriptions.

Quality Research.

To ensure quality research, I established creditability, dependability, and transferability measures (Mertens, 2005), like my pilot studies. For creditability, I employed member checking, triangulation of data, peer checking, and expert checking. I sent the preservice teachers and instructors copies of their interview transcriptions for review to provide as member checks. In addition, I used triangulation to find "consistency of evidence across sources of data" (p. 255) to increase the creditability of my findings. Triangulation of data came from preservice teacher interviews, instructor interviews, and classroom observations. Peers and my advisor verified my codes and themes to ensure their validity. Raymond provided an additional expert check by reviewing and providing feedback for some of my interview questions. I established transferability through a rich, thick description of the participants and setting, as well as with multiple cases. To guarantee dependability, I documented any changes in my data collection and research processes.

In the next chapter, I discuss the findings from my dissertation work and connections to my pilot studies. I provide several quotes from preservice teachers and instructors, as well as classroom observations, to strengthen my results.

CHAPTER IV

RESULTS

Introduction

In this chapter, I detail my new code words and findings from my dissertation study, which answers the following guiding research question (Q1) and sub-questions (Q2-Q5):

Q1 What is the nature of nontraditional and traditional preservice elementary teachers' experiences with and/or perceptions about mathematics and the teaching of mathematics?

Q2 How do nontraditional preservice elementary teachers perceive "mathematics" in terms of standards aligned and nonstandards aligned mathematics in comparison to traditional preservice elementary teachers?

Q3 How do nontraditional and traditional preservice elementary teachers' opinions about "mathematics" evolve (as collective traditional and nontraditional groups and as individuals) throughout a semester long mathematics content course designed to teach preservice elementary teachers in a conceptual format?

Q4 How do nontraditional preservice elementary teachers perceive "mathematics teaching" in terms of standards aligned and nonstandards aligned teaching in comparison to traditional preservice elementary teachers?

Q5 How do nontraditional and traditional preservice elementary teachers' opinions about "mathematics teaching" evolve (as collective traditional and nontraditional groups and as individuals) throughout a semester long mathematics content course designed to teach preservice elementary teachers in a conceptual format?

When I coded my dissertation work, I found some new preservice teachers' ideas (i.e., confidence, disconnect, math definitions, work experience, and Math 100/200/300 courses) that extended my existing pilot study code lists. I discuss these new concepts in the following paragraphs.

Multiple preservice teachers discussed personal confidence issues with doing mathematics and teaching mathematics. These descriptions provide the new code word *Confidence*. In addition, some nontraditional preservice teachers remarked about feeling disconnected from their fellow group members because of age differences so I added a new code word called *Disconnect*.

From my pilot studies, I found that I did not distinguish well between mathematics beliefs and mathematics teaching beliefs so I asked specific questions about these two areas during interviews in my dissertation. I coded participants' evolution of beliefs about mathematics and mathematics teaching separately, as well as added an additional code, *Define math*, about participants' definitions of mathematics. The last two new codes, *Working* and *Schooling*, came from preservice teachers justifications of certain aspects of their beliefs. Some preservice teachers worked in the K-12 school systems and used these experiences to rationalize their reasoning about mathematics and mathematics teaching, which I coded as *Working*. Others explained their belief changes through the Math 100/200/300 teachers, which I coded as *Schooling*. I summarized the final code list alongside the codes from the two pilot studies in Table 7.

Table 7

Summary of Code Words

Pilot study I	Pilot study II	Dissertation
Group work	Group work	Group work
Entertaining mathematics	Entertaining mathematics	Entertaining mathematics
Teacher personality	Teacher personality	Teacher personality
Family	Family	Family
Future teaching	Future teaching	Future teaching
Discovery learning	Discovery learning	Discovery learning
Real world	Real world	Real world
Conceptual	Conceptual	Conceptual
Procedural	Procedural	Procedural
Visual	Visual	Visual
Multiple strategies	Multiple strategies	Multiple strategies
Lecture	Lecture	Lecture
Repetition	Repetition	Repetition
Manipulatives	Hands-on (Manipulatives)	Hands-on (Manipulatives)
	Technology	Technology
	Auditory	Auditory
	Mental mathematics	Mental mathematics
	Connections	Connections
	Research	Research
		Disconnect
		Confidence
		Working
		Define math
		Beliefs about math
		Beliefs about math teaching
		Schooling

I provide findings in the following section for Q1-Q5 that include related quotes from the preservice participants, quotes from the three instructors of Math 100/200/300, and my classroom observations.

With Q1, I found the four themes from my pilot studies, *Standards Aligned Beliefs*, *Nonstandards Aligned Beliefs*, *Senses*, and *Socio-cultural*, persisted in my dissertation work. I was better able to differentiate between mathematics beliefs and mathematics teacher beliefs with my dissertation work so I divided *Standards Aligned*

Beliefs and *Nonstandards Aligned Beliefs* into four themes. The themes included the following: *Standards Aligned Beliefs about Mathematics*, *Nonstandards Aligned Beliefs about Mathematics*, *Standards Aligned Beliefs about Mathematics Teaching*, and *Nonstandards Aligned Beliefs about Mathematics Teaching*. I discovered in Q2 and Q4 the participants' beliefs about mathematics and mathematics teaching, respectively. I categorized their ideas into one of three categories: standards aligned, nonstandards aligned, and standards/nonstandards aligned beliefs. With the last two questions, Q3 and Q5, I found ways and times in which the participants' beliefs about mathematics and mathematics teaching, respectively, evolved.

Findings for Q1

> Q1 What is the nature of nontraditional and traditional preservice elementary teachers' experiences with and/or perceptions about mathematics and the teaching of mathematics?

Q1 contains ideas about the overarching aspects of my dissertation work; the four sub-questions (Q2-Q5) provide insight into specific aspects of Q1. Q1 findings include preservice teachers' thoughts about mathematics and mathematics teaching, as well as belief changes about mathematics and mathematics teaching. The four existing themes of *Standards Aligned Beliefs*, *Nonstandards Aligned Beliefs*, *Senses*, and *Socio-cultural* continued as meaningful themes in my findings for the guiding research question. In addition, I divided the two pilot study themes about beliefs, *Standards Aligned Beliefs* and *Nonstandards Aligned Beliefs,* into four separate themes to incorporate standards and nonstandards aligned beliefs about mathematics and mathematics teaching. I detail the findings about standards and nonstandards aligned beliefs about mathematics with questions Q2 and Q3, and the findings about standards and nonstandards aligned beliefs about mathematics teaching with questions Q4 and Q5. The following narrative focuses on the other two themes, *Senses* and *Socio-cultural*.

Senses.

Similar to the findings from my pilot studies, I discovered that all the preservice participants thought they would utilize multiple senses (i.e., auditory, kinesthetic, visual) to teach mathematics; many valued manipulative use for their own understanding of mathematics; and participants differed in their definitions of entertaining mathematics, which includes items such as manipulatives and puzzles.

Three nontraditional (Nadine, Nancy, and Nicolette) and two traditional (Tasha and Taya) participants described multiple strategies in their responses to the following question during their first interview, "What are the three most important characteristics of good mathematics teaching" (Raymond, 1997, p. 555)? Other participants, such as Terri, also expressed the importance of multiple strategies in teaching mathematics. In the following quote, Terri detailed a typical response among the preservice teachers that different strategies and sensory activities benefit student comprehension.

> It is important to try to hit all of the five senses probably to help students understand. Different students learn different ways and a certain way may help them better. I would help them out whichever way they learn best.

Tasha further commented on how hands-on manipulatives still help her at age 20.

> I would use manipulatives for all elementary ages. Even for me being 20 years old, I find using them helps me understand. It is when you see it instead of just being told it that sometimes it is easier to understand mathematics.

Ms. Hernandez, Tasha's teacher, supported this conjecture in a conversation about how she felt manipulatives might help Tasha learn conditional probability, a topic Tasha struggled with understanding.

> I think she struggled a little bit with the conditional probability in the sense where you have marbles in a bag and you pull one out and keep it. I think adjusting the probabilities for pulling the next one seemed to take her a little bit more to sink. Some of it was conceptual in the sense that it wasn't quite clear initially why the numbers were changing and maybe that's because she maybe is more kinesthetic than some of the other students. That is the case where I tend to notice who is kinesthetic and who's not. Kinesthetic students really want to see it to connect that.

While Tasha described how manipulatives aided her understanding, Norah discussed how they make mathematics learning enjoyable.

> Learning mathematics is fun. I think it is fun learning it because you can use all kinds of stuff. There are so many things you can use to bring mathematics alive in the classroom like manipulatives. Manipulatives make math alive because I like to play.

Taya's idea of entertaining mathematics includes not manipulatives but challenging puzzles.

> Well, I think math is fun. I think different equations can be like puzzles to me. It is something that you might have to spend a lot of time on or you might kind of struggle with figuring it out. In the end, if you do things the way you know they are supposed to be done and you know why you are doing them that way, you usually get the same answer at the end.

During my classroom observations, I noticed signs that Taya might be less interested than her peers when it came to using manipulatives. While most preservice teachers openly worked with Cuisenaire rods in Dr. Flores' class to learn about multiplying and dividing fractions, Taya worked through activities without the aid of manipulatives. Only at the end of the lesson when there was downtime did she pick up the Cuisenaire rods to put them right back down again. I asked Taya during her second interview about this incident, and she said that she did not need manipulatives to understand the mathematics.

Nicolette, like Taya, also mentioned the idea of fun with puzzles but felt that not everyone would agree they are entertaining and compared the concept of puzzles to reform curricula and how not everyone sees mathematics in the same way.

> Math should be fun. Depending on how you see, it can be. Everyone's different, but some people don't like going through puzzles. I don't know if this makes sense, but my kids also do the *Investigations* (reform) curriculum, where you look at ideas multiple ways, rather than just getting one straight answer and that's it. I think that's important because different ways make different sense to people.

Socio-cultural.

Under the *Socio-cultural* theme, participants provided various comments about how their families, past teachers, confidence issues, K-12 work experience, and Math 100/200/300 influenced their beliefs about mathematics and mathematics teaching. As with my pilot studies, the nontraditional preservice teachers spoke about how their families influenced their beliefs about mathematics and mathematics teaching. The following quotes provide examples of how multiple nontraditional participants stress family in their explanations about beliefs, as opposed to the traditional participants with only one mention about family.

Natalya, a mother of two, discussed her change in attitude towards mathematics, which occurred because of age and motherhood.

> Until I came back to college now as an adult, I always thought of math as hard but now I don't. I don't think of it as a breeze or anything but I just think I look at math in a different way, challenging but in a positive way. I used to not like math. The change occurred here at school. It was frustrating to me before, but I think it was because I wasn't patient enough. Being at a different time in my life and helping my children with their homework and realizing math wasn't that hard to begin with influenced my beliefs.

She further commented about the interplay between taking classes and helping her children.

> Last semester, my son and I were learning the same math concept. That was really cool because it helped me explain to him what he had to do.

Nadine, another mother of two, mentioned multiple times during her first interview how she integrated the idea of movies into mathematics to help her younger daughter learn, since movies are what interests her daughter.

> Games are fun. I like puzzles, but I don't think everyone likes puzzles. I don't know if they would think it was fun. Some kids might, if you gave them some. I know my younger daughter would love it. She loves that kind of challenge. My little one would not care about them. Her interest would have to do with Hollywood or movies. You need to find something the kids are interested in.

The only nontraditional participant without children, Norah, discussed her niece during both interviews. In the passage shown below, Norah commented on how she helps both her niece and nephew with mathematics and tries to comprehend how they understand mathematics.

> I help my niece. She will call me. One is in 9th grade and my nephew is in 7th grade. I help both. The way they learn influences the way I learn. I know sometimes she will ask who learned it this way? And there is always someone who says I learned it a different way. I'm like wow, I've never seen this. There might be an easier way, but you are always taught you do it this way. And it is neat to see.

Traditional participants tended not to discuss families, with the exception of Taylor. She described in her first interview how mathematics helped her mother and her figure out a problem.

> I remember one time in high school, me and my mom were trying to figure out something and I said, "Yeah, I totally learned this so we are going to use this." I was figuring out how much something costs.

Besides family, preservice teachers also commented on the effect of a teacher's personality on their general attitudes about mathematics. Tonya described how the mathematics teachers at the university who taught Math 100/200/300 displayed positive attitudes about mathematics, which showed in their teaching.

> The teachers here actually like math and everything like that and they are really good at what they do. Teacher personality does play a role. I think that if someone is really passionate about the subject and that it shows through when they're teaching and that helps a lot. I feel all three teachers here were really passionate about math and knew what they were talking about. They were basically opposite of what I felt in secondary school.

Tasha added personal comments about her current mathematics instructor, Ms. Hernandez, who made the class relevant to future teachers.

> I absolutely love Ms. Hernandez. She is such a good teacher. She's taken she said 6 weeks to teach probability, which is the longest she has ever done it, but I feel like I really understand and would be able to teach to students really well. I feel like she doesn't just teach us math but she teaches us like future teachers who need to teach math. I feel with her that she really tries hard to make sure we understand how things work and makes it relevant to our future career.

From classroom observations, I noticed that Ms. Hernandez discussed with the preservice teachers how the material connected to elementary teaching. She commented in multiple classes that I observed how she works hard to understand several ways students solve a mathematics problem. She connected this idea to the numerous ways the preservice teachers' future students will arrive at answers and encouraged the preservice teachers to be open to the diverse techniques in their classrooms.

On the other hand, Nancy described a time in her past mathematics schooling in which she felt the teacher's unpleasant attitude influenced her and her outlook on geometry.

> I'll never forget when it was time to take geometry. All my friends got a fun guy who was fun to be around. I got a very sour woman, and I struggled and struggled. It really pointed out to me that while my friends were at the same level as I was cognitively, they all got good grades in geometry. Even though I went for

> extra help and tried really really hard, I ended up with a D. That is when I learned how much a teacher plays a role in making or breaking a student.

Norah also mentioned an experience from her past at a community college that upset her so much that she walked out of the class.

> I don’t remember her name, but honestly I even walked out of her class. It wasn’t a hard class. In one of my classes, we talked about behaviorism and I think she was a behaviorist with straight lecture and no hands-on.It was just her attitude and the way she made the class feel. I got fed up one day and just walked out.

I created three new codes with my dissertation work under *Socio-cultural*, the idea of confidence in mathematics and the teaching of mathematics, previous work experience in the public schools, and previous schooling. Initially, I categorized codes under *Socio-cultural* because of their links to how the participants view people in their lives, which connected to their pasts. I felt confidence also fit under that grouping because the preservice teachers often mentioned how certain people in their lives influenced their levels of confidence in doing mathematics and teaching mathematics.

For their second interview, I asked the participants to rate their confidence in doing mathematics with 1 being not confident in doing mathematics to 10 being very confident in doing mathematics. The preservice teachers’ responses ranged from a three to a nine. The two groups responded with average rankings of around 7 each. Table 8 summarizes their initial confidence ratings in doing mathematics.

Table 8

Confidence Ratings about Math (from Oldest to Youngest Participant)

Participant	Rating
Nancy	3.5
Nicolette	7
Nadine	7
Norah	4.5-9
Nita	8.5-9
Natalya	8
Tasha	8
Theresa	3
Tonya	6-9
Taylor	8
Terri	7-8
Taya	7-8

The following narrative details Table 8 and pertinent quotes related to preservice teachers' confidence rankings about mathematics. Theresa and Nancy gave the lowest confidence levels, a 3 and 3.5 respectively, for their abilities in doing mathematics. Theresa described how understanding mathematics ideas in class is difficult for her. She remarked about how she had to take mathematics home to understand the concepts, unlike a classmate who could comprehend the material from class discussions.

> I feel like part of the confidence issue probably comes from the fact that I usually can't get a concept completely in class. I always have to go home and spend so much more time on it. I'm sure I compare it to a lot of other people, a lot of other students' abilities to just pick it up in class, like Heather. She just can pick it up so

> quickly. I'm sure I compare my abilities of having to go home and really work with it to hers, where she just understands. I've never really felt that comfortable. I've done better. In college, I've done a lot better, but in high school, I really didn't do that well, I'd say. In high school, in Ms. Moffitt's class, I think I made a B in both classes. I'm pretty sure I made B's, but they were pretty much always borderline B/C.

I also noticed how Theresa often asked questions of her group mates, especially Heather. Theresa seemed to struggle with a number of topics, including pyramids, prisms, and symmetry. When I asked Ms. Garcia about Theresa's academic struggles, she gave a similar response to Theresa's that focused on how diligent Theresa worked.

> I think that she tries really, really hard to understand the concepts. I know that she is a hard worker, not only from this experience but because I had her in Math 100. She tries really hard to understand the concepts and eventually she generally does, but I feel like she is the type of student who needs to work at it. It doesn't click automatically.

Nancy, the preservice participant with the lowest confidence rating about doing mathematics, explained her desire to finish all her mathematics courses and move on with her life.

> I'm finding that I have less and less patience with it (mathematics) and I hope not to have to take anymore math because I'm kind of getting down to the home stretch now with my time in school and I'm eager to get all the education classes that I can and the practicum. I'm taking two education classes right now and a math and a Spanish and my patience has run out with math. I'm like just get out of it. Just pass me and let's move on.

When I asked Dr. Flores about Nancy's comment, she stated that she knows that Nancy struggles with the mathematics, but she does a good job of coping with her frustrations. Dr. Flores also remarked about a classroom situation, where Nancy struggled working with an activity, similar to base 10 block manipulation; Nancy did not know how to solve a problem differently than the way provided to her.

> She is honestly working to try to understand things. I don't ever see her shut down, which sometimes weaker students do in class. I see her working and asking

> good questions. She is trying to do the conceptual, but it doesn't come naturally for her. For example, today they were looking at first grade students' work…It's really base 10 blocks, only they are just using longs and ones and they have their own algorithms. She had a hard time just looking at it. She didn't know how to express 125 using base 10 blocks. That was really hard for her. She needed individual help on that, even though she was able to begin to understand the front of the worksheet. Going from looking at someone doing it to doing it herself was really hard.

I also noticed during classroom observations that Nancy struggles with the mathematics, but consistently continues working on problems throughout the class.

On the other end of the spectrum, Nita and Norah expressed some of the highest confidence levels with mathematics. Nita explained how she felt confident in her ability to find answers to problems that she might encounter.

> For the applications that I need, I'd say probably a 9 or 8.5. It isn't necessarily because of my retention, but I know how to find answers. I even know in my experience with those lesson plans I wrote for a class, I know I have a lot of the material that I am going to need at my finger tips. I know how to further find information so I feel very comfortable.

Ms. Garcia explained how Nita's confidence in mathematics is founded on success.

> She's doing very well in the course. I don't think she struggles with the concepts. I mean occasionally she will have questions but then we will talk about them or she will talk about them with her group and she seems to really understand.

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Someone else who prided herself in her abilities in mathematics was Norah. With Norah, she believed she could easily do elementary mathematics up to algebra.

> If it was up to division of fractions, I would say 8.5 or 9. I can do algebra, but I will always miss one. I know it. I just get nervous. I think you are going so fast because you know it. I don't bother to go back and check if I did it right.

Besides understanding mathematics, I also asked preservice teachers about their confidence in teaching mathematics at certain grade levels, with 1 representing not confident in teaching mathematics to 10 representing very confident in teaching mathematics. Table 9 lists the participants' responses.

Table 9

Confidence Ratings about Teaching Math (from Oldest to Youngest Participant)

Participant	Rating (at the K-3 grade level)	Rating (at the 4-6 grade level)
Nancy	9	2.5-3
Nicolette	9-10	8
Nadine	10	4-6
Norah	10	10
Nita	10	9
Natalya	9	9
Tasha	8	6-7
Theresa	7	6
Tonya	8	7.5-8
Taylor	5-6	6-7
Terri	8-9	6-7
Taya	9	9

To obtain a better understanding of the differences between the two groups of preservice teachers, I averaged their confidence rankings for K-3 grade levels and 4-6 grade levels. If a teacher gave a ranking that ranged like Taylor's for K-3 grades with a 5-6 out of 10 confidence rating, I averaged her rating into one number, 5.5 out of 10. The nontraditional participants as a whole, when I averaged their confidence rankings about mathematics teaching together, had an average rating of 9.6 out of 10 for teaching K-3 grade levels and 7.3 out of 10 for teaching 4-6 grade levels. The traditional participants' ratings were 7.7 out of 10 for teaching K-3 grade levels and 7.0 out of 10 for teaching 4-6 grade levels.

In addition, I found evidence to suggest that participants' self efficacy about teaching mathematics increased over time only for nontraditional participants' perceptions of teaching K-3 grade levels. I calculated this result, as well as the traditional participants' K-3 and 4-6 grade level confidence for teaching mathematics and nontraditional participants' 4-6 grade level confidence for teaching mathematics by averaging the scores for the participants in each group from each course. For example, I averaged Norah's and Nancy's, the Math 100 nontraditional participants, confidence rankings together. Thus, Norah's and Nancy's confidence ratings of 9 and 10, respectively, for teaching mathematics at the K-3 grade level resulted in a 9.5 combined ranking. I repeated this procedure for all participants in both grade level groupings and compared averages, which resulted in an increasing trend, as stated above, in nontraditional participants at the K-3 grade level. In the following paragraphs, I examine the Table 9 ratings of nontraditional and traditional preservice teachers with related quotes about their concerns about confidence.

Similar to the personal confidence issues with doing mathematics, Theresa gave one of the two lowest confidence ratings with teaching mathematics at the K-3 grade level. She commented about her lack of confidence with mathematics as a factor for her ranking. Theresa also remarked on how with preparation she felt she could teach effectively in the classroom and believed she would be more patient than other teachers who do not struggle understanding mathematics.

> I actually think right now, if I could look at whatever material ahead of time, I would say a 7. That is not because I would consider myself good at math, but

> because if I had the resources, and I've never had a problem taking my time to look at stuff. I'm perfectly aware that I'm not good at math, or I don't feel comfortable with math so I know that and that I need to take extra time to study and get it. Being aware of that is how my learning style is in math. I have struggled in math before so I feel that I can relate to kids who do struggle, whereas someone who might be frustrated with it, I feel like I would be a lot more patient with it because that is exactly how I feel with it. I have no problem taking the time with it so I feel like I could do a good job, if I could prepare my lesson.

Taylor gave the lowest confidence rating for teaching mathematics at the K-3 grade level, but with opposite reasoning from Theresa. Taylor is confident in her mathematics abilities, a rating of 8, which is the reason that she felt mathematics would be difficult to explain to students. She also remarked about how she felt her explanations may be too difficult for others to understand.

> I'd say a 5 or 6 because I don't want to do things that are too much for them to understand. I'm always afraid that I am saying something that doesn't make sense. I feel like it is hard for me to explain a problem, especially if I have already done it. I have to be doing it at that time, and I just want to make sure that they understand it and it is easy enough for them and I'm getting to every student. I just want to be able to explain it in an elementary way instead of what I am actually doing in my head.

When I interviewed Ms. Hernandez, she felt that Taylor explained concepts well to her group mates, including Nicolette who usually took longer to understand topics than most of her group members. I also observed Taylor discussing the answers to her group on multiple occasions. She patiently described concepts to classmates and even re-explained ideas that the group members did not understand. Even though we both felt Taylor taught her table mates well, Ms. Hernandez could see how Taylor could struggle with explaining, since she comprehended the material quickly, especially the algebra component of Math 200.

Three nontraditional preservice teachers (Nadine, Norah, and Nita) rated themselves at the opposite end of the scale. They all gave themselves a 10 and felt secure in their capabilities in teaching mathematics at the K-3 grade level. Nadine detailed how mathematics at that age is easy and enjoyable.

> I would have to give myself a 10 on that one because the concepts are pretty easy. You can use a lot of manipulatives and the reward systems, and kids at that age are excited to learn. You can do more games and make it more fun without the kids getting too crazy.

While Norah simply believed she ranked a 10 because the mathematics at that level is simple, Nita described in detail her beliefs about mathematics teaching. She elaborated on her ranking of a 10 by describing her preparedness through taking courses at the university level.

> I'd say 10 (about confidence in mathematics teaching at the K-3 grade level) definitely because I just feel like I have been given a lot of tools and information at the college level, particularly the 100/200/300. I feel like I have been given enough to where I could successfully direct lessons appropriate for those grade levels.

Nita even connected her response to her children and a trip they took to her daughter's future kindergarten class.

> My oldest is going into kindergarten and I got the opportunity to see what they are going to be using in their math class. It is amazing. I'm so excited. They start addition and subtraction their first month of school in kindergarten. They have the neatest manipulatives that you wouldn't think would be math related. They have these penguins. They have a set for each group and they have different colors and then they have different sizes. She said they start out counting and then we say can you sort them into groups? Then, we learn about groups. I'm very excited about it, especially at that grade level because I think I could contribute to what those grade levels are teaching because I think there is so much out there that you can tie into teaching. Even if you have a small level or small depth of knowledge, there is so much out there, so many resources, especially with the internet, that you can definitely be successful if you have taken these classes.

Even though some preservice teachers rated their skills in teaching mathematics at the K-3 and 4-6 grade levels comparatively close, some described reservations and challenges they felt they would face while teaching at the 4-6 level that differed from the

K-3 grade level. Nancy fit into this category. She ranked her confidence to teach mathematics as a 9 at the K-3 level but a 2.5-3 at the 4-6 grade level. Nancy commented about her lack of explaining mathematics at that level hindered her abilities to tutor fifth grade students in long division. She felt she confused the students more than helped them.

> I would rank myself a 2.5 or 3 because that is about where my ability to explain it might be a little shaky. I actually just for one session worked with fifth graders and we were doing long division. I could tell them how to do it but it was hard for me to explain what it really is. And I thought that this level is getting a little too advanced for me to feel competent with it. That was recently. I volunteered to tutor. I actually just did it once and thought I really am not doing these kids service, if I try to explain from my modeled brain how to do it. I just didn't want to take a chance on confusing them anymore than they already were.

Like Nancy, Nadine ranked herself lower at the 4-6 grade level because of personal mathematical ability and experiences working with mathematics at those grade levels. She described her frustration in trying to help her daughters with their homework.

> My kids are those ages and working with them. They are both actually at the same level because my sixth grader is in a high level math and my seventh grader is in a low level math. My sixth grader at the high level, I can't understand a lot of the stuff she is doing. Well, I do but some of the material I am not sure what she is doing and she can figure it out on her own anyways. When they are asking me to help them with math, I'm not real sure how to do it.

Nadine further added comments about her apprehension in answering student questions at that age. She did not want to have parents of her future students upset with her about her explanations of mathematics.

> They are smarter at that age. That would probably bother me because even my kids ask me questions and I don't know. I'm not intimidated because they are my kids, but then when you have a whole classroom full and you have to meet their parents. They are going to go home and say their teacher doesn't know it.

Not all the preservice teachers believed that mathematics teaching at the 4-6 grade level would be intimidating. Three nontraditional (Norah, Nita, and Natalya) and one traditional (Taya) gave themselves rankings of 9 or 10. Norah simply stated that she felt she could teach mathematics at the 4-6 grade level easily because the mathematics was uncomplicated at that age, the same answer she previously stated for the K-3 grade level. Nita elaborated in detail on how her niece's experiences in sixth grade mathematics with the lattice method for multiplication factored into her answer. Nita discovered that even though she thought the lattice method was more difficult than the standard algorithm, her niece had the exact opposite opinion. Newer methods to Nita, like the lattice method, intimidated her a little.

> I'd say 9 because of personal experience. I look at my niece who just finished sixth grade and I look at some of the stuff she was learning. I don't know that I am concerned about the concepts, but it is how she was taught to do things; things that are foreign to me. The one that I always think of is she learned multiplication by the lattice method. Well, I learned it. We kind of did it at the community college, just as one of our units one day, but I am used to you do this, and do this, and move over one space. How is that easier than the standard algorithm? It is for her. I tried to show her how I did it and she said why would you do that? I am so removed from those newer concepts, even though I have been introduced to the lattice method, I am still not confident because I haven't done it a lot. I think that is some of those more advanced, those upper level concepts for the fourth, fifth, and sixth graders that I am not comfortable with.

The last two participants in the upper ranking, Natalya and Taya, mentioned the use of multiple strategies as the determining reason in their classification of a nine for teaching mathematics at the 4-6 grade level. Even though a 9 ranking is high, Taya felt that she could not give herself a 10 because of her lack of knowledge about all the multiple strategies to teach mathematics.

> I feel like I understand it well enough to explain it, but at the same time there are new strategies and stuff that I keep learning, or different methods of doing things. When we were doing division, there were different division methods. It was funny to me because I always used the standard method but seeing it done different ways. It was something that I learned really fast because our first example with that was looking at a student's work and doing it. I picked it up really fast what they were doing. That was really cool. It was just something that I had never seen before that point and time.

Taya further explained her lack of concern about questions from students at the 4-6 grade level, as opposed to the K-3 grade level. She felt it would be more difficult to answer all the “why” questions that younger students may pose than the older students.

> Concerned about student questions—Really, I think it is harder to answer questions from really small children because…K-3 age children have a tendency to ask why a lot and those are the kinds of questions where you say why do you do this? You can explain it once but they are going to be like why do you do that? A lot of times they keep questioning things because when you think you have fully, to the fullest extent possible. 4-6 might be a little easier to relate to the students.

In her second interview, Dr. Flores described how Taya’s opinion is typical with strong mathematics ability students like Taya who feel they cannot explain easy concepts to students.

The other two new codes, W*orking* and S*chooling*, pertain to preservice teachers’ experiences with K-12 schools as part of their teacher preparation program and experiences with Math 100/200/300, respectively. Only one nontraditional (Natalya) and two traditional (Tasha and Tonya) participants did not have experience working in the public schools. The other preservice teachers tutored, substituted, taught, observed, and/or volunteered, which gave them knowledge about how students think and interact in the classroom. In the following paragraphs, I include preservice teachers’ quotes about how these experiences influenced their thoughts about mathematics and mathematics teaching.

Nadine worked for six years at an elementary school and helped in a first and fourth grade classroom. She described her experience trying to teach an inattentive first grade boy counting and addition.

> In the first grade, I just worked with one boy. I helped him with counting. He didn’t pay attention. I usually took him in another room and I worked with him on

> counting and adding.

She further commented on how she learned that even though teachers may try innovative strategies to teach mathematics, she felt one needs to be careful to include parents in the learning process. They may teach their children differently than the teacher instructed, which could cause problems.

> I think it just shows that kids. What I learned was that they learn from their parents at home too and you have to combine what they are getting at home. If you are expecting the parents to help, they are going to have to use some of those methods that their parents know because they come up with new ways and their parents don't always know the new ways. I learned with the first grader, the only way to get him to do anything was to use manipulatives. He used little bears for counting.

Norah, the participant with the most experience with K-12 schooling, ran a computer lab for 15 years. She remarked about how she taught a lesson about fractions to a fifth grade class using manipulatives. Norah saw the students struggling and decided to use candy to help the students understand fractions.

> I would use manipulatives for older students, say for fractions. You can use those Hershey candies in it. I used to do that with the kids at school. Our fifth graders would be on a computer, even though they weren't learning fractions yet in class. On the computer, they were learning them so they were having trouble with this so I used to buy candies and say, "Okay, guys, if I cut this much from"…I was at a K-5 grade school.

Even though most of the preservice teachers spoke positively about their experiences, Nancy detailed an experience, while substituting, which changed her opinion about group work with older elementary students. She felt the students' group seating arrangement hindered her ability to manage the classroom.

> I honestly don't know if I would do group work with that age. I substituted in a sixth grade [class] and they were pretty much uncontrollable. They should have been sitting in individual seats but they took every opportunity to get out of their seats. I often thought if there were rafters in the room they would have been

> swinging from them. It was a good thing there weren't. I would use group work with the younger ages though.

Traditional participants also shared their memories of working with elementary students. Taylor commented on the teaching techniques she learned from the classroom teacher that included classroom management tips for working with students with disabilities, as well as group work strategies with large and small group dynamics.

> I learned a lot from the teacher, in general. She told me things to remember when I have my own classroom, like classroom management and how to deal with certain students. There was a year that I was there that she had a student that had focusing problems and she was telling me all the stuff that she would do to help him. I learned how to structure my class, like in groups. You have big groups and then you break them up into smaller groups. You started out with the class, the whole class, like specifically and then you break them up into groups and did small specific activities about what you taught.

Another participant, Theresa, also remarked about how she took away new ideas about teaching. She discussed an activity she could teach in her future mathematics classroom.

> I think that one was just the relevance of that because everyone has had pizza and have to figure out how to split the pizza and everyone like that. It helped with the kids just relating to it and I think it made it a little bit easier to learn about it by letting them color in the pizza and put the different toppings they liked and talking about it and stuff. I think it made it more fun and interactive for them to do too. That seemed like it worked really well actually. They were all really engaged with it when I was walking around and talking to them. I think I would definitely use that strategy.

The last new topic under *Socio-cultural* is *Schooling*, where preservice teachers discussed the impact of the Math 100/200/300 on their beliefs systems about mathematics and mathematics teaching. Since preservice teachers often discussed the impact of Math 100/200/300 when discussing changes in their belief systems, I detail specific quotes relating to *Schooling* under Q3 and Q5.

Summary of Findings for Q1.

The findings for Q1 included a description of the preexisting pilot study themes for participants' beliefs about mathematics and mathematics teaching. I created two new themes, *Standards Aligned Beliefs about Mathematics* and *Nonstandards Aligned Beliefs about Mathematics* to add to the four existing themes. In addition, I discussed the connections between the themes *Senses* and *Socio-cultural* in relation to my dissertation study. In the next section, I detail Q2's findings about participants' perceptions about mathematics.

Findings for Q2

> Q2 How do nontraditional preservice elementary teachers perceive "mathematics" in terms of standards aligned and nonstandards aligned mathematics in comparison to traditional preservice elementary teachers?

With Q2, I divided the responses into standards aligned, nonstandards aligned, and combination beliefs about mathematics. I found that three nontraditional participants believed mathematics to be standards aligned; two nontraditional participants believed mathematics to be nonstandards aligned; and one nontraditional participant and all six traditional participants believed mathematics to be a combination of standards and nonstandards aligned. I specify in the following paragraphs quotes to justify my classifications.

While three nontraditional preservice teachers (Nancy, Natayla, and Norah) embraced standards focused opinions about mathematics, there was no evidence that traditional preservice teachers held similar beliefs. The following quotes illustrate the three nontraditional participants with standards aligned mathematics views.

Nancy, the oldest participant, described how she learned surprising concepts about mathematics in her Math 100 class through *ethnomathematics* (D'Ambrosio, 1985). She loved the way people in India counted using their fingers and toes.

> Math is more changeable than not because it is so vast that there is room for many combinations. One of the readings Dr. Flores handed out was really interesting to me. I think it was when you hit India thousands and thousands of years ago it just kind of clicked in my head how the number system, our number system, was probably evolved out of our 10 digits, 10 fingers and 10 toes and that was kind of an exciting and surprising and Wow!

Another nontraditional participant, Natalya, expressed similar beliefs about mathematics as changing and adapting over time.

> I think math is pretty changing. I think at least in the way that it is taught or how much more math you need to know earlier on in your life or understand earlier, at a younger age.

She further described how some of the concepts in the United States astonished her because of her background in mathematics from Norway, her home country.

> I think the topics and methods can both be surprising because some of the topics I haven't had before and I don't know if that's just the difference of country or what, whatever it is, which has made sense because I have heard of a few other people that have had it and I have thought I have never had it.

Norah added in her comments about mathematics the importance of learning from others and the unique methods each person may utilize to arrive at an answer.

> It's surprising because we all learn different ways. We all bring different ways of coming up with the same answer but in different ways. I mean I could have cut that step out and I would have saved paper!

Two nontraditional (Nadine and Nicolette) preservice teachers expressed beliefs about mathematics that aligned with nonstandards views about mathematics. These beliefs included ideas about mathematics as "fixed" and "predictable," two terms Raymond (1997) described as nonstandard or traditional views about mathematics.

Nicolette summarized these views in the comment shown below by stating how the predictable nature of mathematics comes from the fact that mathematics is fixed.

> I feel like math is pretty fixed. I feel like it is pretty fixed because I feel like there's a right or wrong answer. I feel like math is also predictable. Ideally, it's predictable based on math being fixed.

Nadine reiterated Nicolette's sentiments about how mathematics and answers are not ambiguous.

> I think it is fixed because it seems there is only one right answer and the way learning formulas and stuff is the way I have always learned it.

One nontraditional (Nita) and six traditional (Tasha, Taylor, Taya, Terri, Theresa, and Tonya) participants held views that mathematics can be a combination of both standards and nonstandards aligned beliefs about mathematics. During interviews, they expressed opinions that mathematics was fixed and surprising. As an example, Nita stated her dual view of mathematics that included preset components like shapes and the rules for addition, as well as varying characteristics that include the enormity of mathematics as a discipline.

> I'd lean more towards math being fixed, but I do think there is change in it. I guess I look at things that are always going to be. This (a tabletop) is always going to be a circle. It isn't going to be something else. Two plus two will always be four, but I think there is a lot still we don't know about math. There is still a lot of unknowns out there. So I think that is where the changing part comes in is there's still more that we might not understand. We might not understand why this is a circle. There might be more behind this table that we might discover in 10 years, 100 years, something else, so that is where my change comes in. I only think of little kids in elementary school. They are like on the tip of the iceberg and don't know what is ahead of them.

Tasha also felt mathematics contains unchanging characteristics, such as equations and right answers.

> My perception of math is that it is very fixed. That this is the equation and this is the right answer…like just with science. I am in physics and this is how it is. It is

> not like psychology and everything is changing. I don't know that but that is how it seems to me. It is kind of one of those things, where it is either right or wrong.

Even though Tasha detailed ideals about the certainty in mathematics, she also further explained how the conceptual nature of the discipline is amazing.

> I think it is surprising to me. I think about the new things I have learned. That is the surprising part about how, or just learning now how things do work (the conceptual part).

Similarly, to Tasha, Taylor addressed her dual view about certain aspects of mathematics, such as procedures to solve addition and subtraction problems, are fixed, whereas techniques to arrive at those answers are ever-evolving.

> I think it could sort of be both because fixed because there are certain things that are always going to be the way they are, but then you could totally find a different way to do a problem that even a teacher wouldn't know. It's just like a totally new thing so it could change…Fixed to me is like the main procedures: add, subtract, just pretty much those. Strategies to solve are changing.

Taya expressed different ideas about the double role that mathematics can play when I asked her about whether or not mathematics seemed predictable to her. Taya discussed how certain concepts in mathematics can be more surprising to her than others, such as matrices.

> With predictable being a one and surprising being a 10, I would say it depends on what you are doing. It could be anywhere from a three being predictable, but also I find when you learn a new concept and learning the way you can do something and get a specific answer you are looking for, I find it can be very surprising, like a six or an eight. When math is a three, that is more the concepts of adding and subtracting. When math is a six or eight, I find that more with matrices, which are really surprising. Algebra II was something that was one of those discovery learning classes that I had to figure out. Once I figured it out why you did it the way you did it, it was really exciting for me.

Terri also explained the twin nature of mathematics by detailing the astonishing material she learned in class as opposed to the expected ideas from algebra.

> There are different theories but math really has not changed over the years.

> I would say that it is both predictable and surprising. Because sometimes math is predictable but then in math on Tuesday, we learned something else that surprised me about how some other people subtract differently than what I learned. I didn't even know that there was like another way to subtract. Math that is predictable is algebra and equations.

As an additional example from the university mathematics program, Theresa described how the Math 100/200/300 classes influenced her middle ground opinions about mathematics neither as completely fixed nor changing. She felt there are specific answers, but remarked on how Math 100/200/300 made her think more about different strategies to obtain those answers.

> I think probably more in the middle ground between changing and fixed. I think after taking some of these classes here at this university, my belief is kind of more towards the changing side…We are learning about the different ways to teach math so that our students can learn the right answers. I think the way that these classes are kind of structured kind of are in the way that teaches us that there is some flexibility, but I still think there is kind of a set answer, but I don't necessarily like that. That is the part I don't like about math. In a lot of classes, there are, but probably more so with math, there is a set answers, but with these classes, I thought about it in a different way and was made to kind of think about different answers too.

Tonya added that the courses at the college made her see the incredible side of mathematics through innovative teaching techniques. She remarked on how interesting these methods seemed to her.

> I would have to say math is changing because, well, some of it is just set in stone, like certain aspects of it like some of the maybe algebraic equations and the upper level stuff. The stuff that is changing is the different methods or the different approaches maybe to solving. I think math recently (Math 100/200/300 courses) is more surprising because in class, it is like "Oh, wow, I didn't know you could do that" or "Wow, that is a really cool way."

Summary of Findings for Q2.

I found that preservice participants discussed mathematics in terms of standards aligned, nonstandards aligned, or combination of standards/nonstandards aligned beliefs.

I classified nontraditional participants into one of the three categorizations, while all six traditional participants fit into the combinations aligned grouping. In the next section, I detail how participants describe changes and influences on their mathematics beliefs.

Findings for Q3

> Q3 How do nontraditional and traditional preservice elementary teachers' opinions about "mathematics" evolve (as collective traditional and nontraditional groups and as individuals) throughout a semester long mathematics content course designed to teach preservice elementary teachers in a conceptual format?

In the second round of preservice teacher interviews during the last few weeks of the semester, I read the participants their initial comments about mathematics and asked whether their opinions about mathematics changed during the semester or some other time. Participants' responses ranged from no change in beliefs to changes in beliefs coming from Math 100/200/300, family, work experience, or age. I summarize these responses from preservice interviews and include pertinent quotes in the following paragraphs.

Two nontraditional (Natayla, Nicolette) and one traditional (Tonya) participant stated they always held the same ideals about mathematics. Tonya further elaborated on how the university has made her a more reflective person about her mathematics beliefs.

> I kind of always knew my beliefs about mathematics, but I never actually thought about my math views until I came to this university.

One nontraditional (Nancy) and three traditional (Tasha, Taylor, and Theresa) preservice teachers believed their opinions about mathematics remained unchanged from the first to the second interview but due to other events in their lives. All four participants felt that the university's preservice mathematics courses for elementary teachers (Math 100, 200, and/or 300) altered their opinions about mathematics. Nancy, a Math 100 participant, described how Dr. Flores' class changed her standard algorithm view of mathematics.

> Since having come back to school and taking especially 100 where it is all about approaching things from different perspectives because kids may see things from different perspectives. Dr. Flores gave us examples of work that kids had done. They approached it in a completely different way than the standard or what I know to be the standard way to do math.

Tasha commented on how different her experience with mathematics at college was when compared to high school. She described how mathematics was like "a + b = c," which seemed to mean that she felt mathematics was fairly straightforward and procedural. Even though she did well in high school, she believed that she did not understand the mathematics until she came to this university.

> I left high school and I really didn't like math. I always got good grades. It wasn't challenging. It was a + b = c and that is the way it was. I think definitely taking math here has opened my eyes to how it is in your everyday life and the concepts aren't that hard to understand In high school, I did well but didn't understand. Here, I do well and understand. I knew how to do the procedures in high school but now it is material that I can actually take with me and remember.

Taylor reiterated similar ideas about high school as Tasha. Taylor also remarked how she made connections in college between the mathematics concepts, which can be unexpected.

> Before now, I don't know if it was so much predictable because through high school you keep learning and learning until the last couple of years, where you use the same concepts. You are just broadening how to use math so I guess before college here, it was more surprising than predictable. It is not so much predictable even now, but I think I have more knowledge and now I am using that knowledge and putting things onto that knowledge and connecting that knowledge to other ideas, but certain things are still surprising. Concepts I haven't seen is interesting.

Theresa further explained her belief change about mathematics and the role it will play in her future career. She now sees the relevance mathematics holds in her life and understands how important a teacher is in inspiring his/her class.

> Before Math 100, I would have thought of math as more fixed. Through taking 100, 200, and 300, my opinions about math has changed. That isn't necessarily because I like math more but I think I understand. I mean I could have liked math if I had a teacher that had inspired me to really like math or taught it in a different way. I think I understand that part now more. It is not even necessarily that I feel my feeling towards math is necessarily different, but I understand why. I think I just understand why math is important and why it is relevant, and especially why it is important for me as becoming a teacher. I think that has come from this university.

Two nontraditional (Nadine and Norah) and two traditional participants (Taya and Terri) expressed mixed feelings about their evolution of thoughts about mathematics. Norah, like Taya and Terri, felt that in Math 100 they learned specifically about new strategies to solve problems. Nadine and Norah also believed that their work in the public schools and with their families contributed to their beliefs about mathematics. Nadine, Taya and Terri, on the other hand, felt they held certain mathematics views, besides the one about strategies, throughout their lives.

The last participant, Nita, mentioned how aging contributed to her changing beliefs about mathematics that included intimidation and delight.

> I think I have had an epiphany as I have gotten older because I still follow the whole attitude that I don't like math but I don't know why I say that because I do like math. I think I am intimidated by math. But I think the more I know, the more I enjoy it. As I started my college career over, a couple years ago, I changed.

Summary of Findings for Q3.

Preservice participants differed in the influences that affected their mathematics beliefs. Some felt their mathematics beliefs remained unchanged throughout their lives, while others noted the influence of Math 100/200/300 on their mathematics ideas. In addition, some nontraditional participants described the influence of K-12 work experience, family, and/or aging influencing their beliefs. In the following section, I transition from beliefs about mathematics to beliefs about mathematics teaching. In particular, I describe the findings for Q4, the participants' beliefs about mathematics teaching.

Findings for Q4

> Q4 How do nontraditional preservice elementary teachers perceive "mathematics teaching" in terms of standards aligned and nonstandards aligned teaching in comparison to traditional preservice elementary teachers?

When I categorized preservice teachers' views about mathematics teaching for Q4, I used a similar approach to my pilot studies. I grouped participants based on their strong opinions for standards or conceptually driven mathematics teaching (zero nontraditional and two traditional), for nonstandards or procedurally driven mathematics teaching (two nontraditional and one traditional), or for a combination of the two (four nontraditional and three traditional).

Standards Aligned Beliefs about Mathematics Teaching.

I found evidence that zero nontraditional and two traditional preservice teachers (Theresa and Tonya) possessed standards aligned beliefs about mathematics teaching. These two preservice teachers stated views of procedural teaching stifling their mathematical understanding. Theresa explained how conceptual learning can help you arrive at answers, even if you do not see the answer or technique quickly, whereas procedures cause confusion because of mindless steps.

> Conceptual gives you a better understanding of what you are doing and why you are actually doing it. That is probably a better learning style, conceptual is, just because if you are doing a test or if in your future classroom and a student asks

> you a question that you didn't think of, then you can use the conceptual knowledge that you had to kind of work it out and have an answer for them. I think it is much easier to get lost in the procedural, but if you know the conceptual, hopefully, you'd be able to kind of figure out the procedural from there or what the appropriate procedural steps would be.

She further described the limitations of procedural learning with non-routine problems that can arise during testing situations. If you only understand the procedure, then you may be unable to solve problems on tests that look slightly different than you are accustomed to working out.

> I think it is still important to teach the conceptual because there are a lot of exceptions to rules. On tests, you can study it all in one format but you'll encounter questions where you have to know the conceptual understanding of it, and you'll have to look outside the basic way that you have been doing it to figure out the answer.

Similarly, to Theresa, Tonya stressed how she feels about the strengths of conceptual learning and the weaknesses of procedural learning. Tonya felt that conceptual learning resulted in students retaining the mathematics because they understand the topics.

> I have done procedural learning from my previous schooling, and I feel that is why I didn't learn as well as I should have… I think that is the main method (procedural) a lot of my previous teachers used before coming to school here. From actual learning the conceptual way, I think I would probably use more of the conceptual way when teaching because it would stick with students longer. I would probably try to stay away from using procedures as much as possible because it is easier to grasp the concept and understand more where the numbers come from and how it all works (by learning conceptually).

Nonstandards Aligned Beliefs about Mathematics Teaching.

Two nontraditional (Nadine and Nancy) and one traditional participant (Terri) conveyed nonstandards aligned beliefs about mathematics teaching. Nadine expressed her feelings about the value of procedural learning and how easy procedural learning is as compared to conceptual. She remarked that memorization of rules caused less stress for her and procedures were what she valued.

> I can memorize a lot better. I can memorize formulas, which helps me learn it. With math, I like the procedural way because I don't care why. I would probably teach more procedurally. I usually get a headache when I think about why. Learning conceptually was hard. It was really hard to do.

Nadine also commented on the importance of teaching her future students times tables, a nonstandards aligned concept. She discussed a strategy for her future teaching she would utilize to help students grasp this concept.

> One thing I would like to do when I'm teaching is I've noticed kids don't know their times tables or addition tables or any of those very well and…As I said, in second grade, I was learning them and in fourth grade the kids didn't know what eight times eight was. I would like to keep that going and even if I was a second grade teacher, I would want to have a couple problems on the board throughout the day and that would be their problem of the day.

An additional idea with standards aligned mathematics teaching involves the use of group learning. As with earlier comments, Nadine disliked this standards aligned concept and described her frustrations when working in groups. She remarked on how she would not voice her answer to the group in fear of causing conflict.

> Sometimes, I have noticed that I get it but I can't explain it to them (my group) because I'll get an answer and I know my answer is right, but I think they will debate me on it so I let it go. That is why I don't like groups because you get some people who are right all the time and don't want to be wrong, and then you get others who might be right.

Nancy, the oldest participant, also explained how procedural learning is a better fit for her than conceptual learning. She remarked on how she enjoyed the straightforwardness that comes from procedural learning.

> I think I like learning procedural learning better than conceptual learning. Conceptual is so abstract. Abstractness doesn't work for me and procedural kind of gives you a why and how in a more concrete (step by step) way.

Her mathematics teacher, Dr. Flores, discussed with me in an interview how Nancy struggled with the conceptual aspects of Math 100 and how her concerns with conceptual issues may reflect in her future teaching. Dr. Flores further remarked that Nancy commented to her about her worries with future student questions that may be difficult for her to address.

> She struggles with it (the conceptual learning). It is very different, obviously, for her age. It has been a long time since she has been in school. She was very traditionally and very procedurally taught, and it is a stretch…She's concerned about her abilities when she gets out there that she will really be able to deal with these kids who ask questions about things differently than the way she has perceived them. She has some confidence issues around that.

When discussing her future instruction, Nancy also detailed how she would utilize the standard algorithm for addition and division instead of opting to use manipulatives, which Dr. Flores taught during Nancy's Math 100 class. Nancy described how teachers never instructed differently than the standard algorithm for addition so she would use that method for her own instruction.

> I don't even remember any instruction that included blocks or manipulatives with addition. There are times where you go, "Yes, this is the way that is done." I can see me doing that (standard algorithm) and just working through the addition with students instead of using blocks.

One traditional participant, Terri, also commented about her desires to teach procedurally to her future classes. Terri explained how she learned procedurally and how algorithms would be faster than conceptual learning for students. Even though she believed she would teach students both procedurally and conceptually, she focused on the importance of procedures during interviews.

> I've only done the procedural learning. It depends on what I'm learning, whether I like learning using procedures because having a set formula makes things easier sometimes. I think the faster method would be the one they might want to use. I

> would probably show them the step by step first, and then show them pictures and let them choose.

Standards and Nonstandards Aligned Mathematics Teaching.

Four nontraditional (Natayla, Nicolette, Nita, and Norah) and three traditional (Tasha, Taya, and Taylor) preservice teachers expressed strengths of both standards and nonstandards aligned mathematics teaching. Natalya, the youngest nontraditional preservice teacher, stated how she might teach her future students with procedures and visualizations. She further stated that by teaching this way students can see the procedure, which results in students understanding the concept.

> When teaching, I guess I would just introduce both (conceptual and procedural learning) at the same time. You know one at a time every time because I think you might be able to see the procedure. You can see the procedure and envision it because you have already done it and then you can say that it is right because I did this and that's the procedure. Actually, understanding the procedure is very important.

Natalya went on to detail how the future students in her class will learn multiple ways of solving mathematics problems to enrich their mathematics knowledge. She remarked about how true understanding comes from being able to explain mathematics through various methods.

> I just think the way they (students) would have to be able to explain it in a couple of different ways would justify true understanding. It is not just the way that they have read about or been shown on the board, but to show that you have thought about it on your own and have an understanding of it well enough to explain in a couple of different ways. Learning is more than a right answer because if you can't explain how you got the answer, then you can't teach it to anybody else.

Natalya held several views that characterize standards aligned mathematics, but she saw drawbacks of some of them in her own learning, such as group work. She described to me a detachment between her and the younger preservice teachers in her group who discussed "childish" issues, such as dorm fights, parties, and boys.

> I'm the oldest (in my group), and I fairly perceive things a little differently than they do and they're freshman and they're girls too, and you know, they are girls who fight and empty each other's shampoo bottles and stuff. So you know, but the work itself is fine but I think you notice the other stuff when you're older maybe. Last semester, my group talked a lot about dorms and this party and that boy, this, that, and the other and I feel really old now.

When discussing Natalya's view of group work, Ms. Hernandez nodded in agreement with all of Natalya's comments from the above quote.

Nicolette explained how teaching her daughter using base 10 blocks allows her to see the value in conceptual learning, as well as makes more sense to her daughter than the procedural learning.

> I taught my daughter the base 10 block system, my 6-year-old first grader, on the blocks and I could tell the procedural way she didn't understand, but the blocks, she got. I would do some of both conceptual and procedural teaching of mathematics. I think the conceptual is good and I would stress it. I would teach conceptual first based on seeing my kids and seeing little kids, in general. I just think it makes more sense.

Ms. Hernandez spoke in her two interviews about Nicolette's need to learn material conceptually. She detailed how Nicolette will ask her group to slow down and explain topics conceptually so she can understand.

> I think she does really well. There's some people in her group that think procedurally and really don't want to spend the time on the conceptual part. She is the person that says, "Okay, why did you do that?" She really tries to force them to stop and walk through the conceptual part with her. I wish every group had somebody like that. She's really good about making them stop and put some thought into it and walk it through for her because she needs the time and the effort to go through it that way.

I also saw Nicolette, during classroom observations, repeatedly tell her group to stop so she could understand the steps. When I asked Nicolette in an interview about her role in the group dynamic, she expressed opinions similar to Ms. Hernandez and my observations.

> I think it is important to understand the material because a lot of times the concepts build so if I don't understand the first one then I know I am going to have problems down the road. I tell the group to wait because I want to understand. I am vocal to try to understand.

Even though Nicolette believed conceptual learning is important, she also described how lecture would play a role in her future mathematics classroom.

> In my future class, I would always like to start out with a little bit of a lecture, rather than just cold turkey because I feel I learn a lot better that way.

Ms. Hernandez reiterated in an interview Nicolette's sentiments when Ms. Hernandez discussed how Nicolette seeks reassurance from her on a regular basis.

> Nicolette asks her group and gets somewhere and she needs me to reassure her that she's right. She is looking to me more for the reassurance of authority than for clarification.

The third nontraditional participant with combination beliefs about mathematics teaching was Nita. Nita articulated the importance of a complete understanding of mathematics through different avenues of learning.

> I think to get a full understanding, you have to see all different aspects of math. You need to look at the procedural. You need to look at the conceptual, and you need to look at real life scenarios to tie everything together. It completes the picture.

Though Nita liked standards aligned ideas about mathematics teaching, she disliked certain aspects also, such as group work. Nita described a disconnect with her group members, where she worried about childcare and her mortgage, not dorm life. She felt she sacrificed a lot by coming to school and wanted to obtain as much knowledge about the subjects taught that she could.

> It's just very hard for me (to work in groups). It's very apparent that I am at a totally different point in my life. That's just because I'm a nontraditional student. It has nothing to do with the dynamics of the group. It's very hard for me to come to class and sit and talk about what's going on in the dorms. I'm like I have a

> mortgage and two kids. My biggest worries are $300 a week in daycare so I can go to school. When I go to class, I'm like, "Okay, I want to get as much as I can in this class." I want to absorb, discuss, work through, whatever. I want to spend my time wisely. I'm paying a premium, not just the class as a class but I'm away from my kids and paying for daycare.

Ms. Garcia, Nita's instructor, felt surprised about Nita's remarks about group work. She offered the following explanation about how well Nita seemed to work in her group.

> It's a little bit surprising. I mean she seems to make the group assignments work. She seems to make that dynamic happen. As far as I could tell, she didn't necessarily have a problem with it besides one time.

I also thought Nita liked her group and the work because she seemed to always contribute and help her table mates and ask questions of them. The only time I felt she distanced herself included an event towards the end of the semester where the preservice teachers could choose a partner at their table to work a group assignment. She told the other group members to work together, and she would work by herself. In a later interview, I asked her about that incident. She made the following comment about her dislike for group work and negative experiences working with younger classmates.

> I hate group assignments. I hate group projects. I was fine not being asked to be in a group. I have had so many group assignments this semester. I've learned a lot about working. I'm 33 years old. I've learned a lot about working with 19 and 20 year olds in multiple classes.

Norah, the last nontraditional participant to hold a combination view about mathematics teaching, discussed the dual importance of conceptual and procedural learning. She conveyed how important conceptual learning has been to her in Math 100 and how her future students may feel the same way. For example, Norah detailed how fractions finally made sense to her through Math 100 techniques.

> I've never experienced conceptual learning in my classes before. I've done the fraction bars in Math 100. I liked it because I'll be honest with you. I was always having trouble with fractions, and I got it the other day by picturing it so it was

> like, "Oh, I get it now." I might even use this activity in my own class…I would use conceptual learning a lot in my classroom. You are going to have students who will understand it both ways, maybe some students just by looking the first time are like, "Oh, oh, I know." But then you are going to have those like me who are going to be sitting there like, "I don't know how she got that answer." And so if you break it down or draw it, maybe even with fraction bars.

Dr. Flores commented on Norah's value she places on conceptual knowledge, which is also in agreement with my observations of Dr. Flores' class.

> I think she is very conceptually oriented because she focuses on understanding and the questions she asks tend to be more along the lines as "Have I been able to think about this correctly?," as opposed to, "Is the answer correct?," or "Is the procedure correct?"

One time in class, Dr. Flores gave the preservice teachers a list of word problems about whether the remainder of a division problem mattered in certain situations. Dr. Flores stopped at Norah's table and listened to how Norah explained how the problem produced multiple answers for her, depending on the circumstances. One of the problems described how a shipman took seven passengers across on a ferry. Norah described how she would not go back to pick up the remainder of passengers because it would cause more pollution than was necessary. Dr. Flores laughed but seemed to understand that Norah thought differently about the problems and cared more about the mathematics than just a right answer.

Even though Norah liked conceptual learning, she felt that procedures with topics, such as division, helped students understand common mathematics facts. She talked me through a simple example of 45 divided by 5.

> There was one topic I liked using procedures with. It was with division. Our teacher showed us step by step. I liked it. I was able to figure out that if I put 45 here (as the dividend), my remainder (when I put a quotient of 9) is going to be zero.

The first traditional participant with combination aligned beliefs about mathematics teaching, Tasha, expressed how important knowing “why” is to her future teaching career. She remarked that she would always desire to be able to explain to students how mathematics worked.

> I like learning that way (conceptually) because you understand why and it is one of those things, where you’re not just told this is how it is because how are you going to explain that to a student? I am not going to tell one of my students that this is just how my teacher told me so this is how it is.

Tasha even went further to explain the importance of also incorporating procedural learning into her teaching, especially for older students in college and in their future careers.

> In my own class, I think I wouldn’t stress conceptual or procedural more because they are both important. The procedural helps when they get into high school and when they get to college. You can’t bring your blocks with you all the time. They need to understand the procedure, especially if you are at a job and you are the accountant and you can’t bring the blocks.

Taya, one of the two youngest traditional participants, described how conceptual understanding is important but procedures can help make the mathematics work quicker than going through the conceptual thought process for every value. She believed that conceptual learning should be taught first and that procedures then could be used to make the problem-solving efficient.

> As a teacher, I would definitely do both. It is important to know the procedures and know how to make things shorter because sometimes students don’t always see that. They are okay with doing the conceptual way and individually adding every single thing up (say with multiplication and repeated addition) but a lot of times it is very useful also to know a quicker procedure…I would definitely teach the conceptual way first. I think it is more important to understand what you are doing before you actually try to do different tricks with it.

Lastly, Taylor discussed how she felt that several teachers use only procedures to teach their classes, in which students do not have to know why the mathematics works. She remarked on how she would want to teach more than just procedures so that students would know multiple ways to solve problems, as well as the conceptual underpinnings of the mathematics.

> Many teachers just give you a formula and you do this because that is the way it is and they don't explain why and so I think I am going to want to have my students learn why it is that way because they might understand it better. I definitely want them to learn how to do it in a few different ways and I think you can do that by using group work or having somebody else teach somebody else how to do something and pictures and manipulatives always help too. I definitely would not just give them the information.

Even though Taylor detailed the importance of standards aligned approaches to mathematics, she also expressed the need for the balance between procedural and conceptual comprehension. She felt that some mathematics topics may need to be taught one way more than another but struggled to give me an example.

> I would want to try to stress the conceptual and procedural the same because that is what it should be because I don't think you should have one more than another, unless there is a certain topic that needs one more than the other. I don't know an example of it but maybe.

Summary of Findings for Q4.

Q4's findings indicate that preservice teachers' beliefs about mathematics teaching fit into one of three categories: standards aligned, nonstandards aligned, and combination of standards/nonstandards aligned. I found that I could group traditional participants into one of the three groupings, while nontraditional participants fit into only the nonstandards aligned or combination of standards/nonstandards aligned classifications. For the final question Q5, I discuss the participants' reasoning for changes in beliefs about mathematics teaching.

Findings for Q5

> Q5 How do nontraditional and traditional preservice elementary teachers' opinions about "mathematics teaching" evolve (as collective traditional and nontraditional groups and as individuals) throughout a semester long mathematics content course designed to teach preservice elementary teachers in a conceptual format?

During the second round of preservice teacher interviews, I also read the participants their initial comments about mathematics teaching and asked whether their opinions about mathematics teaching changed during the semester or some other time. Participants' responses ranged from no change in beliefs to changes in beliefs coming from Math 100/200/300, family, or work experience. I summarize these responses from preservice interviews and include pertinent quotes in the following paragraphs.

I found evidence that zero nontraditional and one traditional (Taya) preservice teacher felt their beliefs about mathematics teaching were unchanged. Taya discussed this opinion in the context of how group work can be helpful. She remarked on how she oftentimes worked independently but saw the value in multiple opinions and strategies through group interaction.

> I think those are probably the beliefs about math teaching that I have always held. I have worked, maybe not better independently, but I have always been good at working independently. I acknowledge no individual is perfect so sometimes it is helpful to be in a group and to hear multiple opinions and different perspectives about that.

Three nontraditional (Nancy, Nicolette, and Nita) and five traditional (Tasha, Taylor, Terri, Theresa, and Tonya) believed the university's mathematics courses (Math 100/200/300) influenced their opinions about mathematics teaching. In her second interview, Nancy stated how Math 100 influenced her opinion about mathematics. She now felt that mathematics can contain some, maybe not much, standards aligned views about teaching, such as group work and manipulatives. She now saw value in seeing others perspectives and the usefulness of hands-on approaches to learning.

> I think I have changed [my opinion about mathematics teaching] because I have had time over the duration of the class to see the other approaches, the other visualizations, the other means. My mind probably opened up a little bit. I think I would probably try group work in the older grades also and if it took a bad turn, then I would change my approach. I think it is probably always worth a try now, since I learned so much from groups. I also think I might loosen up a bit [about manipulative] and use them in both because I found them useful in Math 100.

Terri expressed a view, shared by Nita, Tasha, and Tonya, that the reformed mathematics at the university is different from their upbringing. Terri stated the limitations in her early mathematics schooling that included only procedural learning with no visualizations.

> When I was taught math until now, there was really no visual. Teachers didn't show any pictures. They just gave us the procedure. Since going here, I have been exposed to groups, manipulatives, etc.

Taylor and Theresa also discussed how they now believe that conceptual learning is vital to student comprehension. Theresa detailed how her attitude change from procedural to conceptual learning occurred and how important conceptual learning, true understanding, is to her in the following comment.

> Before coming to this college, I think I would have thought only procedures were important. I don't think I really understood what conceptual was and that it really is why something works, not just like this is the idea. It is like why something works. I don't think I understood that. I think I would have probably been more focused on procedural knowledge.

Even though the university experience of Math 100/200/300 aided several participants' views about mathematics, two nontraditional preservice teachers (Natalya and Nicolette) felt their children affected their belief about teaching mathematics. Natalya commented how her children's learning styles changed her beliefs about teaching. She detailed how her son thinks similarly to herself with a visual influence, while her daughter learns procedurally.

> I think my son has changed my opinions and the way my children see it and how I see it clicks for them and they get it and what's helped them learn it. And they actually both learn in different ways. My son is visual like me and my daughter is really procedural like my husband who was also a math major. It just kind of clicks for her. And I just see her go so she is not that visual at all.

Nicolette mentioned the idea of the university helping her, in an earlier quote, but she also discussed her children's influence on her belief system, especially the use of group work in the public school system. Unlike some parents, Nicolette commented, she believed that her high achieving children benefit from helping others with their mathematics work.

> They (my beliefs about mathematics teaching) have changed since I have graduated high school. They have changed in watching my kids and taking these classes. I think group work is good. My kids are pretty high. I think it is good for them. I know parents sometimes when their kids are a little higher end (gifted) they hate it because their kid is not getting anything because they are just sitting. I think it is good. It reinforces what you think when they work.

Two nontraditional preservice teachers (Nadine and Norah) felt that working in the public school system influenced their view about mathematics teaching. Nadine first discussed the importance of memorization throughout school.

> I think the procedural is something I always had because it was something easy for me to memorize these things.

Nadine went on further to say that by working in the public schools, she learned that manipulatives can help students learn mathematics.

> I have had to work with kids and that's the only way they would be able to see it. It was sometimes the only way to keep their interest because other times they would just be spacing off for something.

Similarly, to Nadine, Norah detailed how working as a computer aide in the schools for 15 years molded her opinion about mathematics teaching.

> I think I got my opinions more when I worked in the schools because before that, it is just the way I know it and it is because of the teachers that I had.

Summary of Findings for Q5.

Similar to the responses to Q3, preservice participants described influences to their mathematics teaching beliefs coming from K-12 work experience, family, and Math 100/200/300 classes. One traditional participant also felt their beliefs never changed throughout her life. The greatest influence for most traditional participants was the Math 100/200/300 sequence of classes. In the following section, I summarize all five research questions and provide models to synthesize the findings.

Summary of Findings

In Table 10, I summarized the findings for participants' beliefs about mathematics and mathematics teaching and participants' final course grades in the Math 100/200/300 course they were enrolled in during this study.

Table 10

Preservice Teachers' Summary Data

Preservice teacher	Age	Classification mathematics	Classification mathematics teaching	Mathematics course (with instructor)	Final grade
Nancy	53	Standards	Nonstandards	Math 100 (Flores)	C
Nicolette	36	Nonstandards	Standards/ Nonstandards	Math 200 (Hernandez)	A
Nadine	34	Nonstandards	Nonstandards	Math 300 (Garcia)	B
Norah	34	Standards	Standards/ Nonstandards	Math 100 (Flores)	C
Nita	32	Standards/ Nonstandards	Standards/ Nonstandards	Math 300 (Garcia)	A
Natalya	31	Standards	Standards/ Nonstandards	Math 200 (Hernandez)	B
Tasha	20	Standards/ Nonstandards	Standards/ Nonstandards	Math 200 (Hernandez)	A
Theresa	20	Standards/ Nonstandards	Standards	Math 300 (Garcia)	A
Tonya	19	Standards/ Nonstandards	Standards	Math 300 (Garcia)	B
Taylor	19	Standards/ Nonstandards	Standards/ Nonstandards	Math 200 (Hernandez)	A
Terri	18	Standards/ Nonstandards	Nonstandards	Math 100 (Flores)	B
Taya	18	Standards/ Nonstandards	Standards/ Nonstandards	Math 100 (Flores)	A

The table suggests a possible difference in nontraditional and traditional participants' final course grades. Two nontraditional and four traditional participants made A's; two nontraditional and two traditional participants made B's; and two nontraditional and zero traditional participants made C's. Of the six participants who made A's, one held nonstandards aligned beliefs about mathematics, five held standards/nonstandards aligned beliefs about mathematics, while one held standards aligned beliefs about mathematics teaching, and five held standards/nonstandards aligned beliefs about mathematics teaching. Of the four participants who made B's, one held standards aligned beliefs about mathematics, one held nonstandards aligned beliefs about mathematics, and two held standards/nonstandards aligned beliefs about mathematics, while one held standards aligned beliefs about mathematics teaching, two held nonstandards aligned beliefs about mathematics teaching, and one held standards/nonstandards beliefs about mathematics teaching. Of the two participants who made C's, both held standards aligned beliefs about mathematics, while one held nonstandards aligned beliefs about mathematics teaching and one held standards/nonstandards aligned beliefs about mathematics teaching.

I created figures to synthesize the specific influences on nontraditional and traditional preservice teachers' beliefs about mathematics and mathematics teaching that included comments they explicitly mentioned during interviews. The four main lines going into the inner belief ovals are all solid lines since they represent major bonds/themes in my research. The bold words and lines associated with each figure correspond to the connections the participants specifically discussed in interviews influencing their belief systems. In addition, the number after each bold word represents the number of participants who specifically mentioned that code word during interviews as an influence on their beliefs. All models provide a visual way to see the factors that affect preservice teacher views and what factors are more influential for nontraditional and/or traditional participants.

In addition to the lines themselves, the thicknesses of the lines in the mathematics and mathematics teaching beliefs figures coincide with the number of preservice teachers who remarked during their interviews about a concept under one of the themes. Thus, the greater the number of preservice teachers who commented about how a certain aspect influenced their beliefs about mathematics and/or mathematics teaching, the thicker the arrow.

In the following paragraphs, I first discuss the nontraditional and traditional participants' beliefs about mathematics and then detail the nontraditional and traditional participants' beliefs about mathematics teaching. In Figures 4 and 5, I provide models that synthesize themes for the nontraditional and traditional preservice elementary teachers' views about mathematics.

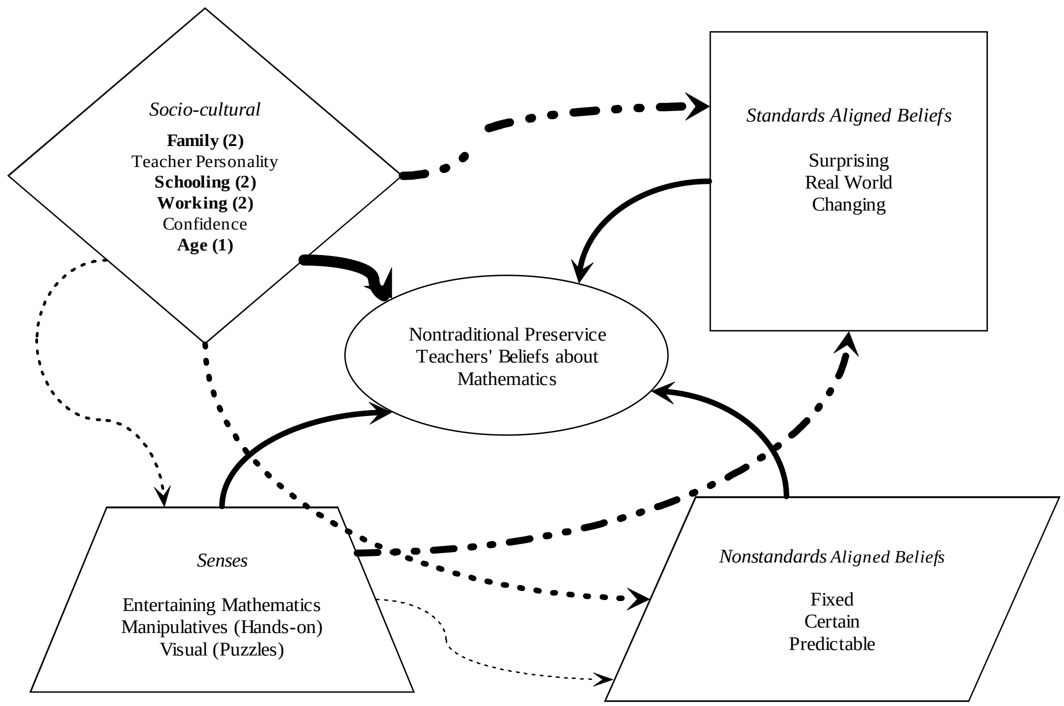

Figure 4. Model for nontraditional participants' beliefs about math.

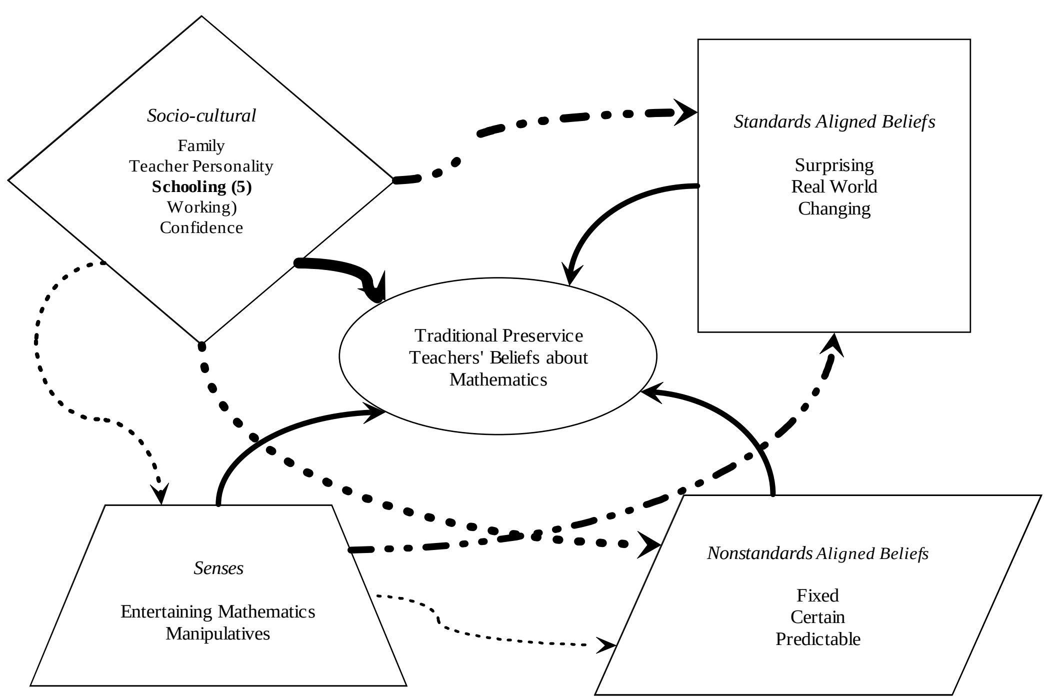

Figure 5. Model for traditional participants' beliefs about math.

These first two models represent a combination of participants' responses about their beliefs about mathematics. The models are similar in structure to the models I used in my pilot study research about participants' beliefs about mathematics and mathematics teaching. I modified the earlier pilot study figure by utilizing ideas from Raymond (1997) about mathematics, including mathematics as "fixed, predicable, real world, certain, surprising, and changing." I also changed the aspects under *Senses* to *Entertaining mathematics*, *Manipulatives*, and *Visual* because these were the only ideas about the five senses the participants commented about during interviews.

On inspection, the figures seem similar to one another. The link from *Socio-cultural* to *Standards Aligned Beliefs about Math* and the link from *Senses* to *Standards Aligned Beliefs about Math* are the same thickness, while the links from *Socio-cultural* to *Senses* and *Senses* to *Nonstandards Aligned Beliefs about Math* differ only slightly from one another. The biggest difference between the two models comes from the link from *Socio-cultural* to *Nonstandards Aligned Beliefs about Math*. To help explain this difference, I describe in the following paragraph my rationale for creating the arrows' thicknesses.

I utilized a system (See Table 11) for determining the thickness of the arrows by analyzing transcription data and keeping track of the participants who remarked about a particular link between codes. I specifically examined comments made under the code words of *Beliefs about math*, *Define math*, and *Entertaining mathematics* to tally participants' responses about their beliefs about mathematics. Analysis of the comments under *Beliefs about math* provided the bulk of my findings, but I also discovered that participants commented a substantial number of times about their beliefs while discussing the definitions of mathematics and fun mathematics so I included those codes in my analysis.

Table 11

Connections between Themes of Participants' Beliefs about Math

Participant category	Initial theme	Theme impacted	Participant names
Nontraditional	Socio-cultural	Senses	Nicolette, Nita
Nontraditional	Socio-cultural	Nonstandards aligned beliefs about math	Norah, Nadine, Nancy, Nita
Nontraditional	Socio-cultural	Standards aligned beliefs about math	Nadine, Nancy, Natalya, Nicolette, Nita, Norah
Nontraditional	Senses	Standards aligned beliefs about math	Nadine, Natalya, Nancy, Nita, Norah
Nontraditional	Senses	Nonstandards aligned beliefs about math	Nita
Traditional	Socio-cultural	Senses	Taya, Taylor, Theresa
Traditional	Socio-cultural	Nonstandards aligned beliefs about math	Tasha, Taya, Taylor, Terri, Theresa, Tonya
Traditional	Socio-cultural	Standards aligned beliefs about math	Tasha, Taya, Taylor, Terri, Theresa, Tonya
Traditional	Senses	Standards aligned beliefs about math	Taylor, Tasha, Taya, Theresa, Tonya
Traditional	Senses	Nonstandards aligned beliefs about math	Taya, Terri

From Table 11, it is apparent that the biggest difference comes from the fact that only four nontraditional participants discussed how socio-cultural factors influenced their beliefs about nonstandards aligned mathematics, while all six traditional participants discussed this link.

Similarly, for participants' beliefs about mathematics teaching, I created two additional models (See Figures 6 and 7) that detail the similarities and differences between the beliefs systems of the nontraditional and traditional participants.

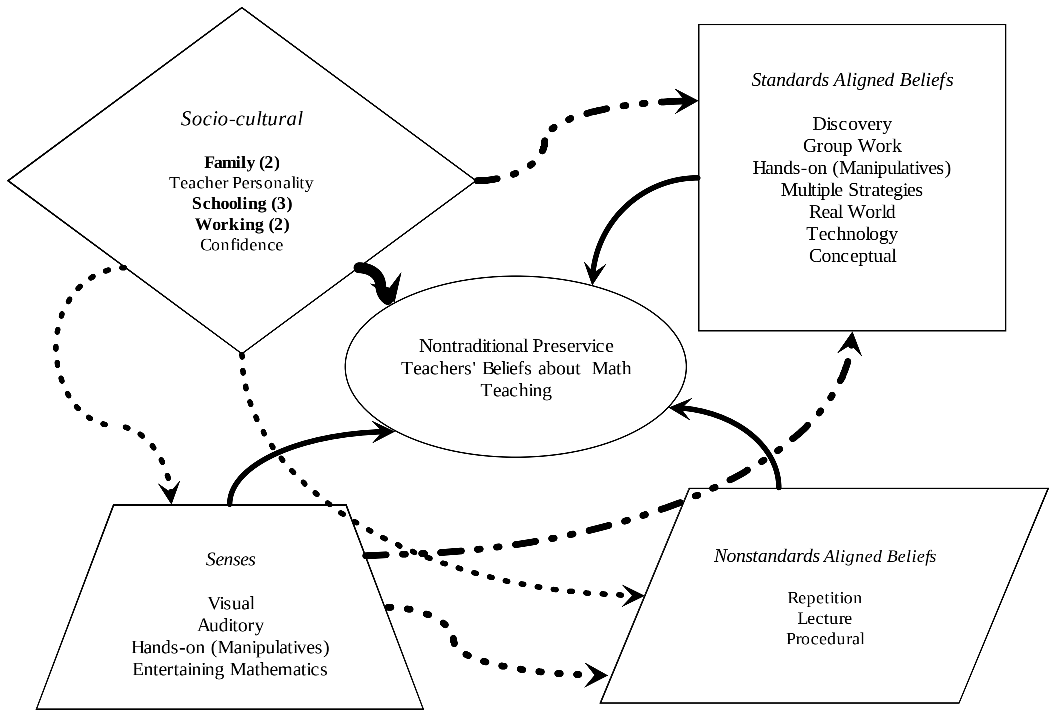

Figure 6. Model for nontraditional participants' beliefs about math teaching.

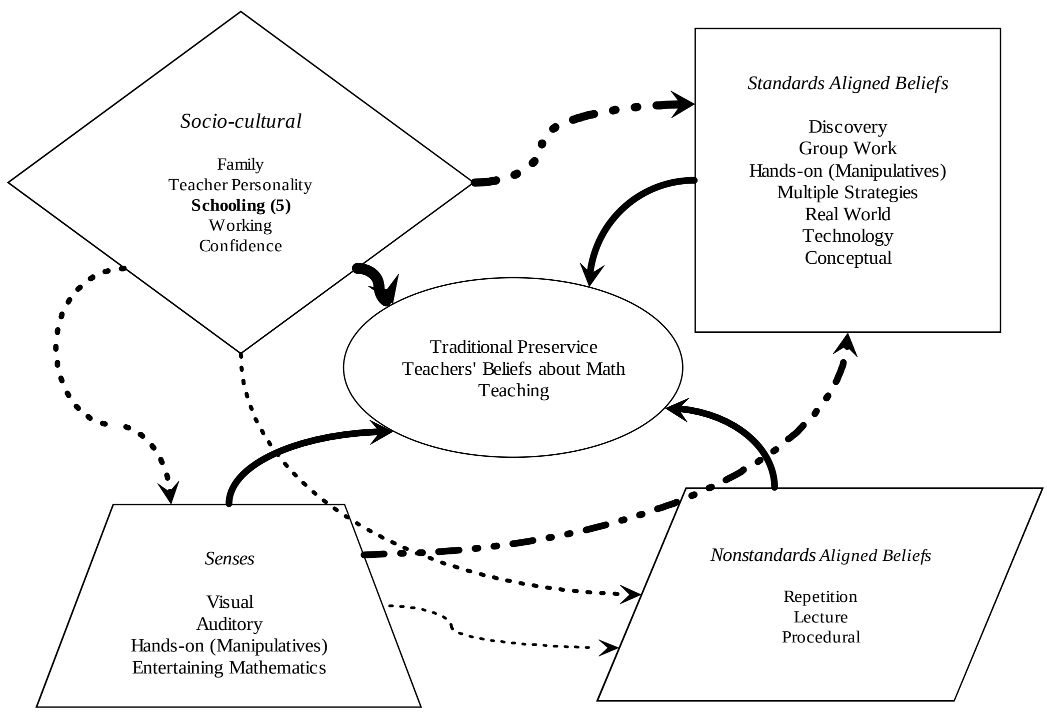

Figure 7. Model for traditional participants' beliefs about math teaching.

The models resemble the earlier ones I created about participants' beliefs about mathematics, except that I included more terms from the pilot study research under the themes, such as repetition, lecture, conceptual, procedural, auditory, group work, multiple strategies, and group work. As with the participants' beliefs about mathematics, there are two links of the same thickness, the link between *Socio-cultural* and *Senses* and the link between *Senses* and *Standards Aligned Beliefs about Math Teaching*. Two other links differ slightly, the link from *Socio-cultural* to *Nonstandards Aligned Beliefs about Math Teaching* and the link from *Socio-cultural* to *Standards Aligned Beliefs about Math Teaching*. The greatest variation comes from *Senses* to *Nonstandards Aligned Beliefs about Mathematics Teaching*. Again, I can examine my tally system (See Table 12) to determine the thickness. I analyzed transcription data and kept track of the number of participants who remarked about a particular link between codes. I specifically inspected remarks under the code words of *Beliefs about mathematics teaching* and *Entertaining mathematics*. Analysis of the comments under *Beliefs about mathematics teaching* provided me the bulk of my tally findings, but I also found that participants commented a substantial amount about their beliefs while discussing fun mathematics so I also included those codes in my analysis.

Table 12

Connections between Themes of Participants' Beliefs about Math Teaching

Participant	Initial theme	Theme impacted	Number of participants
Nontraditional	Socio-cultural	Senses	Nadine, Nancy, Nicolette, Nita
Nontraditional	Socio-cultural	Nonstandards aligned beliefs about math teaching	Nadine, Nancy, Natalya, Nita
Nontraditional	Socio-cultural	Standards aligned beliefs about math teaching	Nadine, Nancy Natalya, Nicolette, Norah
Nontraditional	Senses	Standards aligned beliefs about math teaching	Nadine, Nancy, Natalya, Nita, Norah
Nontraditional	Senses	Nonstandards aligned beliefs about math teaching	Nadine, Nancy, Nita, Nicolette
Traditional	Socio-cultural	Senses	Tasha, Taya, Theresa, Tonya
Traditional	Socio-cultural	Nonstandards aligned beliefs about math	Tasha, Theresa, Tonya
Traditional	Socio-cultural	Standards aligned beliefs about math	Tasha, Taya, Taylor, Terri, Theresa, Tonya
Traditional	Senses	Standards aligned beliefs about math teaching	Tasha, Taya, Taylor, Terri, Theresa
Traditional	Senses	Nonstandards aligned beliefs about math teaching	Taya, Terri

The table indicates the biggest difference in nontraditional and traditional participants' links among themes comes from the fact that four nontraditional participants and only two traditional participants discussed the link between *Senses* and *Nonstandards Aligned Beliefs about Mathematics Teaching*.

Summary of Models.

All figures indicate that the Math 100/200/300 sequence, otherwise known as S*chooling*, affected several nontraditional and traditional preservice elementary teachers' values about mathematics and mathematics teaching. Even though the mathematics sequence of classes affected both groups of participants, the nontraditional participants also explicitly stated other influences besides Math 100/200/300, such as family and K-12 work experience. Nita even mentioned the idea of maturity, which she felt played a role in her views.

The figures also detail the six main themes: *Standards Aligned Beliefs about Mathematics*, *Nonstandards Aligned Beliefs about Mathematics*, *Standards Aligned Beliefs about Mathematics Teaching*, *Nonstandards Aligned Beliefs about Mathematics Teaching*, *Senses*, and *Socio-cultural*. Certain ideas persisted for all the preservice teachers, including ideas about strategies and fun mathematics. All preservice participants expressed an interest to teach mathematics using different strategies, including hands-on manipulatives. Most of the preservice teachers (11 of them) felt mathematics should be entertaining. The link between *Socio-cultural* and *Senses* is the same thickness for nontraditional and traditional participants in relation to their beliefs about mathematics teaching. Even though the link is the same thickness, the reasoning for creating the link is different; the nontraditional participants discussed their family, while the traditional participants commented about their past teachers. In the final chapter, I will discuss the connections between these findings and the literature, as well as implications of my research for research and teaching.

CHAPTER V

DISCUSSION

Introduction

In this chapter, I summarize my study, detail limitations of my research, discuss connections between my research and related literature, and provide implications of my research to teaching, policy, and research. I also offer ideas for future research into the mathematics preparation of nontraditional and traditional preservice teachers.

Summary of Research

I conducted a qualitative case study of 12 preservice elementary teachers (6 nontraditional and 6 traditional) to analyze the following research questions:

Q1 What is the nature of nontraditional and traditional preservice elementary teachers' experiences with and/or perceptions about mathematics and the teaching of mathematics?

Q2 How do nontraditional preservice elementary teachers perceive "mathematics" in terms of standards aligned and nonstandards aligned mathematics in comparison to traditional preservice elementary teachers?

Q3 How do nontraditional and traditional preservice elementary teachers' opinions about "mathematics" evolve (as collective traditional and nontraditional groups and as individuals) throughout a semester long mathematics content course designed to teach preservice elementary teachers in a conceptual format?

Q4 How do nontraditional preservice elementary teachers perceive "mathematics teaching" in terms of standards aligned and nonstandards aligned teaching in comparison to traditional preservice elementary teachers?

> Q5 How do nontraditional and traditional preservice elementary teachers' opinions about "mathematics teaching" evolve (as collective traditional and nontraditional groups and as individuals) throughout a semester long mathematics content course designed to teach preservice elementary teachers in a conceptual format?

Nontraditional participants consisted of preservice elementary teachers who ranged in age from 31 to 53, while traditional participants consisted of preservice teachers who ranged in age from 18 to 20. I selected four participants from each of Math 100, Math 200, and Math 300 classes. The instructors of each of the preservice teachers had taught their respective course in previous semesters and espoused to teaching philosophies that embraced standards and nonstandards based beliefs about mathematics teaching. I interviewed each preservice participant two times (approximately 45 minute interviews) and each instructor two times (approximately 30 minute interviews). In addition to interviews, I observed the participants' in their respective Math 100/200/300 classes twice a month. This amounted to observing two 50 minutes classes per month in Math 100/200, as well as observing two 75 minutes classes per month in Math 300.

From the interview transcripts, I coded the data using *NVivo* and utilized content analysis (Merriam, 1998) to look for overarching themes in the data. The following discussion provides an examination of the findings within the context of related literature.

Discussion of Findings

I found six themes in the data: *Standards Aligned Beliefs about Mathematics*, *Nonstandards Aligned Beliefs about Mathematics*, *Standards Aligned Beliefs about Mathematics Teaching*, *Nonstandards Aligned Beliefs about Mathematics Teaching*, *Senses*, and *Socio-cultural*. In the following paragraphs, I explore these themes and the links between my findings and the literature.

Beliefs about Mathematics.

In my previous pilot studies, I struggled with teasing out the preservice elementary teachers' views about mathematics and mathematics teaching. For my dissertation work, I utilized some of Raymond's (1997) questions, where she distinguished between mathematics and mathematics teaching through a series of questions. My findings showed that two nontraditional participants (Nicolette and Nadine) held nonstandard aligned views about mathematics; three nontraditional participants (Nancy, Norah, and Natalya) held standards aligned; and one nontraditional participant (Nita) held a combination of nonstandards and standards aligned views of mathematics. All traditional preservice teachers believed mathematics to be a combination of standards and nonstandards aligned ideas. Since all three Math 100/200/300 instructors espoused to teaching philosophies that incorporated procedural and conceptual values, the instructor views might have influenced the traditional participants. This influence may be stronger for traditional participants than nontraditional participants because older participants may know what they believe and have held their belief systems for a substantial period of time, while younger people may still be trying to seek out what they believe. This finding is supported by Pajares' (1992) article, where he discussed how individual's recently formed beliefs are more likely to change than their long-held beliefs.

As seen in my models, both groups of participants showed similar links between *Socio-cultural* and *Standards Aligned Beliefs*, *Senses* and *Standards Aligned Beliefs*, *Senses* and *Nonstandards Aligned Beliefs*, and *Socio-cultural* and *Senses* with their beliefs about mathematics. Most participants made links between socio-cultural ideas and nonstandards and standards aligned mathematics, which could be explained through their exposure to Math 100/200/300 instructors who valued both procedural and conceptual knowledge. From classroom observations, I watched how all three instructors taught both types of mathematics to the preservice teachers and explained the value of both to their understanding of the subject.

The biggest difference came from the link from *Socio-cultural* to *Nonstandards Aligned Beliefs about Math*, where four nontraditional participants and all six traditional participants mentioned this connection. The two nontraditional participants who did not create this link via interviews were Natalya and Nicolette. Even though I categorized Nicolette's views about mathematics as nonstandards aligned, she never discussed socio-cultural factors as the reasons for her views. On the other hand, I classified Natayla's views about mathematics as standards aligned, which fits well with her comments about mathematics that center around standards aligned concepts that she learned through the Math 100/200/300 sequence.

Another finding from my models that is not surprising to me was the weak link in both models from *Senses* to *Nonstandards Aligned Beliefs about Math*. It does not seem like a natural link to connect fun mathematics to procedural mathematics concepts, but Nita, Taya, and Terri did. I classified all three participants as combination beliefs about mathematics, which could explain why they feel that procedural mathematics could be entertaining. Taya and Terri, the two youngest participants, might alter their opinions about fun mathematics once they have taken all three preservice elementary mathematics content courses.

Through the course of time, whether it be the Math 100/200/300 sequence or some other time, several of the preservice teachers in interviews gave socio-cultural reasons for a change in their mathematics beliefs. Two nontraditional and five traditional participants believed the Math 100/200/300 sequence transformed their opinions about mathematics. These findings support Steele's (1994) mixed methods work, where she also discovered that several preservice elementary and middle school teachers changed their opinions about mathematics during a semester long mathematics methods course. Results from the mathematics beliefs surveys she gave at the beginning and the end of the semester showed that 18 out of the 19 preservice participants felt repeated practice of mathematics facts was unnecessary for student understanding and children could learn mathematics on their own. I also asked participants about giving students repetitious work, but most participants did not have strong opinions either way, which could be because they were not enrolled in a mathematics methods course and had not thought much about the idea of drill practice.

Similar to my research, some of the participants in Steele's (1994) study mentioned manipulatives and group work as helping them to understand mathematics. Steele interviewed 5 of the 19 preservice teachers to gain further insight into their beliefs systems. All five participants commented about times during the course, such as working with division with fractions, in which their thoughts about mathematics changed to more discovery oriented views. In my work, not all participants felt the Math 100/200/300 changed their opinions about mathematics, which might be due to the bigger sampling I took or the fact that all of Steele's participants were students in her class and felt they should respond in a certain way.

When I investigated participant confidence in doing mathematics for both groups of preservice teachers, I did not find a difference. Both groups' confidence levels averaged out to roughly 7 out of 10, where 10 represents very confident in doing mathematics. This finding is encouraging for instructors, like me, of preservice teachers who often hear preservice elementary teachers complain that they cannot do mathematics; in actuality, many of the participants in my dissertation study felt like they can do mathematics.

Beliefs about Mathematics Teaching.

Similar to my work with beliefs about mathematics, I utilized some of Raymond's (1997) questions in relation to mathematics teaching, which I also used during my second pilot study. I found that two nontraditional (Nancy and Nadine) participants and one traditional (Terri) participant held nonstandards aligned opinions about mathematics teaching; two traditional participants (Theresa and Tonya) held standards based beliefs about mathematics teaching; and four nontraditional (Nicolette, Norah, Nita, and Natayla) as well as three traditional (Tasha, Taylor, and Taya) participants held combination beliefs about mathematics teaching. These multiple opinions about mathematics teaching support the work of Crespo (2003), Eisenhart et al. (1993), Raymond (1997), Thompson (1984), and Vacc and Bright (1999). All of these articles examine preservice or inservice teachers' mathematics teaching, with teaching strategies that ranged from conceptual to procedural. A main difference between these researchers' work and mine consisted of the fact that they all included student teaching or field experience in their investigations, which could be a future research extension to my work.

Since all of the instructors of the preservice teachers believed in the importance of both conceptual and procedural learning, it is not surprising that most of the preservice teachers also felt this way. This finding supports the mixed methods work of Philippou and Christou (1998) and how preservice teacher programs can influence teacher attitudes about mathematics. Unlike my work with mathematics content-only courses, Philippou and Christou conducted their research with preservice teachers who took two mathematics classes about mathematics history and one about mathematics methods.

As seen from my models, both groups of participants showed similar links between *Socio-cultural* and *Senses*, *Socio-cultural* and *Nonstandards Aligned Beliefs*, *Socio-cultural* and *Standards Aligned Beliefs*, *Senses* and *Standards Aligned Beliefs*, *Senses* and *Nonstandards Aligned Beliefs* with their beliefs about mathematics teaching. Even though the Math 100/200/300 sequence does not specifically contain pedagogy as part of the course, its influence can be seen in several of the preservice teachers' responses. The majority of both groups of participants discussed how socio-cultural factors, including the Math 100/200/300 sequence, influenced their thoughts about standards and nonstandards aligned mathematics teaching. Similar to the mathematics beliefs models, I feel this connection could be due to the influence of the instructors' dual importance they place on their own teaching of both procedural and conceptual learning. Thus, the preservice teachers who have not taken a mathematics methods course utilize their recent experiences with mathematics teaching, which includes combination beliefs. This fact also coincides with several of the preservice teacher comments during interviews, where they mentioned they had never thought about how they would teach their future mathematics class. After taking a mathematics methods course, they may have had more time to contemplate how they plan to teach.

The biggest difference between the two mathematics teaching beliefs models was the same major difference found in the mathematics beliefs model, the link from *Senses* to *Nonstandards Aligned Beliefs about Math*. With the beliefs about mathematics teaching model, four nontraditional (Nadine, Nancy, Nita, and Nicolette) and two traditional participants (Taya and Terri) made this connection. The link could be due to the fact many of these participants held mathematics and/or mathematics teaching beliefs that were nonstandards aligned. Thus, they felt that fun mathematics could include such ideas as repetition, lecture, and procedures. On the other hand, Norah, Natalya, Theresa, and Tonya, the four participants with standards aligned mathematics or mathematic teaching beliefs, did not make this link.

Analogous to the findings with beliefs about mathematics, several preservice teachers (three nontraditional and five traditional) expressed changes in beliefs about mathematics teaching due to the Math 100/200/300 sequence. These findings are also supported by the work of Steele (1994) and Swars, Smith, Smith, and Hart (2009), which I will detail in the following paragraphs.

After taking a semester-long preservice mathematics methods course, Steele (1994) found that the preservice teachers in her class believed mathematics teaching now included such concepts as modeling and discovery. One participant in Steele's study even used the phrase "tour guide" (p. 21) to describe his/her role in teaching students. Also, similar to my work, participants discussed at the beginning of the semester, their experiences in mathematics classes and their plans for teaching mathematics, where several of the comments contained nonstandards aligned viewpoints. Then, at the end of the semester, these views about teaching evolved into more standards aligned sentiments.

Swars et al. (2009) and my work contain related findings about how preservice teachers' opinions about mathematics teaching changed due to a university teacher program. Swars et al. conducted a mixed methods longitudinal study over four semesters, including preservice teacher experiences in the schools and student teaching, with a cohort of 24 preservice elementary teachers. They utilized four different survey instruments given at varying times throughout the courses, as well as interviewed 6 of the 24 participants. Even though their program included such aspects as methods courses and experiences in the public school and ours included only instruction over mathematics content, both created changes in several preservice teacher opinions about mathematics teaching.

Along with beliefs about mathematics, the preservice teachers also discussed their confidence levels for teaching mathematics. In contrast to the work done by Swars et al. (2009) that indicated that preservice elementary teachers' self efficacy about teaching mathematics increased as they progressed in their mathematics methods courses, I found support, detailed in chapter 4, to suggest that participants' self efficacy about teaching mathematics increased only for nontraditional participants for teaching K-3 grade levels. This finding may be because the preservice teachers have only taken courses in the Math 100/200/300 sequence, which are mathematics content courses for preservice elementary teachers. Once the preservice teachers take their mathematics methods course, they may have more confidence in teaching mathematics.

In addition, the nontraditional participants as a whole, as stated in chapter 4, reported a confidence rating of 9.6 out of 10 for teaching mathematics at the K-3 grade level and 7.3 out of 10 for teaching mathematics at the 4-6 grade level. The traditional participants' confidence ratings were 7.7 out of 10 for teaching mathematics at the K-3 grade level and 7.0 out of 10 for teaching mathematics at the 4-6 grade level. Thus, it seems that age, not Math 100/200/300 course, may influence self efficacy for teaching mathematics. A possible reason for the confidence could be the fact that 5 out of 6 of the nontraditional preservice teachers were parents and most of them helped their children with mathematics. Nontraditional participants would often comment about how they enjoyed learning the same mathematics concepts their children were learning. Nicolette even discussed how she would take extra activities home to her children to work on, since they were learning the same material. The other nontraditional participant, Norah, aided her niece and nephew in mathematics. None of the traditional participants mentioned tutoring of family members, which could lead them to not feel as confident with mathematics, since they do not regularly help elementary-aged children in mathematics. A further explanation for the nontraditional participants' higher confidence levels in teaching mathematics could be due to the fact that most of the nontraditional participants had some experience working in the K-12 school systems, such as being a volunteer (Nicolette), substitute teacher (Nancy), computer lab helper (Norah), and teacher's aide (Nadine).

An additional finding related to the participants' confidence in mathematics teaching stems from social cognitive theory. When teachers possess low self efficacy in providing instruction, they "may avoid planning activities they believe exceed their capabilities, not persist with students having difficulties, expend little effort to find materials, and not reteach in ways students might understand better" (Schunk, 2004, p.119). Thus, students who may have traditional preservice teachers as instructors, who may hold lower self efficacy in teaching mathematics, may have a higher chance of receiving less demanding instruction than those students who have nontraditional preservice teachers as instructors.

Another contrast between my work and related research (Swars et al., 2009) is the following:

> Prospective teachers who had stronger beliefs in their abilities to teach mathematics effectively generally had more cognitively oriented beliefs toward the teaching and learning of mathematics. (p. 58)

With my research, the participants with the highest confidence rankings in their ability to teach mathematics (Norah, Nita, Natalya, and Taya) did not possess standards aligned beliefs about mathematics teaching. Tonya and Theresa held standards aligned beliefs about mathematics teaching but ranked 7 and 10 (out of 12) in confidence levels about teaching mathematics. This finding may come from the fact that Tonya and Theresa struggle with doing mathematics but both see that learning conceptually is critical to understanding mathematics. They both value teaching mathematics for comprehension, rather than doing senseless steps, even though they do have some confidence issues surrounding teaching mathematics. This discrepancy could also be because the participants in my research had not student taught, unlike the preservice teachers in the work of Swars et al. By going through the student teaching process, the self efficacy levels of the teachers could change, especially the traditional participants who have had little experience with K-12 schools, which could explain the inconsistencies in these findings.

All participants believed in the importance of teaching mathematics in ways that incorporate all of the senses. Similarly, all of the participants described the idea that mathematics should be fun, which supports Collopy's (2003) and Gellert's (1998; 2000) research on how some preservice elementary teachers expressed views that entertaining mathematics is an important aspect in their future teaching of mathematics. While Collopy's work contained interviews and observations, Gellert's work consisted of student journals, which both differed from my data collection of preservice teacher interviews, instructor interviews, and classroom observations.

Additional Findings about Connections between Beliefs and Grades.

A combined finding about the mathematics beliefs and the mathematics teaching belief classifications came from Nancy's categorizations. She is the only participant with a classification of standards aligned and a classification of nonstandards aligned. Specifically, Nancy held standards aligned beliefs about mathematics and nonstandards aligned beliefs about mathematics teaching. This discrepancy might come from the fact that Nancy willingly learned conceptually, but she still explained her future mathematics teaching procedurally. When I asked Dr. Flores about Nancy's proposed procedural teaching techniques Nancy discussed during our interviews, Dr. Flores did not act surprised because Nancy's mathematics background from childhood consisted of only procedural learning. Nancy felt more comfortable with her years of procedural learning, as opposed to her newly acquired conceptual learning.

Though Nancy believed mathematics consisted of standards aligned beliefs, she and Norah obtained the lowest grades, C's, out of the 12 participants. Both were enrolled in Math 100. Nancy's low grade may be due to her attachment to procedural methods. She openly tried to learn new conceptual strategies but struggled with the material. During an interview, Dr. Flores commented on Nancy's difficulties with her class because Nancy learned procedurally growing up and struggled with the conceptual material.

> I talked to Nancy today at the end of class about it. She struggles with it. It is very different, obviously, for her age. I think she is even older than I am so it has been a while for her. It has been a long time since she has been in school. She was very traditionally and very procedurally taught, and it is a stretch. My feeling about Nancy is she is not fighting the stretch like some students really don't like you presenting math differently than their comfort zone.

On the other hand, Norah embraced the standards aligned methods, but Dr. Flores replied in an interview that she felt Norah's mathematics background was not strong and struggled with the material. Dr. Flores further commented that her difficulties were different from Nancy's.

> Norah definitely struggles. I wouldn't say she struggles the same way as Nancy. She struggles in a good way. I'm impressed with Norah because she'll stay after it on a regular basis and ask me a question…Her background, I think, is weak. I think it is weaker in some ways than someone like Nancy, but I also think she is much more oriented towards understanding things conceptually.

Another possible explanation for their low grades could be a finding from Becker (2001). He utilized data from the *Teaching*, *Learning*, and *Computing* (TLC) survey of over 4,000 teachers of Grades 4-12 nationwide and discovered that, "Teachers with the most constructivist teaching philosophies are stronger users of computers" (p. 11). My communications with Nancy and Norah consisted of either phone calls or meetings in person, since neither used email; they were the only participants I have ever had that I could not correspond to via computer. Since neither Nancy nor Norah utilized email, they might have struggled more with the standards aligned material in the Math 100/200/300 sequence.

Limitations of My Research

Even though I found several areas of discussion and links to literature, my study includes three main limitations that I think are important to interpreting my results. They include bias, participant sampling, and interview protocol.

I gathered, coded, and created tables of themes of all of the data for the research, which could lead to bias in my analysis of the data (Patton, 2002). To mitigate potential bias, I utilized member checking, expert checking, triangulation of data, and peer debriefing (Schwandt, 2001). After I transcribed the interviews, I sent them to participants for revisions and/or additions to their original answers. The participants could add to their answers if they wish or alter any responses they felt needed changes for whatever reason. My advisor, as well as Raymond, examined certain interview questions and provided feedback. In addition, my advisor acted as an expert check and fellow graduate students acted as peer checks for any other questions that arose.

Triangulation of data occurred through classroom observations, preservice interviews, and Math 100/200/300 instructor interviews. During classroom observations, I noted participant behavior and asked the participants (both instructor and preservice teachers) about my observations. Preservice teachers commented about their own thought processes and beliefs, as well as their instructors' teaching philosophy and other participants who were enrolled in their class and whom they worked with in the class. As a third line of triangulation, the Math 100/200/300 instructors detailed their opinions about my classroom observations and preservice teacher comments.

The second limitation of my study included the fact that all my participants, instructors and preservice teachers, volunteered (Patton, 2002). These participants may not be characteristic of all the preservice teachers and instructors at the university under research. These types of participants may be more motivated and higher achieving than the typical preservice teacher and Math 100/200/300 instructor.

Lastly, a third limitation of my research is my interview technique. I used a semi-structured interview practice described by Merriam (1998). Through this interview procedure, I oftentimes asked participants multiple follow-up questions. These types of questions could be leading without me consciously knowing. Thus, participants might answer a certain way because they feel I led them to answer a particular way. Though my research contains some limitations, I found possible benefits to teaching and research that come from my work, which I explore in the following paragraphs.

Implications for Teaching and Policy

In the discussion that follows, I detail three ways in which teachers can incorporate findings from my research into their classroom instructions and one way to incorporate into institutional policy. These include offering activities involving family members as classroom protocol, providing additional tutoring and cohort support for nontraditional preservice teachers, and giving traditional preservice teachers extra support and early experiences with teaching children to aid possible self efficacy concerns about teaching mathematics.

The first implication for teaching includes ways for mathematics instructors of preservice elementary teachers to incorporate family connections in the curricula of their courses. These links may make the class more relevant to preservice teachers, which in turn may lead to higher achievement rates of preservice teachers, especially those with children. For example, instructors may include group projects where preservice teachers are required to teach family members of group members certain lessons that preservice teachers develop. Preservice teachers can also teach a lesson to a family member of a group member and reflect on the strengths and weaknesses of their delivery and ways to improve their own teaching skills and mathematical knowledge. Since nontraditional preservice teachers in my study often talked about family members as justification for their beliefs, all preservice teachers may benefit from this type of activity.

I also found evidence that the Math 100/200/300 sequence influenced traditional preservice teachers more than nontraditional preservice teachers in terms of their beliefs about mathematics and mathematics teaching. In my experience of teaching preservice teachers, I found it sometimes difficult to hold class discussions with preservice teachers who would not entertain the idea of differing opinions to their own. Therefore, these findings could help teachers, like me, to be reminded of the fact that preservice teachers, especially nontraditional preservice teachers, may not agree with the reform ways of teaching mathematics. These nontraditional preservice teachers may have had more experiences working with children than traditional preservice teachers have had. Based on those experiences, the nontraditional preservice teachers may have already formed their opinions about mathematics and mathematics teaching. This awareness could help ease reform instructors' concern about not feeling adequate as teachers because they have not affected the opinions of a particular group of preservice teachers.

I also found that nontraditional preservice teachers may have held higher self efficacy towards teaching elementary mathematics than traditional participants. Instructors could investigate this fact in their own classroom and let preservice teachers journal or discuss their concerns with mathematics teaching in hopes of abating some of the apparent fears and challenges that especially the younger preservice teachers may feel. Instructors could also provide activities that they feel could ease the preservice teachers' apprehension toward teaching mathematics. This could be done by allowing preservice elementary teachers to read and write about research conducted in this field and provide an open forum for discussion. Through reading about others' struggles and successes in the classroom, preservice teachers might raise their own self efficacy levels.

In addition, I found the nontraditional preservice teachers, Nadia (42), Nancy (53), Naomi (34), and Norah (34), obtained the worst grades in the Math 100/200/300 sequence. Instructors of these courses may find it useful to encourage these preservice teachers to seek additional support in the form of tutoring. Both of the oldest participants, Nancy and Nadia, discussed with me how they regularly sought help from the instructor or on-campus tutoring services. Some of the preservice teachers, such as Natalya and Nita, seemed to like to work with preservice teachers similar in age to them. Instructors could be cognizant of this fact and may take steps to have older preservice teachers work together some in class or through a few group assignments. This might include an institutional practice to have a cohort of nontraditional students working together to help build support systems with one another.

Implications for Future Research

By conducting this research, I discovered other avenues of exploration through quantitative, qualitative, and mixed methods that researchers may use for investigating traditional and nontraditional preservice teachers' preparation in elementary mathematics. In the following discussion, I describe six potential follow-up studies that include influences of family, Math 100/200/300 courses, entertaining mathematics, multiple teaching strategies, and inservice teaching.

I found a link between nontraditional preservice elementary teachers' beliefs about mathematics and mathematics teaching and family influence. A researcher may investigate this bond further by developing a preservice elementary teacher course, where preservice teachers include their family in their learning experience. A quantitative research question the researcher could study may be, "Does family-focused activities relate to higher preservice elementary teacher achievement?" The study would be a quantitative study of two classes, where participants would take a pre- and post-test over the content of the course; the treatment would be a series of participant developed lessons the participants would teach to family members over the content in the course. At the end of the semester, the preservice teachers would complete a post-test to see which group of preservice teachers had the highest percent gains in scores. These results would provide a quantitative component to my dissertation finding about the effects of family on preservice teachers.

A case study extension of the above study consists of the addition of participant interviews to answer the research question of how do family-focused projects influence preservice elementary teacher learning in a mathematics content course designed for preservice elementary teachers? A researcher could interview 8-12 preservice teachers, half with children and half without children. A difference may be found in these two groups of participants or even among preservice teachers with older versus younger children.

Findings also suggested that the Math 100/200/300 sequence influenced the traditional preservice teachers' opinions more than the nontraditional participants did. A quantitative research question to follow-up this finding is, "Does age influence preservice elementary teachers' transformation in their beliefs about mathematics and mathematics teaching in mathematics content courses designed for preservice elementary teachers?" This could be a longitudinal study that spans the three course sequence (Math 100-Math 300). At the beginning of each of the Math 100/200/300 semester classes, preservice teacher participants complete an attitudes survey with questions about their views on certain topics related to mathematics and mathematics teaching. The following are suggested sample questions with a scale of 1 to 5 with 1 being strongly disagree to 5 being strongly agree.

1. I feel confident teaching mathematics at the K-3 grade level.
2. I do not feel confident teaching mathematics at the 4-6 grade level.
3. I feel confident in doing simple mathematics (i.e., addition, subtraction, multiplication, and division).
4. I feel mathematics is surprising.
5. I do not feel mathematics problems can be solved using multiple strategies.

A researcher could see what belief changes occur after every semester and see if age does play a role in these beliefs.

Even though preservice teachers might have explained their teaching strategies using only one method, all of the participants expressed the belief that they would utilize multiple strategies in the classroom. A qualitative case study could include the research question what is the nature of preservice teacher instruction about (insert mathematics topic). This teaching idea resembles a project preservice teachers completed during my first pilot study collection. I selected topics for preservice teachers to teach, and they developed a lesson plan, activities, and homework. Then, they taught the lesson to their fellow classmates. This study would include classroom observations of the day the preservice teachers conducted their lesson, as well as pre- and post-interviews that centered on the lesson presentation.

My study supported Gellert's (1998; 2000) research about how some preservice teachers believed entertaining mathematics instead of substantial mathematics is the important aspect of a mathematics lesson. A quantitative study to confirm this finding is centered on the research question, "Does preservice elementary teachers sacrifice sound mathematical knowledge for entertainment benefits?" A researcher could provide preservice elementary teachers with a survey about mathematics teaching beliefs. Sample questions consist of the following with a scale from 1 to 5 with 1 being strongly disagree to 5 being strongly agree.

1. I would use a mathematics activity even if the mathematics content was lacking, if I felt the lesson was interesting.
2. Mathematics lessons should always be entertaining for the students.
3. Mathematics lessons cannot be fun and full of mathematics content.

As a connection to Raymond's (1997) work that provided much insight into my own research, a researcher could extend my dissertation work by following preservice elementary teachers into the classroom to see if there are any inconsistencies in their belief systems. Raymond found that Joanna's, Raymond's participant, "practice was more closely related to her beliefs about mathematics content than to her beliefs about mathematics pedagogy" (p. 550). Through a mixed methods study, a researcher could answer the research question what differences exist between teachers' beliefs about mathematics and/or mathematics teaching and their teaching practices?

Data collection could include surveys about beliefs about mathematics and mathematics teaching. The pre-survey would be prior to participants obtaining jobs in the K-12 schools, while the post-survey would be after a year of teaching. The researcher could also observe the participants and document changes in their teaching from their survey answers. In addition, the researcher could interview the participants to provide further information about their survey responses.

Through my dissertation and pilot study work, I have added to the literature findings about traditional and nontraditional preservice elementary teachers' perceptions about mathematics and mathematics teaching. Before my work, I had not seen any research about preservice elementary teachers and the examination of age differences on beliefs systems about mathematics and mathematics teaching. I have learned a substantial amount from this dissertation work, but there is still much more to examine and study. I am excited about the opportunity to take what I have learned in both teaching and future research into my own classroom to potentially benefit preservice teachers of all ages.

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APPENDIX A

PARTICIPANT INFORMATION SHEET

Participant Information Sheet

Name: ____

Age range (Circle): 18-20 21-24 25-29 30-35 above 35

If you have children, please list their ages: _______________________________

CONTACT INFORMATION:

Email Address: ____

Phone: ____

APPENDIX B

INFORMED CONSENT FOR PARTICIPATION IN RESEARCH

Informed Consent for Participation in Research

Project Title: A Case Study of Pre-service Elementary Teachers' Development in Reflecting: A Pilot Study
Lead Researcher: Ann Wheeler, Graduate Assistant, School of Mathematical Sciences;
Research Advisor: Dr. Hortensia Soto-Johnson, Ph.D, School of Mathematical Sciences;

We are interested in examining your development in thinking about mathematics and teaching mathematics. For example, we want to know how you would approach a certain mathematical topic with elementary students. In order to conduct this investigation, we request your permission to contact you about three possible interviews. The interviews will consist of a 30-minute meeting to discuss your current pre-service elementary mathematics course. This conversation will be audiotaped, but only the people listed above may listen to the contents of the audio. To further help maintain confidentiality, computer files of interviews will be created and names will be replaced by pseudonyms. Your name will not appear in any professional report of this research.

The risks of participation in the study are no greater than those associated with taking a college mathematics course. By participating in this study, mathematics educators may have a better understanding of the way traditional and non-traditional pre-service elementary teachers think about mathematics and mathematics teaching. Nonparticipation or withdrawal from the study will not affect your standing in the class. If you do choose to participate, you will not be identifiable in final report(s) about the study. Participation is voluntary. You may decide not to participate in this study and if you begin participation, you may still decide to stop and withdraw at any time. Your decision will be respected and will not result in loss of benefits to which you are otherwise entitled. Having read the above and having had an opportunity to ask any questions, please sign below if you would like to participate in this research. A copy of this form will be given to you to retain for future reference. If you have any concerns about your selection or treatment as a research participant, please contact the Sponsored Programs and Academics Research (omit contact information).

Thanks in advance for your participation.

Sincerely,

.______________________________ __________
Researcher's Signature Date

______________________________ __________
Research Advisor's Signature Date

______________________________ __________
Participant's Signature Date

Participant's Full Name (Please Print)

APPENDIX C

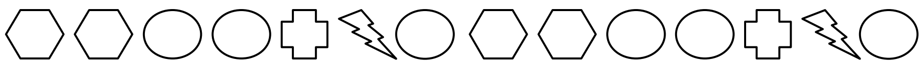
SAMPLE MATH 200 FINAL EXAM QUESTIONS

Sample Math 200 Final Exam Questions

1. Use a pan balance and algebra to solve the equation $4x + 9 = 3x + 13$. Be sure to draw the pan balance and to write the associated equation and equality-preserving action for each step. (6 points)

2. Give an example of a list of numbers where the IQR (interquartile range) would be 35. (10 points)

3. Consider below the repeating pattern. (6 points)



   a. What shape will be in the 592^{nd} position?

   b. How many circles will appear between and including the 1^{st} and the 2090^{th} position?

4. For the following scenarios, state whether the probability can occur or not. If it is possible, give an example of a scenario that would have the given probability. (12 points)
   a. 5/2
   b. 0%
   c. 1

5. Suppose you are tossing one six-sided die **twice** in a row. (6 points)
   a. What is the probability of getting a result that is divisible by 3 (i.e., a number that when divided by 3 has a remainder of 0)?

   b. What is the probability of not getting a number divisible by 3?

6. Write and solve a probability problem where P(A or B) = a % from 70%-90%. State the problem and then show work to find the answer. (10 points)

7. Create a set of 12 numbers in which the range is 32, the mode is 10, and the mean is 16. (6 points)

8. (5 points) Tim is considering opening a running shoe store in a local town. The town's population is 80,000. Before opening his store, Tim decides to conduct a survey to determine how many people in town are interested in running. He is mostly concerned about having the right proportions (percentages) of certain populations. Based on the above information, what kind of sampling should he conduct?

APPENDIX D

SAMPLE REFLECTION

Sample Reflection

A great way to find ideas to use in your classroom lessons is to search the Internet.

1. Find three websites that discuss slope.

2. Make a copy of the first page of each website to turn in with your reflection.

3. Write a 3-5 paragraph essay that addresses the following points:

    a. Summarize the information on the websites.
    b. Describe what you learned about teaching from the websites. (For example, were the websites good tutorials to learn about slope? Did they present the material in ways that were conducive to learning?)
    c. Describe whether and why you feel prepared to teach these ideas, and
    d. Describe what we have done in class (in your groups or as a whole) to reinforce the ideas presented in these websites.

APPENDIX E

TEACHING OPPORTUNITY: PROBABILITY

Teaching Opportunity: Probability
GROUP PROJECT (40 points):

For the last part of the semester, each table will have the opportunity to present a topic about probability. This project will let you have a chance to develop a lesson plan with an activity and homework. I will give each table a topic with a date to present, and we will spend Wednesday and Friday developing this lesson. You will be given an entire class period to teach your lesson.

Each lesson should include:

- A **typed lesson plan** that you will give me the day you teach, which at least should include lesson objectives (e.g., "Students will be able to…"), materials you will need to teach, and an instructional plan that details what you plan to accomplish during the class (10 points)
- A **class activity** (10 points)
- A **typed homework assignment** that I will grade (10 points)

You will also be graded (10 points) for presentation. I will grade based on how prepared I feel you are with the material and how you conduct the class.

Experimental probability (Nov. 5—Mon.)
Terms—sample space, event

Theoretical probability (Nov. 7—Wed.)
Concepts--complement of an event, probability of getting a 0 or a 1

Fair and unfair games (Nov. 9—Fri.)
Multi-stage probability (Nov. 12—Mon.)
With replacement—independent events
Tree diagrams

Multi-stage probability (Nov. 14—Wed.)
Without replacement—dependent events
Tree diagrams

Permutations (Nov. 16—Fri.)
Organized list, tree diagram, table

Combinations (Nov. 19—Mon.)
Organized list, tree diagram, table

The Monty Hall Problem and other applications (Nov. 26—Mon.)

APPENDIX F

SAMPLE QUESTIONS FROM MS. HERNANDEZ’S MATH 100 TESTS

Sample Questions from Ms. Hernandez's Math 100 Tests

1. (9 points) List the first 5 multiples of the following numbers, not including the given number.

    a. 12

    b. 19

2. (5 points) Explain using your knowledge of the models of division why it does not make sense to divide by zero.

3. (10 points) Find the Least Common Multiple and the Greatest Common Factor for the numbers 72 and 96.

4. (8 points) For the number 4,618.726935

    a. Round to the tenths place.

    b. Round to the tens place.

5. (9 points) Using the meaning of fractions, explain why $\frac{4}{6} = \frac{4*3}{6*3}$. DO NOT use multiplication by 1. Draw a picture to support your explanation.

6. (12 points) Compute the exact solutions to the following problems using mental math. Show how you grouped the numbers to show your thought process.

    a. $2 \times 21 \times 2 \times 25$

    b. $17 + 26 + 13 + 24$

7. (10 points) Evaluate each product. Write the answers in both exponential form and positional (expanded) form.

    a. $3^7 \times 3^2$

    b. $2^9 \div 2^4$

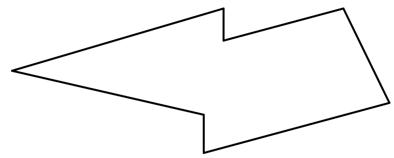
APPENDIX G

SAMPLE QUESTIONS FROM DR. RAMIREZ’S AND WHEELER’S FINAL EXAMS

Sample Questions from Dr. Ramirez's and Wheeler's Final Exams

Directions: Answer each of the following questions. Be sure to show your work where applicable in order to receive full credit. Form, grammar, and good taste will be graded. Be sure to complete all calculations.

1. What is the angle sum of the following polygon? Explain your reasoning. <u>Measuring the angles does not count!</u> **(4 pts)**



2. If one angle of a parallelogram measures 120° then what are the measures of the remaining three angles? Explain your answer. **(4 pts)**

3. Miguelito says, "It is possible to have an isosceles trapezoid with an angle measuring 70° and another measuring 100°." Is Miguelito correct? Why or why not? How do you reinforce his thinking if he is correct? How do you correct his misconceptions if he is incorrect? **(5 pts)**

4. If a circle has a circumference of 8π cm, then what is its area? **(5 pts)**

5. Suppose that the length of the shortest side of a 30-60-90 triangle has a length of 6, what is the length of the other two sides? **(6 pts)**

APPENDIX H

PILOT STUDY I: SAMPLE INTERVIEW I QUESTIONS

Pilot Study I: Sample Interview I Questions

1. How would you think you might foresee yourself teaching mean, median, and mode to your future class?

2. What does activity based learning mean to you?

3. What do you like about activity based learning?

4. What do you dislike about activity based learning?

5. Would you foresee yourself using activity based learning in your future classroom?

6. What does group learning mean to you?

7. What do you like about group learning?

8. What do you dislike about group learning?

9. Would you foresee yourself using group learning in your future classroom?

10. What does discovery based learning mean to you?

11. What do you like about discovery learning?

12. What do you dislike about discovery learning?

13. Would you foresee yourself using discovery based learning in your future classroom?

14. Based on your responses, is there a different way you would have approached teaching mean, median, and mode?

APPENDIX I

PILOT STUDY I AND II: SAMPLE INTERVIEW QUESTIONS

Pilot Study I and II: Sample Interview Questions

1. Thinking back to your elementary schooling, are there any mathematics teachers that stand out to you as good teachers? Explain why you would categorize them in this way.

2. Thinking back to your elementary schooling, are there any mathematics teachers that stand out to you as poor teachers? Explain why you would categorize them in this way.

3. Thinking back to your secondary schooling, are there any mathematics teachers that stand out to you as good teachers? Explain why you would categorize them in this way.

4. Thinking back to your secondary schooling, are there any mathematics teachers that stand out to you as poor teachers? Explain why you would categorize them in this way.

5. Are there any past mathematics teachers at the college level that stand out to you as good teachers? Explain why you would categorize them in this way.

6. Are there any past mathematics teachers at the college level that stand out to you as poor teachers? Explain why you would categorize them in this way

7. If you had an elementary student who refused to learn conceptually, how would you respond?

8. If you had an elementary student who refused to learn procedurally, how would you respond?

9. Describe how you would teach an elementary student about (insert Math 1/2/300 topic).

10. If an elementary student just could not understand the concept of (insert Math 1/2/300 topic) through conceptual learning, how would you further help him/her comprehend the concept?

11. What do you believe are the main goals, or objectives, you should get across to your elementary students during a mathematics lesson?

12. Do you feel that a central purpose of mathematics lessons should be the concept of fun? Explain.

APPENDIX J

PILOT STUDY I: SAMPLE INTERVIEW III QUESTIONS

Pilot Study I: Sample Interview III Questions

1. From previous interviews you said that you like group work, to teach 50% conceptual and 50% procedural, real world problems, going vague to specific with questions, have students to put answers on the board, have fun lessons, and have students to not be afraid to ask questions. Would you still agree with these statements?

2. Looking back on the lesson you helped with teaching on theoretical probability, do you feel you included

    a. group work? Explain briefly.
    b. 50% conceptual and 50% procedural learning? Explain briefly.
    c. real world problems? Explain briefly.
    d. asking vague questions(with no context) to specific questions (that talked about scenarios)? Explain briefly.
    e. have students work on the board? Explain briefly.
    f. had a fun lesson? Explain briefly.
    g. have students not afraid to ask questions? Explain briefly.

3. If you did not include some of the above, why did you not?

4. If you taught this lesson by yourself, would you have done anything differently? Explain.

APPENDIX K

PILOT STUDY II: SAMPLE INTERVIEW II QUESTIONS

Pilot Study II: Sample Interview II Questions

Demographic information
IDLA emphasis:
Age:
From the first set of interviews, certain concepts kept coming up from my 8 interviews. For the first part of this interview, I will state certain topics, and I would like you to tell me in what ways the terms have meaning for you and your future teaching.

1. What does conceptual learning mean to you?
    a. Have you experienced conceptual learning? How do you know?
    b. If so, in what ways?
2. What does procedural learning mean to you?
    a. Have you experienced procedural learning? How do you know?
    b. If so, in what ways?
3. How, if any, will lecture play a part in your future teaching of mathematics?
4. Do you foresee using group learning in your future teaching of mathematics?
    a. In what ways?
    b. Same ability versus different ability grouping
5. What does discovery learning mean to you?
    a. Have you experienced discovery learning? How do you know?
    b. If so, in what ways?
6. How, if any, will real world problems play a part in your future teaching of mathematics?
    a. Why?
7. Is hands-on learning important to you as a student?
    a. As a teacher?
8. In what ways, if any, do manipulatives play in a mathematics classroom?
9. Define fun in respect to mathematics?
    a. Should mathematics be fun? Why?
10. What role do you feel a teacher's personality plays in teaching mathematics?
11. Do you believe in using multiple ways to teach a concept?
12. How much should teacher help students in solving problems?
13. Is teacher organization important to you?
    a. In what ways?
14. Should students always be able to visualize mathematics?
15. How do you feel about teacher and student presentations in a mathematics classroom?
16. How do you envision teaching your future classroom to ensure that your students are learning conceptually?
17. How, if any, would you teach mathematical concepts differently K-3 versus 4-6?
18. For the last part of the interview, I want to know how you might address the following concept to your future elementary student.
    MATH 100—addition of 2- and 3-digit numbers (23 + 199)
    MATH 300—rigid motions of the plane (rotation, reflection, and translation)

APPENDIX L

SUMMARY OF CODE WORDS AND USAGES

Summary of Code Words and Usages

Code Words	Quotes
Auditory	The rhymes help me remember things (hit flow get….)
Participants mentioned teaching strategies involving auditory (hearing).	Bought a CD—about adding (singing)
Connections	Applying different subjects
Participants commented about relating mathematics to other disciplines, such as science and history.	Maybe write about it. Can relate to science- Ex., butterfly lesson (symmetry and science) For example, if I do the ocean—and pull everything into that so they are learning all these different subjects through one main theme. I remember in 2nd grade , we had an old west theme which was so cool. We did a play with the literature. It was like the western movement and learning history. It was really cool because we were incorporating everything we were doing in two weeks on intense studying. We had a big dinner. It was something like a pilgrimage and all our parents came and we put on a play. We made these books. It was cool because we got to experience the culture. It was really neat.
Discovery Learning Participants talked specifically about discovery learning, which often included discussions about the meaning of conceptual learning.	He would be like, okay figure this out. He really hadn't taught us anything to figure it out with. I know experimenting with some ideas is good, but in that class we didn't know anything to start off with so that was really hard. I like discovery learning because it helps you remember the material better because you figured it out by yourself. It is more than the teacher talking about the concept because you won't remember it (teacher lecture) as much as discovery learning To some extent, I would use discovery learning (in my teaching). (Ex., like filling the shape to find the volume) Some topics would be better for discovery learning I would probably use discovery learning it would just be hard to figure out when and what.

Entertaining Mathematics Participants discussed mathematics and/or mathematics teaching as entertaining or fun.	I think that is when we started learning algebra, and she made it really fun by me being able to understand it. When I understand the math, it is fun. I want to make math fun. Growing up, everybody said that they hate math and are horrible at math. I think I had a math teacher one time that was too into making math class fun that we didn't learn anything. Fun is activities where they learn something Fun parts need to be brought into a math class because a lot of people don't like math and activities are one way of doing that Algebra is fun, but other people don't think so…
Family Participants commented on their family, such as siblings, parents, and children.	I was talking to my dad about this the other day. We were talking about how boys usually weren't interested in math, and he was saying about how you could use football stats to figure out how fast the pass is and to convert into different units. What is cool is when they start applying that outside of school with their family because I have a 7-year-old sister and she will come home and be like it's probably snowed 7 inches right now and that is this many centimeters. She applies everything she is learning and it is really neat to see that. And she is like, "I learned the metric system." It is very interesting to see that in a second grader After have son—put more effort in, don't know teacher style or attitude change Have 3-yr-old—can't spend hrs working on a problem that is wrong, and spend another lengthy amount on time 4 year old son—totally sees how blocks help (can see with child and he uses blocks)
Future Teaching Participants discussed their future teaching methods.	I would have a set amount of time to go over the shapes everyday—maybe flash cards. I would do hands-on…Maybe not blocks because blocks are 3-D and if you are just learning the shapes…Maybe have them come up and draw a square on the board or cut a square out of their papers

	For 4-6—If I made flash cards, I would make them more complex by adding in equilateral and isosceles. I don't remember which grade that is when you do those but would in properties. I would have students come up with properties on their own after going over basic stuff. I think when you are first starting a concept, you can lecture about it for a little bit. Lecture is such a funny word when thinking about elementary school. I think it would be cool to like how you have a glass of water (soda) and see how much volume they are putting into themselves. That's kind of hard. (hands-on)
Group Work Participants mentioned ideas about group work or group learning.	They definitely need to so maybe I would group them or partner them with a friend in the class that was interested and that cared. I think it would be better to talk to them one on one or with a few kids because for me learning best is me, a few other kids, and the teacher. That's just the easiest way. You're not afraid to talk if you have questions and you're not intimidated to talk to the teacher because you're not alone. I think I would structure my class with groups, maybe not every day. It would depend on what we are learning or doing, but I would definitely incorporate it into my classroom. My favorite class is 5^{th} grade. We all had are separate desks, which were grouped together. That worked really well because it wasn't like rows. I liked working with the 5 people and getting to know the people. I might do that grouping in my class It depends on what we are learning whether students will be in groups. I feel there are some things you need to do on your own or you won't learn it, but there are other activities that are better as a group and learn different methods.
Hands-on = (Manipulatives) Participants mentioned ideas about hands-on or manipulative	If I try to push another way of teaching, it may not make sense in their head, when blocks could. 4-6—I would push how you need to actually learn the procedures then just looking at blocks because by then your mind has developed further and more stuff makes sense I would do hands-on.

use (three dimensional objects). These comments often included discussions about conceptual learning.	Maybe not blocks because blocks are 3-D and if you are just learning the shapes. Maybe have them come up and draw a square on the board or cut a square out of their papers Blocks would be good I think, but that is going into 3-D shapes, which is a sphere not a circle, which is a big mistake that people do confuse those, which is odd though. 4-6—If I made flash cards, I would make them more complex by adding in equilateral and isosceles. Maybe some hands-on, not as much The Feeley Box—I really liked that idea because you do need to know the concepts of the shapes. That could be like for the higher level because you do need to know the properties of the shapes. (4-6). Hands-on is good for shapes but I don't know how you would do hands-on for algebra Manipulatives fall more under conceptual because that is more of a hands-on type of thing and that's more of concepts.
Lecture Participants discussed the use of lecture in the classroom.	He (teacher) came everyday with his notes and wrote his notes on the board and explained as he went. I think when you are first starting a concept, you can lecture about it for a little bit. Lecture is such a funny word when thinking about elementary school. I would lecture about 10 minutes because they don't have a long attention span.
Mental Mathematics Participants talked about the idea of mental mathematics or estimation in their heads.	Then, go into the idea of making 199 into 200 and 23 into 22 and then make the problem much easier (using mental math)…So, add ones, carry the one over (first way to teach) Then, show the mental math They could also round 200 + 22. It is the mental math and nicer numbers I think mathematics is very visual but also when it comes to real world issues when you are using math in your head and stuff or when you are at the grocery store because you need to know how to do that stuff too.

Multiple Strategies Participants discussed the idea of multiple ways to teach or learn mathematics topics.	He was really clear. If you didn't understand something, he would go over it another way (or later go over it in another way.) Even if the student tried and still refused, I might try different ways. I would use multiple ways to teach a concept because people learn different ways I don't want to be predictable (not always procedures). One day use a certain manipulative and the next day to change it up (maybe visual or algorithms or discovery).
Procedural Participants discussed procedural learning using terms, such as procedural, algorithms, and/or procedures.	K-3—If that's the way they're remembering it, then that's okay. Because when you're that young, you're trying to remember to do things a certain way. 4-6—I would push how you need to actually learn the procedures then just looking at blocks because by then your mind has developed further and more stuff makes sense I would probably use procedural more at the elementary level because I feel it is more helpful than just here is the basic stuff and learn the rest on your own. Step by step will be easier for littler kids. I would then move into the procedures.
Real World Participants discussed real world application problems that involved ideas about cooking, household items, and other daily activities in their lives.	Maybe have them try to recognize different shapes in their daily lives. That is another good thing for students who say they will never use math in everyday life that I just thought of. Oh, that table is like a circle. Make them see the connection, which may make them remember better than just blocks. We were talking about how boys usually weren't interested in math, and he was saying about how you could use football stats to figure out how fast the pass is and to convert into different units. I use to hate word problems, but that is really the only way to incorporate real world and to actually use someday. K-3—Have everyday items they see as a cereal box is a rectangle or maybe ask them to find a rectangle in their house (hands-on)
Repetition Participants mentioned the	He would say 2 words at the beginning of class and then tell us our homework and then we would work on homework all day. Lot of wkshts in class—walked around and help

use of multiple worksheets, examples, or problems to learn concepts.	She gave lots of examples and worksheets, similar to high school Lot of examples and lots of hmk (visual)
Research Participants detailed research related ideas, such as books and articles they have read about mathematics. Also, participants commented on researching topics to gain understanding about topics.	K-3—It will be very simple. Just the other day, Dr. () was talking about Chinese students and how they go over addition problems repeatedly that when they get to 3 digit problems it is a lot easier K-3—I would do some research to see if there was any alternatives to working with blocks. Maybe you can work with M and M's. If they didn't want to work at all with hands-on, then I would sit down with that student and come up with a worksheet that we could do together Liping Ma--Talk about Dr. () discussion with student who was from China
Teacher Personality Participants described their past teachers and the role of teacher personality in the classroom.	He didn't run the class well and all of us in the class were confused about what he said and how he said things. I didn't really learn anything. He cared about us as people, not just you need to learn math. He would do notes and everything, but when we had questions, he really wouldn't answer anything. He would be like, okay figure this out. He really hadn't taught us anything to figure it out with. I know experimenting with some ideas is good, but in that class we didn't know anything to start off with so that was really hard. Right, I think it is more about how the teacher liked me. That was a big deal and is still a big deal. I don't want my teachers to not like me or help me. The teacher knew I was a good student and would take notes for those who needed someone to take notes. You have to be having fun. You can't put one of those teachers who is bored with it. You have to make it seem like it is not horrible. It's not horrible, but students still think it is so you have to trick them into thinking it is not. Maybe not trick them….

Technology Participants commented on computer programs, videos, and lab equipment.	Things like Geometer's Sketchpad, where you move one point and it moves the entire picture. I remember watching nerdy videos, which were cool. It was good to get away from the blackboard. Also, bringing up those questions and finding those questions. For example, if you are doing a unit on rocks or something, you could bring in a magic school bus movie or something Went to lab and find different shapes (technology) Infomercial on Newton's Law
Visual Participants commented on the use of two-dimensional objects, such as drawings or worksheets with pictures.	Maybe have them come up and draw a square on the board or cut a square out of their papers Maybe do drawing also, not cutting out. You shouldn't always have to visualize math because it depends because there are different visuals For ex,. For geometry, there are blocks Algebra and basic addition it is harder to visualize besides just numbers on paper

APPENDIX M

PARTICIPANT INFORMATION SHEET

Participant Information Sheet

Name:____

Age:__________

Class Instructor/Time:_____________________________________

If you have children, please list their ages:_______________________

CONTACT INFORMATION:

Email Address:____

Phone:____

Days of the week and times best for interviews:___________________

APPENDIX N

INFORMED CONSENT FOR PARTICIPATION IN RESEARCH (PRESERVICE)

Informed Consent for Participation in Research (Preservice)

Project Title: Traditional and Nontraditional Preservice Elementary Teachers' Perceptions about Mathematics and Mathematics Teaching

Lead Researcher: Ann Wheeler, Graduate Assistant, School of Mathematical Sciences

Research Advisor: Dr. Hortensia Soto-Johnson, Ph.D, School of Mathematical Sciences

We are interested in examining your development in thinking about mathematics and teaching mathematics. For example, we want to know how you would approach a certain mathematical topic with elementary students. In order to conduct this investigation, we request your permission to contact you about two interviews. The interviews will consist of 45-60 minute meetings to discuss your current pre-service elementary mathematics course. This conversation will be audiotaped, but only the people listed above may listen to the contents of the audio. Audiorecordings will be erased after the work has been published. To further help maintain confidentiality, computer files of interviews will be created and names will be replaced by pseudonyms. Your name will not appear in any professional report of this research. In addition, we request to have a copy of your final grade in your Math 100, Math 200, and/or Math 300 course to see your progression in the courses, as well as have permission to ask your mathematics instructor about your work and their opinion about the way you think about mathematics.

The risks of participation in the study are no greater than those associated with taking a college mathematics course. By participating in this study, mathematics educators may have a better understanding of the way traditional and non-traditional pre-service elementary teachers think about mathematics and mathematics teaching. Nonparticipation or withdrawal from the study will not affect your standing in the class. If you do choose to participate, you will not be identifiable in final report(s) about the study. Participation is voluntary. You may decide not to participate in this study and if you begin participation, you may still decide to stop and withdraw at any time. Your decision will be respected and will not result in loss of benefits to which you are otherwise entitled. Having read the above and having had an opportunity to ask any questions, please sign below if you would like to participate in this research. A copy of this form will be given to you to retain for future reference. If you have any concerns about your selection or treatment as a research participant, please contact the Sponsored Programs and Academics Research Center (omit contact information).

Thanks in advance for your participation.

Sincerely,

______________________________ __________

Researcher's Signature Date

______________________________ __________

Research Advisor's Signature Date

______________________________ __________

Participant's Signature Date Participant's Full Name

APPENDIX O

INFORMED CONSENT FOR PARTICIPATION IN RESEARCH (INSTRUCTORS)

Informed Consent for Participation in Research (Instructors)

Project Title: Traditional and Nontraditional Preservice Elementary Teachers' Perceptions about Mathematics and Mathematics Teaching
Lead Researcher: Ann Wheeler, Graduate Assistant, School of Mathematical Sciences
Research Advisor: Dr. Hortensia Soto-Johnson, Ph.D, School of Mathematical Sciences

We are interested in examining your teaching structure and beliefs about teaching. In addition, we want to learn more about the participants' interactions in class. In order to conduct this investigation, we request your permission to contact you about two interviews. The interviews will consist of 30 minute meetings to discuss your current pre-service elementary mathematics course. This conversation will be audiotaped, but only the people listed above may listen to the contents of the audio. Audiorecordings will be erased after the work has been published. To further help maintain confidentiality, computer files of interviews will be created and names will be replaced by pseudonyms. Your name will not appear in any professional report of this research.

The risks of participation in the study are no greater than those associated with taking a college mathematics course. By participating in this study, mathematics educators may have a better understanding of the way traditional and non-traditional pre-service elementary teachers think about mathematics and mathematics teaching. Nonparticipation or withdrawal from the study will not affect your standing in the class. If you do choose to participate, you will not be identifiable in final report(s) about the study. Participation is voluntary. You may decide not to participate in this study and if you begin participation, you may still decide to stop and withdraw at any time. Your decision will be respected and will not result in loss of benefits to which you are otherwise entitled. Having read the above and having had an opportunity to ask any questions, please sign below if you would like to participate in this research. A copy of this form will be given to you to retain for future reference. If you have any concerns about your selection or treatment as a research participant, please contact the Sponsored Programs and Academics Research Center (omit contact information).

Thanks in advance for your participation.

Sincerely,

.______________________________ __________

Researcher's Signature Date

______________________________ __________

Research Advisor's Signature Date

______________________________ __________

Participant's Signature Date Participant's Name

APPENDIX P

SAMPLE QUESTIONS FROM DR. FLORES' MATH 100 FINAL EXAM

Sample Questions from Dr. Flores' Math 100 Final Exam

1. (3 points) Use the distributive property to determine 44 X 18.
2. (3 points) If 9 counters are a whole, show how many are in five-thirds of a set. Explain.
3. (3 points) Jim ate 1.5 cups of cereal. The bag says that a single serving is ¼ cup. How many servings did Jim eat?
4. (3 points) Use the partial products algorithm to calculate 32 X 47.
5. A stock whose value goes from $46.78 to $71.54 has increased by what percentage?

APPENDIX Q

SAMPLE QUESTIONS FROM MS. HERNANDEZ’S MATH 200 FINAL EXAM

Sample Questions from Ms. Hernandez's Math 200 Final Exam

1. (4 points) Suppose you are tossing two dice. Imagine getting a 2 on one die and getting a sum of 11 on the second die.

   a) Are these mutually exclusive (disjoint) events? Why or why not?

   b) Are these independent events? Why or why not?

2. (6 points) Consider the following arithmetic sequence: 7, 11, 15, 19, 23, ...

   a) Describe how the sequence is changing.

   b) Find the 10th term in the sequence.

   c) Write an expression for the nth term in the sequence.

3. (6 points) Consider the following set of numbers: 99, 100, 81, 60, 88, 86, 81, 76, 84, 78, 72, 93, 89, and 78.

   a) Find the median.

   b) Find the mode.

   c) Find the mean.

4. (6 points) If you are rolling two dice:

   a) Find the probability that the first die is a 5 and the second die is a 1.

   b) Find the probability that the at least one of the dice is a 2.

5. (6 points) Construct a set of data for each collection of properties below. Be sure to verify that your data set satisfies all of the properties.

(a) Mode = 6	Range = 10	Median = 8	N = 4
(b) Median = 9	Mean = 10	Mode = 13	

APPENDIX R

SAMPLE QUESTIONS FROM MS. GARCIA'S MATH 300 FINAL EXAM

Sample Questions from Ms. Garcia's Math 300 Final Exam

Directions: Answer each of the following questions. Be sure to show your work, where applicable. Make sure all work is mathematically accurate. Wherever necessary, round to the nearest hundredth. Use complete sentences and appropriate vocabulary when asked to explain. I will be grading form, grammar, and good taste.

1. If one angles of a parallelogram measures 55°, then what are the measures of the remaining three angles? *Explain your answer.* (3 pts)

2. Pick ONE: Answer one, and only one, of the following questions. You will not earn additional points by responding to both. Use illustrations to support your explanations. (3 pts each)

   A. Describe one way in which you could convince a student that the formula for the area of a kite is (diagonal one × diagonal two) ÷ 2.
   B. Describe one way in which you could convince a student that the formula for the area of a trapezoid is (base one + base two) × height ÷ 2.

3. A square-based prism has a height of 50 ft and the perimeter of the base is 240 ft. What is the volume of the prism? (3 pts)

4. True or False? If the statement is <u>false</u>, state "false" and give a *mathematical reason* as to why it is false. If the statement is <u>true</u>, state "true" and *give a mathematical reason* as to why it is true. Use illustrations to support your reasoning. (3 pts each)

   a. A square is a type of isosceles trapezoid.

   b. If a quadrilateral has two pairs of equal sides, it is a parallelogram.

APPENDIX S

DISSERATION: SAMPLE INTERVIEW I QUESTIONS

Dissertation: Sample Interview I Questions

1. Thinking back to your elementary schooling, are there any mathematics teachers that stand out to you as good teachers? Explain why you would categorize them in this way.
2. Thinking back to your elementary schooling, are there any mathematics teachers that stand out to you as poor teachers? Explain why you would categorize them in this way.
3. Describe how you might teach your mathematics class.
4. What role does lecture have in the mathematics classroom?
5. How does your role as a parent influence your beliefs about mathematics?
6. How does your role as a parent influence your beliefs about mathematics teaching?
7. Have you helped your children with mathematics (for parents)? Explain.
8. Do you know how your child learns mathematics? Explain.
9. What roles, if any, do a society's and/or teacher's culture play in the teaching of mathematics?
10. What roles, if any, do senses (hearing, touching, seeing) play in teaching of mathematics?
11. What does conceptual learning mean to you?
12. What does procedural learning mean to you?
13. "What do you think is the most effective way to teach mathematics" (Raymond, 1997, p. 555)?
14. "What are the three most important characteristics of good mathematics teaching" (Raymond, 1997, p. 555)?
15. Do you foresee using group learning in your future teaching of mathematics?
    a. In what ways?
16. What does discovery learning mean to you?
    a. Have you experienced discovery learning? How do you know?
    b. If so, in what ways?
17. Is hands-on learning important to you as a student?
    a. Is hands-on learning important to you as a teacher?
    b. In what ways, if any, do manipulatives play in a mathematics classroom?
18. Define fun in respect to mathematics. Should mathematics be fun? Why?
19. Have the mathematics courses that you are taking here been able to help you help your children with their mathematics courses? Explain.
20. "What do you think mathematics is all about" (Raymond, 1997, p. 555)?
21. Describe the degree you feel mathematics is
    a. "dynamic/static,
    b. predictable/surprising" (Raymond, 1997, p. 561).
22. "What most influences your mathematics beliefs" (Raymond, 1997, p. 555)?

APPENDIX T

DISSERTATION: SAMPLE INTERVIEW II QUESTIONS

Dissertation: Sample Interview II Questions

1. Have you ever worked with children (substituting, observing, etc.) in a school setting? Explain.
2. If you had an elementary student who refused to learn conceptually, how would you respond?
3. If you had an elementary student who refused to learn procedurally, how would you respond?
4. What do you believe are the main goals, or objectives, you should get across to your elementary students during a mathematics lesson?
5. How much should a teacher help students in solving problems? Explain.
6. Should students always be able to visualize mathematics? Explain.
7. Can mathematics always be done using a formula or procedure? Explain.
8. How do you envision teaching your future classroom to ensure that your students are learning procedurally or conceptually? (depends on participant's preference) Explain.
9. Describe your experience with groups in your Math 1/2/300 class.
10. Describe your experience working with manipulatives in your Math 1/2/300 class.
11. Prior to university classes, what experiences do you have working in groups?
12. Prior to university classes, what experiences do you have working with manipulatives?
13. On a scale of 1 to 10 with 1 being not confident with mathematics to 10 being very confident with mathematics, how confident are you in your mathematics abilities? Explain.
14. On a scale of 1 to 10 with 1 being not confident with teaching mathematics to 10 being very confident with teaching mathematics, how confident are you in teaching mathematics at the K-3 grade level? Explain.
15. On a scale of 1 to 10 with 1 being not confident with teaching mathematics to 10 being very confident with teaching mathematics, how confident are you in teaching mathematics at the 4-6 grade level? Explain.
16. "What kinds of math do you do?" (Raymond, 1997, p. 555).
17. "What do you think mathematicians do when they do math?" (Raymond, 1997, p. 555).
18. Do you see mathematics as an "unrelated collection of facts and rules"?
19. What evidence would you need to see in order to believe that a student understands a topic in mathematics?
20. Is there "only one way to learn mathematics"? (Raymond, 1997, p. 557).
21. How, if any, has your attitudes about mathematics evolved from
    a. The beginning to the end of the semester?
    b. Any time during your life?
22. How, if any, has your attitudes about mathematics teaching evolved from
    a. The beginning to the end of the semester?
    b. Any time during your life?

APPENDIX U

SAMPLE OBSERVATION FORM

Sample Observation Form

Lesson Topic:

Name(s) of participant in class:

Seating of participants:

- Groups?
- With other traditional/nontraditional preservice teachers?
- Interaction with group members
    - Preferences in interactions—other traditional or nontraditional
    - Role in group
    - Attitude about group work

Instructor's approach to teaching:

- Facilitator?
- Lecturer?
- Use manipulatives?
- Technology?
- Answers preservice teacher questions?
- Multiple strategies?

Participant's attitude about mathematics

- Frustrated?
- Enjoyment?
- Procedural?
- Conceptual?

APPENDIX V

INSTRUCTOR INTERVIEW I

Instructor Interview I

1. What is your teaching philosophy with respect to Math 100/200/300?
2. How do you feel the following preservice teachers are performing in your class (academically)? Explain. Do they struggle with any particular concepts?
3. How do you feel the following preservice teachers think about mathematics? Explain.
4. How do you feel the following preservice teachers utilize group work? Explain.

APPENDIX W

INSTRUCTOR INTERVIEW II

Instructor Interview II

1. How well you think the following participants do in your class academically? Do they struggle with anything? How do they feel about conceptual/procedural?
2. How do you think the following preservice teachers utilize group work?
3. What do the following preservice teachers feel about manipulatives?
4. How would you respond to these quotes from preservice teachers' interviews and classroom observations?
5. Have you noticed any changes in the semester in the preservice teachers, whether good or bad? Explain.

APPENDIX X

TONYA'S INTERVIEWS

Tonya's Interviews
1st Interview: Tonya Math 300 Date: 2/6/2009

1. Thinking back to your elementary schooling, are there any mathematics teachers that stand out to you as good teachers?

No, not really. Maybe my third grade teacher but I don't really remember math very much from that grade…I remember the multiplication stuff…I liked that part of it. I liked just understanding the multiplication…It was probably my teacher was the reason I liked multiplication because she was my favorite teacher. Personality was why she was my favorite. She was just there for her students.

2. Thinking back to your elementary schooling, are there any mathematics teachers that stand out to you as poor teachers?

No.

3. Thinking back to your secondary schooling, are there any mathematics teachers that stand out to you as good teachers?

No.

4. Thinking back to your secondary schooling, are there any mathematics teachers that stand out to you as poor teachers?

Yes, all of them. I feel like I have not had a good math teacher because in seventh grade I started out with a good one but then I got moved up and the teacher was horrible and I felt like I didn't learn anything and I am not confident in a lot of my math…I got moved up to a higher level math…Algebra I am really comfortable with but Geometry and Pre-Cal and all that…My teachers…I just really didn't learn anything. And Stats I did well in because it was one of those self-taught….What I didn't like about these teachers were their teaching styles and I didn't feel like they really knew what they were talking about because my eighth grade math teachers' major was in French and she had a math minor. They just didn't know how to teach math and I felt they just weren't really there for the students either because you would go and talk to them and get help and it was just hard…It was lecture and then my teacher I had for two years, she would do a problem and do it wrong but not erase it. She would try to figure out where she made a mistake and just start from there and I can't. It's really hard for me to do that so…A lot of my math teachers really didn't know how to get the mathematics across to the students.

5. What mathematics courses have you taken at a community college?

No.

6. What mathematics courses have you taken at a college?

100—Ms. Hernandez, 200—Ms. Smith, 300—Ms. Garcia

7. Are there any past mathematics teachers at the college/community college level that stand out to you as good teachers?

Yes, all of them are good teachers. They actually like math and everything like that and they are really good at what they do…Teacher personality does play a role. I don't know…I think that if someone is really passionate about the subject and that it shows through when they're teaching and that helps a lot too….I feel all three were really passionate about math and knew what they were talking about…Basically, opposite of what I felt in secondary school, extremely so.

8. Are there any past mathematics teachers at the college/community college level that stand out to you as poor teachers?

No.

9. Do you feel it is important to make A's in a mathematics class? Explain.

I do. With all classes, I feel it is important to make A's.

10. What other obligations besides schooling do you have (e.g., family, job, commuting)?

No kids. I am looking for a job, I used to have one. I babysit a lot, probably about 13 hours on the weekend and I'm looking for a part-time job. I'm taking 15 hours. I don't commute.

11. Describe how you might teach your mathematics class.

I want to teach third grade. I don't know why. It just seems like the perfect age to me…I don't know if my favorite teacher has something to do with it. I don't know…I would probably do groups. It is what all of the teachers here have done, and I feel it has worked really well. I would probably use manipulatives when I teach…I like the hands-on. I am very hands-on. I am very visual…I guess I would try to figure out the different learners in the class and try to incorporate all of that.

12. What role does lecture have in the mathematics classroom?

I don’t know. I would play it by ear because of my past experiences. I mean I would probably do some lecture and then some activity to follow up that lecture. The lecture length would have to depend on what it is, I think.

13. What roles, if any, do a society’s and/or teacher’s culture play in the teaching of mathematics?

I do think it is important for the teacher to bring their culture into the classroom and for the teacher to bring in their students’ cultures into the classroom. It is important to be inclusive of all backgrounds.

14. What roles, if any, do senses (hearing, touching, seeing) play in teaching of mathematics?

Seeing, hearing, and hands-on are all important because they kind of play off of each other because if you see it and then do it, it kind of reiterates what you just learned. And then hearing it of course.

Extra Question: Do you see a manipulative and a visual on the board as being 2 separate things?

I feel writing on the board is different than having a manipulative because I think writing on the board is more seeing it done. And then using the blocks is more like physically seeing yourself, do it yourself as opposed to letting someone else just show you. I would probably use both in my classroom. I think they are both are pretty important but I would say the hands-on is probably a little bit more important from my own experience of liking hands-on more.

15. “What do you think is the most effective way to teach mathematics” (Raymond, 1997, p. 555)?

Probably just, I don’t know how to describe it. Making sure, because in my math classes here, I was taught exactly what a number is, and using the different methods that they use and everything because if I was taught those methods like in math then I would maybe remember it more. So all the tricks and stuff they have and doing that just to make it easier to comprehend and it if you don’t understand this way, show a variety of techniques, I guess. By tricks, I mean like the lasso method or different methods to understand.

16. "What are the three most important characteristics of good mathematics teaching" (Raymond, 1997, p. 555)?

Well, passion for the subject. Knowledgeable or knowledge about the subject and, I guess, there for your students so if the students have questions and they don't blow you off.

17. Do you foresee using group learning in your future teaching of mathematics?
    a. In what ways?

See above

18. How do you feel about assigning repetitious work to students?

I guess it would depend on what it is, depending on whether I thought they understood it or not. If they did understand it, I would do it just to make sure. If they didn't understand, I would do it also to make sure they understood. I probably wouldn't do it as much if they had a grasp on it.

19. "Good mathematics teaching entails, or depends on—
    a. A good textbook/use of manipulatives
    b. Teacher direction/student participation
    c. Teacher effort/student effort
    d. Explicit planning/flexible lessons
    e. Helping students to like mathematics/helping students see mathematics as useful" (Raymond, 1997, p. 563)

a. Using manipulatives are important from previous.

b. Both are important because I think the student and teacher kind of play off of each other, learning and everything. I think teacher/student interaction is important because their interacting with their students…because I think that the teacher interacting with their students making sure they understand and then the student can trust the teacher and ask questions and feel comfortable and then everything.

c. Both are important because the teacher is putting an effort to teach the students and the students, I think, in order to grasp something would benefit more if they were trying in their class, as well.

d. Both are important because I think it is good to have a set plan but then also to be flexible because they may need longer or they may understand something really fast so you can move that faster.

e. I think if they understand they kind of like it. I don't really know. I don't think it is okay if they don't understand to not try. I think everyone should try. As a teacher, I think I would try to inspire students to like math because I think math is fascinating, even though I might not necessarily be super strong or as strong as I would like to be in it. Just sharing that passion or fascination or learning.

I think it is important to help students see math as useful because from past experiences. We would sit in class and say, "When would we ever use this?" "Why do we have to bother learning this?" If you can see when you would ever potentially use this here then, and "Oh, this can be useful."

20. How, if any, would you teach mathematical concepts differently K-3 versus 4-6?

Probably, I don't remember everything that was taught in those grades, but yeah, I think so. Well, I think the younger probably goes more the hands-on a lot. And the older, more, I don't know…I probably would do some hands-on with the older. I would use groups with both ages because of the importance I feel comes from groups. I never liked groups until I came here. I feel my groups here are useful.

Within my own group, I feel we work well together. My role in the group is equal asking and answering questions.

21. What does conceptual learning mean to you?
    a. Have you experienced conceptual learning? How do you know?
    b. If so, in what ways?

I learned mainly the conceptual way at the university.

22. What does procedural learning mean to you?
    a. Have you experienced procedural learning? How do you know?
    b. If so, in what ways?

I have done the procedural learning from my previous schooling, and I feel that is why I didn't learn as well.
I have done it a lot. I think that is the main method a lot of teachers use or my previous teachers before the university.
From actual learning the conceptual way, I think I would probably use more of the conceptual way because it would stick with them longer. It depends if I would use the procedural way. I would probably try and stay away from using it as much as possible, some things, I don't know. I think some things I would have to use the procedural. I feel the conceptual is more important because just from sitting in the classroom and just doing it these ways and it is like you are seeing it in your brain, I don't know, it is like easier to grasp the concept and you understand more where the numbers come from and how it plays into it as opposed to this and this and this and not really understanding the background behind it.

23. What does discovery learning mean to you?
    a. Have you experienced discovery learning? How do you know?
    b. If so, in what ways?

I don't think I learned this way. I guess I feel a little bit with the geometry book, but I don't know about the synthesis goes with that.
I probably wouldn't use discovery learning when I taught because personally I would like to have some direction on where I'm going, because..I know Spanish is completely irrelevant to this but in my Spanish class we teach our self and then we go it. If I had that knowledge of how to do something and then do the homework and then go over it,it would be a lot easier and easier to understand.

24. Is hands-on learning important to you as a student?
    a. Is hands-on learning important to you as a teacher? See above
    b. In what ways, if any, do manipulatives play in a mathematics classroom?

Yes, it is important to me as a student because, I think, if you actually see it and are doing it in front of you, it's a lot easier to understand and comprehend what you are doing, as opposed to just someone showing you.

25. Define fun in respect to mathematics. Should mathematics be fun? Why?

Fun with math to me is algebra because I think it is really fun trying to figure out the equations what x is, like the basic algebra. I think everything I did in Math 100/200 (manipulatives included) are fun.

I think math should be fun because I guess it is more fun and exciting to go to…the more apt you would go to class and want to learn, more motivating.

26. “What do you think mathematics is all about” (Raymond, 1997, p. 555)?

The word that comes to me is numbers…I don’t know how to define math actually…Math is the concept of numbers. I just think of numbers.

27. Describe the degree you feel mathematics is
    a. “dynamic/fixed,
    b. predictable/surprising
    c. applicable/aesthetic” (Raymond, 1997, p. 561).

a. I would have to say math is changing because, well, some of it is just set in stone, like certain aspects of it…like some of the maybe algebraic equations and everything, like the upper level stuff, and the stuff that is changing is like the different methods or like the different approaches maybe to solving.

b. I think math recently…it is more surprising because in class, it is like “Oh, wow, I didn’t know you could do that” or “Wow, that is a really cool way.”

c. I feel math is relevant because we use it every day with balancing a checkbook or that is the only example that is coming to mind right now. We use it.
I feel math is beautiful when it works out and when you are sitting down and like doing something, it is. It is fascinating how it works and plays together.

28. “What most influences your mathematics beliefs” (Raymond, 1997, p. 555)?

Probably, a lot of my beliefs and everything have changed since coming to the university and in my math classes because before I hated math and just everything, and I wasn’t confident in it and through the courses, and Geometry I’m still kind of struggling with but, it was just really fun learning it and just exciting. Prior to the university, I liked Algebra but the secondary teachers’ personality had a lot to play with me not liking math.

Extra Question: How would you teach shapes?
I would definitely have them cut out the shapes and everything and have them identify the shapes and name the properties of the shapes (angles, sides). If I had older students, it would probably be different. I don’t really know. I can’t even imagine teaching Geometry. I would probably still have some hands-on because even in here that helped me a lot, doing the cut outs and understanding the angle bisector and everything like that

so I guess that would be a different approach. I would be a lot more in-depth with older students.

Interview 2: Tonya

Name: Tonya — Highest math class in high school: PreCal/Stats
Date: 4/15/09 — 100-B (Hernandez), 200-A (T--)

1. Have you ever worked with children (substituting, observing, etc.) in a school setting? Explain.

No. I worked in a church (preschool) for 2 ½ years, a year ago.

2. Have you ever had a mathematics class in another country? Explain.

No.

3. Do you ever have to miss school because of family obligations? Explain.

No. I missed a lot from being sick this semester. I missed one of Ms. Garica's classes. I do feel I got behind.

4. If you had an elementary student who refused to learn conceptually, how would you respond?

I wouldn't be upset by it. I might try to devise something else that would be a better learning tool for that student and then I would also try and push the blocks a little bit because I am very visual….I just think it is something really good. I would definitely try and do something else because it sounds like they are not very visual so maybe something more auditory. I would push a different strategy.

5. If you had an elementary student who refused to learn procedurally, how would you respond?

I would be very supportive of that because I think that creates a lot of issues if the teachers don't cater to their students so I would support that. I think I would always be okay with them using blocks to add. I can't imagine teaching at the high school setting.

6. If an elementary student who could not understand the concept of (Pyramids/prisms) through conceptual learning, how would you further help him/her comprehend the concept?

That is hard. I would probably try and explain it to them in a different way without using manipulatives or actually…Yeah, explaining them in a different way and maybe try and use them but maybe not push them almost so to concentrate more on a different strategy as opposed to using the manipulatives. (Similar to above where I might try auditory). I think eventually sometimes it clicks because there is this initial block (confusion) they need to get over.

7. What do you believe are the main goals, or objectives, you should get across to your elementary students during a mathematics lesson?

I would push….I noticed that the group work has helped a lot. All the math classes have been group based. The good thing about groups is that it is team building and you are playing ideas off of each other and helping each other kind of learn. Not everyone sees something the same way and so I think that is really important because it helps open the students' eyes to different strategies. As much as the group projects are important, I think also doing the quizzes by yourself to see if they are comprehending everything. The individual accountability is important to make sure everyone is on the same page because when you are in a group, you might think you understand it and you take the quiz and it is entirely different. Also, depending on what you are teaching also the procedures, if procedures are necessary.
I definitely like the conceptual over the procedural. I would stress conceptual more in my class because I noticed that has helped me a lot, just from previous experience.
I think certain subjects you need the procedures because there are certain things you need procedures for so in that aspect I would say yes, I would. I guess more of the equations based are what I would call necessary. There are also ways you can show when we were doing the cylinders and filling them up with rice and coming up with the equations and stuff like that. At some point, you will need the equations.
Manipulatives are my number one because just from working in the groups and me as a personal learner and how my previous math teachers did not do that at all and I think it would have helped a lot.

8. How much should a teacher help students in solving problems? Explain.

I definitely think they should be there and help but I also think they should have the student work on the problem and if they are not understanding and they are struggling, they should have the teacher come in and help out. Basically, if a student needs help and they see the student needs help, then they should help them. I would probably not tell them the answer because I want to help them try to get to the answer and so…That is my main reason.

9. Should students always be able to visualize mathematics? Explain.

I think it depends on the person and how they learn. As a teacher, I would always try to make it to where they could visualize math because of the importance I see in it because seeing is believing.

10. Can mathematics always be done using a formula or procedure? Explain.

No, I don't think so. I can't think of any specific examples but I don't think so. Some of it you would just need to conceptually see.

11. How do you envision teaching your future classroom to ensure that your students are learning both procedurally or conceptually? (depends on participant's preference) Explain.

Obviously, I would stress the conceptual. Obviously, have more activities that would do that because from Ms. Garica's class that we did with the patty paper and parallel lines. I don't know how you would show that on a test exactly because I wouldn't really necessarily…Because I would go off of the activities that I did when testing like that. I wouldn't make them redo that activity on a test. Maybe ask, I really like how Ms. Garica has us do the activity and she asks if a student does this, what activity could you do. I would give them something similar because I wouldn't want to completely blind-side them on a test and be like, "Hey we never did this. This is completely opposite of what we did."…And ask questions that aren't so procedural in nature.

12. Describe your experience with groups in your Math 300 class.

I really enjoy the groups, just because we all build off of each other. All of us contribute in the group setting. A girl is absent a lot and oftentimes she won't show up and she has the sheet that we need to turn in. That is kind of stressful. As long as everyone is there and contributing, it is beneficial. I feel Nita is the leader of the group. She is really good. She took a class that is really similar to this class and kind of knows everything so I think that helps out a lot and she does explain, if you don't understand something and say, "Hey I don't understand," and she will explain. She will explain more than the answer. She will show us how to do it. I go to her or Martha for help. I think I ask more questions in the group because geometry is my weak point.
Nita said she would work by herself—It might have been because there were three of us so she decided to work by herself, but I know that…Because I know when we had our partner project coming up, she said, "Oh, you two work together," so it might be that when the other girl is not there…I know that she is really fast and efficient so that might be it too. I know that she feels she is always correct. I know she will admit when she is wrong, but I think she is worried that we are all going to mess it up so I think that is also why she works alone.

13. Describe your experience working with manipulatives in your Math 200 class.

They definitely have helped me learn the material. It is definitely not as heavily stressed as it was in 200. I think geometry is different. They are not stressed as much in 300 but I feel I have learned from them. It seems like everything in 200 had hands-on manipulatives. We also did a lot with 100 because it is easier because it is before you are learning..

14. Prior to the university classes, what experiences do you have working in groups?

I don't think I did in math. Maybe in my Spanish class in partners, but it was never really stressed.

15. Prior to the university classes, what experiences do you have working with manipulatives?

No, I had horrible math teachers.

16. On a scale of 1 to 10 with 1 being not confident with mathematics to 10 being very confident with mathematics, how confident are you in your mathematics abilities? Explain.

It depends on which one, probably a 7 because algebra I am really good at so I would say an 8 and any topic in 200. Geometry I would say a 6/7. Some of the topics I feel really strong about but the stuff we are learning now I don't really remember doing it. It's not completely new to me, but it feels like it because I remember my geometry teachers going out of class and not knowing anything. Pictures aren't my problems. It's the concepts in geometry that are confusing. 100 I would also say 8 or 9. I am confident in those.

17. On a scale of 1 to 10 with 1 being not confident with teaching mathematics to 10 being very confident with teaching mathematics, how confident are you in teaching mathematics at the K-3 grade level? Explain.

Probably an 8. In class, I was remembering/trying what I was doing in third grade because that is the grade I want to teach and so like multiplication and like everything. It kind of goes back to the 100 so it is the material and like the concepts that we learned and how we really teach I feel I could do that. 100 gave me the tools to be confident.

18. On a scale of 1 to 10 with 1 being not confident with teaching mathematics to 10 being very confident with teaching mathematics, how confident are you in teaching mathematics at the 4-6 grade level? Explain.

Maybe a 7 1/2 or 8. I'm hoping I don't end up teaching that age. I feel with the elementary I feel pretty confident. Anything higher than that, I feel I probably couldn't be the best math teacher (higher than sixth grade). The questions they pose would not bother me. I don't know what it is.

19. What types of technology, if any, would you use in your mathematics class? Explain.

Calculators-Depending on what it is, I think calculators have taken over in a derogatory way because I can't do simple math. I can't do 8 + 4. You got so used to using a calculator. With certain things I would allow it, but I would try and stay away from it. I think add/subtract/multiply/divide should be done without calculators. The basic math is definitely no calculators but if it is not the basic math and I feel the calculator is necessary.

Math programs—Certain math websites I noticed are really helpful like math is fun so maybe have them look at that as a resource and show it in class. I would be okay with using math programs in the classroom.

20. "What kinds of math do you do?" (Raymond, 1997, p. 555).

I probably do more conceptual. Actually coming here I learned that basically everything was procedural growing up and then they strayed towards conceptual and I have just noticed that that is really helpful for me because procedures are just memorizing and conceptual you are actually seeing and kind of understanding the concepts, which help you understand the procedure better almost. Procedural was K-12. Now, at the university I learned more conceptual. In these classes, I'm learning more conceptual because I feel you could probably teach it better too if you are teaching it that way as opposed to having them memorize it. Ms. Hernandez really stressed that and how everything was procedures and how kids nowadays like me, I don't remember a lot of things I learned just in high school because it was more procedures because I don't really own it.

21. Do you see mathematics as an "unrelated collection of facts and rules"? (Raymond, 1997, p.556)

I definitely feel math is more interrelated, mostly just because you use it every day. Ideas in 100 I might use in 200. They just build on top of each other.

22. What evidence would you need to see in order to believe that a student understands a topic in mathematics?

Probably being able to explain it to me or another student. It is more than an answer.

23. Is there "only one way to learn mathematics"? (Raymond, 1997, p. 557).

No. (See above)

24. How, if any, has your attitudes about mathematics evolved from
    a. The beginning to the end of the semester?
    b. From Math 100 to Math 2/300?

Dynamic/Fixed—dynamic because the approaches are different to solving (same)
Surprising/Predictable—surprising because learn new approaches I learned (same)
I got the opinions by taking the math classes at the university. I guess I kind of always knew but I never actually thought about it until I came here. (more of a reflective thing)

25. How, if any, has your attitudes about mathematics teaching evolved from
    a. The beginning to the end of the semester?
    b. From Math 100 to Math 2/300?

Conceptual learning is easier to understand. I understand numbers more from learning conceptually. I feel I would use conceptual learning more in a classroom. I would actually stay away from procedural learning. I feel that is the way I learned mostly from my previously learning and it wasn't good—(same).
I would try to stay away from procedural (same).
Groups—like groups (same) I never liked groups until I came here.
Manipulatives—liked (same)
I have had these opinions that I had here at the university and being in productive groups and learning manipulatives and the stress of conceptual learning at the university.

26. For the last part of the interview, I want to know how you might address the following concept to your future elementary student.

Math 300-pyramids and prisms

I would definitely do the cut-outs and have them cut out the pyramids and prisms because it is a lot easier to visualize because you can count the edges, faces, and vertices.
It is a lot more beneficial than maybe doing the activity where you have the shapes and the bags and have them describe them to group members and have them guess. Not only having them to describe them but having them to see it in their head for themselves. I struggled a bit with this activity. It helped though. Nita helped me.
That would be the biggest part with pyramids and prisms.
And then maybe introducing Euler's Formula and have them fill out the shapes and like the hexagonal shape and how many faces, vertices, and edges and doing that between the pyramids and prisms and doing comparisons between the two and the patterns in the charts.

APPENDIX Y

NATAYLA'S INTERVIEWS

Natalya's Interviews
1st Interview: Natalya 2/6/2009 Math 200

1. Thinking back to your elementary schooling, are there any mathematics teachers that stand out to you as good teachers?

No. I grew up in Norway, and we had the same teacher for everything. I didn't have one specific math teacher. I didn't have one that stuck out at all. I moved 14 years ago. I was born and raised there but I have been here for 14 years.

2. Thinking back to your elementary schooling, are there any mathematics teachers that stand out to you as poor teachers?

Not that I can remember.

3. Thinking back to your secondary schooling, are there any mathematics teachers that stand out to you as good teachers?

Not especially.

4. Thinking back to your secondary schooling, are there any mathematics teachers that stand out to you as poor teachers?

I don't know about poor, but impatient. They wanted you to get it right away and hurry up and get going and if you didn't get it, you were kind of a pain because you didn't understand.

5. What mathematics courses have you taken at a community college?

Not at a community college

6. What mathematics courses have you taken at a college?

I took math courses at another college. I took Algebra. It was a long time. It was Algebra I and II, I assume. I took Math 100 here with Mr. Rogers and Math 200 with Ms. Hernandez. That is the only courses in math I took here.

7. Are there any past mathematics teachers at the college/community college level that stand out to you as good teachers?

I think they have all been good. They have all been different, but all good. The one I had a long time ago, he really was thorough and made sure you understood. Mr. Rogers, I liked his way of teaching. He didn't explain as much. He was more because it made sense, which it doesn't always make sense when you don't have a Ph.D. in math. And I really enjoy Ms. Herandez's way of teaching. She really takes her time and makes sure we understand everything and feels like, or at least gives the impression, that she feels like she hasn't done her job if we don't know what we are doing. Out of the three approaches, I like Ms. Hernandez's approach a lot.

8. Are there any past mathematics teachers at the college/community college level that stand out to you as poor teachers?

No.

9. Do you feel it is important to make A's in a mathematics class? Explain.

I would like to make A's in those classes but I just want to pass the class.

10. What other obligations besides schooling do you have (e.g., family, job, commuting)?

I am taking 12 hours. I have two children. I have no job. I drive from another town (20 minutes).

11. Describe how you might teach your mathematics class.

My ideal age would be little—Kindergarten or 1st because for me I want to be a positive influence at the beginning of school because I have seen it with my own kids how one has had a really great start, and the other one was kind of unlucky and how that has made a huge difference in how they view school. I think I would do a little bit of individual and group work, depending on what the topic was. I would definitely use hands-on manipulatives in class. I would use them as often as I could so there is a visual because I think that helps a lot.

12. What role does lecture have in the mathematics classroom?

I probably wouldn't lecture too much, for the 2 minutes that could concentrate and then take it from there. I would definitely do activities.

13. What roles, if any, do a society's and/or teacher's culture play in the teaching of mathematics?

I do. I do believe it is important to bring in both the teacher's and students' culture into the math classroom because I think it is also a way of teaching about other things too, just knowledge they can have from that and also being able to identify to whatever culture they might go back to, if they are from another culture, and they come here, but they ultimately go back and then they have some of that. They can identify with that and don't feel so left out or whatever.

14. What roles, if any, do senses (hearing, touching, seeing) play in teaching of mathematics?

I think senses are very important, but I think it is all individual students. I think it is important to offer all so that everyone has an extra chance in understanding or seeing the concept or whatever.

Extra Question: Do you feel visual (blackboard) and manipulatives are both important in a classroom?

I think you need both because you can do it on the chalkboard but if they can do it, physically do it, that might make for better understanding. I think the manipulative is probably, I don't know about more important, but at least as important.

15. "What do you think is the most effective way to teach mathematics" (Raymond, 1997, p. 555)?

I have no idea. If they….present the material in a positive, not say this is going to be really hard and a lot of people don't understand, but say, "This is going to be fun and we are going to make sure you understand it and I'm going to help you and I'm here if you need in extra help." That kind of way so that it is a positive experience, and it is not just, "It is going to be so hard and I already don't want to do it."

16. "What are the three most important characteristics of good mathematics teaching" (Raymond, 1997, p. 555)?

One would be knowledge of teaching and then knowledge of the subject, obviously, but I think of teaching first and of the subject and like I said presenting it in a positive manner.

17. Do you foresee using group learning in your future teaching of mathematics?
    a. In what ways?

See above.

Extra Question: How do you feel about your group?

I feel the group I am in is useful. I think we kind of work as a group. I'm the oldest, and I fairly perceive things a little differently than they do and they're freshman and they're girls too, and you know, they are girls who fight and empty each other's shampoo bottles and stuff. So you know, but the work itself is fine but I think you notice the other stuff when your older maybe. Last semester, my group talked a lot about dorms and this party and that boy, this, that, and the other and I feel really old now. I find the groups pretty positive and for me, since I haven't been in high school or whatever for so long, it helps me get a fresh perspective on how their teaching now because they didn't do that 13 years ago when I was in high school. They're learning new, different things, or harder things, or whatever, different ways. For me, I think it is beneficial to see how they learn all the stuff and what their background is in it because a lot of the stuff I forgot a long time ago.

My role in the group—I feel am a little bit of both (asking and answering questions).

18. How do you feel about assigning repetitious work to students?

I think I would assign repetitious work if I felt that I needed the extra…or…if my students needed the extra work or if I needed to get it down then that is beneficial to me, absolutely. It has been beneficial in my own experience.

19. "Good mathematics teaching entails, or depends on—
    a. A good textbook/use of manipulatives
    b. Teacher direction/student participation
    c. Teacher effort/student effort
    d. Explicit planning/flexible lessons
    e. Helping students to like mathematics/helping students see mathematics as useful" (Raymond, 1997, p. 563)

a. I think it is a combination. I think it depends on how you would use that good textbook though. The textbook could be great but it depends on how you use it. A beneficial way to use the textbook would be I think I would have to go through it and make sure my students also thought it was a good textbook to keep their interest in it and not to say that this is really fascinating to me and I really like the way this is presented in this book, but then they go to read it and understand it and they're like, "That doesn't make any sense to me and you said this was a good textbook."

b. Both are important because if students aren't participating, then how can you get through to them? I think teacher direction is important because it is important to be specific and answer any questions before you send them off to do their work and not just assign random homework and just assume they can figure it out or use the fantastic textbook you have selected to give them.

c. I think it depends mainly on teacher effort because the students will want to make the effort if you do or if they see you do.

d. I think you have to have an explicit lesson plan, but I think you have to be flexible in that plan if not everybody is on board and you can't because you don't want to leave anyone behind and not understanding or not having grasp the concept. I would be willing to spend another week or whatever on whatever it would be.

e. I think it is important to help students to like math, but I think they will like math if you present it in a more positive way.
I think it is important to help students see math as relevant. Well, everybody says that all you use is plus/minus, but then you have to see that you might have to figure this out or you have this. Yeah, everyday life you don't have to figure out that much algebra or geometry or whatever but you use it eventually.

20. How, if any, would you teach mathematical concepts differently K-3 versus 4-6?

I wouldn't really teach math concepts differently, not if they understood it the first time. Both ages I would use manipulatives, maybe not as much in the older grades, but if there was a topic they didn't understand then that would be useful absolutely I would. I probably would use group work in both. I think I probably would use group work more in the older classes because the little ones get distracted very easily.

Worked in the public school system?
No.

21. What does conceptual learning mean to you?
    a. Have you experienced conceptual learning? How do you know?
    b. If so, in what ways?

I have learned that way in 100 and 200. Yeah, I like learning this way but I like learning procedurally better.

22. What does procedural learning mean to you?
    a. Have you experienced procedural learning? How do you know?
    b. If so, in what ways?

Yes, I have learned using procedures and I feel it was a common way I learned. I think to memorize a formula is easy to actually understand it. For me personally, if it has a formula, I do much better than with probability or doesn't have a specific x, y, z (this is how it is approach).

I would probably use procedural and conceptual learning in my own teaching, I would think, depending on the topic. I don't know which one I would use more. I would probably teach the procedural first because I am more comfortable with it. I could probably explain it better that way. Then, maybe some conceptual.

23. What does discovery learning mean to you?
    a. Have you experienced discovery learning? How do you know?
    b. If so, in what ways?

I have not learned this way. Depending on the topic, I might use discovery learning to see how their problem solving skills might be. I would probably try it out and see how the results are and then use it if I found it to be successful.

24. Is hands-on learning important to you as a student?
    a. Is hands-on learning important to you as a teacher?
    b. In what ways, if any, do manipulatives play in a mathematics classroom?

I have benefited from it a lot, especially last year. We haven't done too much hands-on. We haven't done much hands-on. We've done a little bit this year but last semester we did a lot of hands-on with the manipulatives and stuff. I found it to be helpful to have the visual.

25. Define fun in respect to mathematics. Should mathematics be fun? Why?

A fun aspect of mathematics is understanding it and having that "Ah-hah. I get it." Manipulatives are part of the process. I don't think, "Heee, Rocks" or anything like that. More the understanding through using manipulatives.

I think math should be fun because it has gotten a bad rap for not being fun and there's no reason why it shouldn't be fun. I don't think anyone is set up to not understand math. It is just the way it is brought to you and how it is presented and what attitude you got to presenting.

26. "What do you think mathematics is all about" (Raymond, 1997, p. 555)?

I have no idea what it means to me. I don't know. Until I came back to college now as an adult, I always thought of it as hard but now I don't. I don't think of it as a breeze or anything but I just think I look at it in a different way, challenging but in a positive way, not negative like "Uh, I can't do this before I even see it," like it used to be. I used to not like math. The change occurred at the university. It was frustrating to me before, but I think it was because I wasn't patient enough. Being at a different time in my life and helping my children with their homework and realizing it wasn't that hard to begin with influenced my beliefs.

27. Describe the degree you feel mathematics is
    a. "dynamic/fixed,
    b. predictable/surprising
    c. applicable/aesthetic" (Raymond, 1997, p. 561).

a. I think math is pretty changing. I think at least in the way that it is taught or how much more math you need to know earlier on in your life or understand earlier, at a younger age.

b. I think math is more in the middle probably because a lot of it, personally, I feel is predictable—the kind of easier stuff. But then the new, well what is new to me, is surprising, things I haven't thought of before probably, maybe (in Ms. Herandez's or Mr. Smith's class). I think the topics and methods can both be surprising because some of the topics I haven't had before and I don't know if that's just the difference of country or what, whatever it is, which has made sense because I have heard of a few other people that have had it and I have thought I have never had it.

c. Relevant—mentioned above
Beautiful—I think you can see beauty in math in the formulas and how it just comes together and how there is an answer. There is certainity.

28. "What most influences your mathematics beliefs" (Raymond, 1997, p. 555)?

My mom most influenced my beliefs about math. She loves math. She's a math whiz and also my husband.

(For participants with children)

29. How does your role as a parent influence your beliefs about mathematics?

I think they reflect off of me onto my children, which is why I have tried to be positive about math, so it is not one of those groany, “I can’t do this. I don’t want to do this” kind of thing.” I think it is fun to see how they think and to see how that develops, the way they think about it and how they count. Especially my daughter, who is in 1st grade, and I watch her do her addition and how she counts and the way, in her own way without me having done anything, how she adds that up and by counting. I know this probably sounds funny but if she had, you know 4 + 6, she would go 6-7,8, 9, 10 instead of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. She has figured out just to start with 6 and go 7, 8, 9, 10 and Oh, 4, and whatever it was. And that’s kind of fun for me to watch and how that clicks for her, all on her own.

30. How does your role as a parent influence your beliefs about mathematics teaching?

I think I would teach math with a different kind of patience. I don’t know that I would teach it differently but I think I might have more patience than someone who doesn’t have children or a different kind of patience. I don’t know. I think once you see how frustrated kids are and they just don’t get it and they don’t mean not to get it. They just don’t.

31. Have you helped your children with mathematics? Explain.

I have, every week. My son is in sixth grade and he struggles mostly with word problems, collecting all the information to get it so we try to find ways to break it down so he can get all the information.
My daughter likes to do it on her own. She just, if there is a word she can’t read herself, she’s very independent, and I have to help her with that. Like I said, she usually, she does fine so far, and she likes it. She thinks it is fun and she has little math workbooks at home that she likes to do. Math is not my son’s favorite subject. He is more an artistic kid.

32. Do you know how your child learns mathematics? Explain.

My daughter is independent. My son needs more guidance. He hasn’t struggled with math, ever, but he has needed that path to follow, and my husband and I help him with that.

33. Have the mathematics courses that you are taking here been able to help you help your children with their mathematics courses? Explain.

Yes, last semester my son and I were learning the same thing. That was really cool because it helped me explain to him what he had to do.

Extra question: How would you teach experimental probability?

I would teach semi-close to the way Ms. Herandez has been doing, maybe at a little bit of a lower level but still with the manipulatives and with seeing it and not just thinking, okay the 4 heads and the 4 tails and how would that fit into that—the actually doing that. And with the cards too or pulling something out of a bag or whatever and seeing how that actually works and not just imagining it. I would absolutely still use manipulatives at different levels (grades).

Interview 2: Natalya

Name: Natayla Highest math class in high school: 12th (Norway)
IDLA emphasis: Early Childhood Date: 4/10/09
100—B/B+

1. Have you ever worked with children (substituting, observing, etc.) in a school setting? Explain.

No

2. Have you ever had a mathematics class in another country? Explain.

Yes. My entire public school. I only went to college here.
I don't know that the math is different than here. I think their….I went to school here when I was in fourth or fifth grade and I came back and I had come further than they had when I got back to Norway. But all in all, I think it is pretty much the same stuff that is covered in K-12 here. I can see from my children also that this is true.

3. Do you ever have to miss school because of family obligations? Explain.

No, I haven't. I work my schedule out with my husband and he seems to think he can handle it. I missed two classes for training. I don't feel I was behind because I missed.

4. If you had an elementary student who refused to learn conceptually, how would you respond?

I don't think I would be upset at all. Everybody learns in different ways and I think if that child or if I thought that child could learn the way they wanted to learn, then I would definitely encourage that and go through the steps and see how that would work for them. But if it didn't work out, then I would probably go ahead and say why don't you go ahead and try it with these blocks or whatever and see if that would help you in solving it the way you want to solve it better, but I wouldn't be upset.

5. If you had an elementary student who refused to learn procedurally, how would you respond?

No, I don’t think I would, but I think at some point you have to wean them a little bit off of the blocks so that is not something they feel they have to have to be able to solve something. Maybe teach them a way to see it in your mind. Do you see the blocks? Here’s this pile and here’s this pile and have them think it out instead of having them to physically do it.
I think probably fourth or fifth grade it is good to not have the blocks anymore.

6. If an elementary student who could not understand the concept of (patterns) through conceptual learning, how would you further help him/her comprehend the concept?

I think I would try to break it down for them. Make it a smaller picture first and then add stuff to make them see the big picture all together. It could be easy for us and complicated for them. We had a flower with a center and 6 things on the outside and four petals or something and maybe break it down and say here’s what this is. This is the center and these are the dots. Here are these and here’s what these are. Break it down and say oh, that is how it all fits together and learn to look at it in pieces, not one big awhhh.

7. What do you believe are the main goals, or objectives, you should get across to your elementary students during a mathematics lesson?

Ms. Herandez uses, Multiple Strategies, groups, conceptual and procedural
My main goals—I think a lot of the same things. There isn’t just one set way. It’s my way or the highway and you don’t get to think on your own or whatever. I wouldn’t want to have that because kids are smart regardless and they will find a way to think about it and figure it out on their own way and….encourage them to think on their own. And to also make sure it is within the guidelines or formulas or whatever… That there are several ways to look at it and understand it and still get the right answer.
Groups? I think I would probably do groups and your own work. I think groups are a really good way of getting everybody’s ideas, and if your idea is maybe not getting the answer, then another person’s idea is missing what you came up with or whatever, and that is really important. But I think you also need to learn to do it on your own.
Procedural/conceptual-I think both are good. I think it would depend on the topic as to which I would prefer because I think some topics would need more conceptual than others.
I think I would probably start with the conceptual to just see it because to me, and that is just me putting my idea or thought onto the kids, but to me it would make more sense to see it before, which would maybe help you understand it better when you read it or show them formulas or whatever.
And then whichever method they pick, I would be fine with.

8. How much should a teacher help students in solving problems? Explain.

My son hates me for this because I make him sit there and figure it out. He wants the answers. He wants me to give him the answer, but I will never give him the answer and he hates it. I would be that same way with my class because you are not ever going to learn to think it out for yourself. You are just going to sit and wait for the answer and give up if you don't get it. If you work on it and think about it, take a breather and calm down and look at it again. It will be there.
Ms. Herandez never gives us the answers. She asks questions like how did you get to that or have you thought about this. And be encouraging about it and say here is a different way to think about it or little pointers that are subtle that aren't really going to give you anything.

9. Should students always be able to visualize mathematics? Explain.

To me for me, it helps me out. I visualize math. It helps me, but everybody's different. I think it is easier to understand, if you see it and you don't need something physical there (mentally).

10. Can mathematics always be done using a formula or procedure? Explain.

I don't know. Probably yeah. It has set steps that you can take to get the answer you need or the right answer, not the answer you want.

11. How do you envision teaching your future classroom to ensure that your students are learning both procedurally or conceptually? (depends on participant's preference) Explain.

I don't know. I guess just introduce both at the same time. You know one at a time, everytime…
Because I think you might be able to see the procedure. You can see it and envision it because you have already done it and then you can say, Oh, that's right because I did this and that's the procedure for that because I did it. Actually, understanding the procedure. The hand/brain thing that kind of clicks.

12. Describe your experience with groups in your Math 200 class.

Yeah, it is a pretty good group. I think I learn somewhat from others in my group. Some more than others, maybe…I think students would come to me (I am more quiet) if they have questions. A couple of the people I do look to for help, like Tasha. Her and I work really well together. We had 100 together too and we were in the same group there too. We know each other from math class. We are both really strong in the class.
We have three roommates in our group. They don't always get along and so there is a lot of shampoo switching and toothpaste throwing and clothes stealing that….It isn't math related and I'm older and so really girls? Tasha is not one of those. She also has been in

college for a little while longer than they have. It is the end of the world to turn 19, so uh? Okay. It is the end of my teenage years. That happened a while ago so really….
I help more than get help from others or explains more. I'm definitely more thorough. They make fun of me for writing really long explanations.
I'm a little quiet because I try to sit back and not be like the mom because I think when you are the older one it just kind of happens so I just want to sit back and if they come to me that's fine.

13. Describe your experience working with manipulatives in your Math 200 class.

We did the dice and cards and all that stuff. We haven't done any more after that, I don't think. To me, it is helpful for my own learning because I am a very visual person and it clears things up for me right away. Like I said earlier, for me on a test, it is like, oh, yeah, that is when we did this. I can see it in my head.

14. Prior to university classes, what experiences do you have working in groups?

Math classes-no
A couple of English classes I did group work and a women's studies class. I didn't feel the group work was productive because I am one of the people who ends up with most of it because I don't just let it fly. I'm thorough so for group work, it's just like Ughhhh. I have to do everything because it just all lands on me automatically.
Norway—no group work

15. Prior to university classes, what experiences do you have working with manipulatives?

No

16. On a scale of 1 to 10 with 1 being not confident with mathematics to 10 being very confident with mathematics, how confident are you in your mathematics abilities? Explain.

I used to feel maybe a 2, but since now I'm older and come back. Now, I am a lot more comfortable so I would probably say an 8. Prior to the university, I used to think I couldn't do it and that would reflect in my grades.
I just got frustrated and had very little patience for figuring it out, just like my son. Odd, how that happened. But now, I get it and it makes sense.

17. On a scale of 1 to 10 with 1 being not confident with teaching mathematics to 10 being very confident with teaching mathematics, how confident are you in teaching mathematics at the K-3 grade level? Explain.

I'm very confident, a 9. I feel like I have a handle on it and I understand it well enough to explain it and in different ways.

18. On a scale of 1 to 10 with 1 being not confident with teaching mathematics to 10 being very confident with teaching mathematics, how confident are you in teaching mathematics at the 4-6 grade level? Explain.

9 (same reasons)

19. What types of technology, if any, would you use in your mathematics class? Explain.

I don't know anything about any math computer programs. I'm pretty green when it comes to that but I think calculators, if need be.
I wouldn't let them have a calculator if it is like addition, sub., etc. For addition, subtraction, multiplication, and division, I don't think kids should have a calculator. I think they need to learn all of that in their head. It's just the basics (not big ones—1-10). I would absolutely be open to let students learn mathematics through math programs on the computer.

20. "What kinds of math do you do?" (Raymond, 1997, p. 555).

Procedural/Conceptual?
I very visual (conceptual) so to me I like to learn it that way, if I can. If not, if I have the procedure, I can usually fair pretty well.

21. Do you see mathematics as an "unrelated collection of facts and rules"? (Raymond, 1997, p.556)

I think they are pretty related. A lot of them intertwine and you have to go by a bunch in different problems and different topics. I think they are related. I might use algebra in a geometry class and have other rules that I need to abide by.

22. What evidence would you need to see in order to believe that a student understands a topic in mathematics?

I just think the way they would have to be able to explain it in a couple of different ways, not just the way that they have read about or been shown on the board, but to show that you have thought about it on your own and have an understanding of it well enough to explain in a couple of different ways. It is more than a right answer because if you can't explain how you got the answer, then you can't teach it to anybody else.

23. Is there "only one way to learn mathematics"? (Raymond, 1997, p. 557).

No. (See above)

24. How, if any, has your attitudes about mathematics evolved from
    a. The beginning to the end of the semester?
    b. From Math 100 to Math 2/300?

Dynamic/Fixed—changing (same)
Surprising/Predictable (middle—predictable for easier, surprising when learn something new or new way to figure something out—same)
I have always held these beliefs about math.

25. How, if any, has your attitudes about mathematics teaching evolved from
    a. The beginning to the end of the semester?
    b. From Math 100 to Math 2/300?

Procedural then conceptual—now it is conceptual then procedural
Teach both depending on topic (same)
Easier to understand procedure (no now that I have done a lot more conceptual, I think conceptual is easier)
I think my son has changed my opinions and the way my children see it and how I see it clicks for them and they get it and what's helped them learn it. And they actually both learn in different ways. My son is visual like me and my daughter is really procedural like my husband who was also a math major. It just kind of clicks for her. And I just see her go so she is not that visual at all.
That might be really why you hold this dual opinion….(my comment)
My husband is a civil engineer.
We both help our children with math but in really different ways. Because for him, he sees it, and he knows the procedure and he says that this is just how it is done. That is his way of doing it because that is just how it is.
I'm more like, "If you do this"…or, "How about we do this?" He's like, "No, this is how it is." Don't ask questions, this is just how it is. And my daughter is a lot like him. She is like that is how it is and no questions asked about that. She is like I know. This is how it is.
I think I am more helpful to my son. Well, my daughter is only seven. She kind of takes care of business on her own and gets it right so I let her. My son struggles more. She is ahead in math in school, but he still has to think on his multiplication. It doesn't just click.

26. For the last part of the interview, I want to know how you might address the following concept to your future elementary student.

    Math 200—patterns

I think, I hope I would do it a lot like Ms. Herandez does it. I mean honestly at their level, but….I like how she has broken it down to a very, very basic and moving it slowly to a much broader picture. We started with easier patterns and what comes next. That was like the beginning and then there are three pieces missing and then what would the 10th piece be? What would the 50th piece be or what shape would it be? What about the 100th? How would you figure it out? And so you broke it down to the very, very basic level. Manipulatives? Maybe, at least pictures. I don’t know. Maybe, depending…I think it might depend on the class and the level of the class as a whole. I think you have to kind of feel it out. I would be open to it. The visual the could see the pattern instead of saying triangle/square/triangle/square. Now, what’s next?

APPENDIX Z

MS. HERNANDEZ'S INTERVIEWS

Ms. Hernandez's Interviews

Interview 1: Ms. Hernandez Math 200 3/5/2009

1. What is the teaching philosophy for Math 300?

I really want to engage the preservice teachers in Math 200 at a very basic level of understanding. They have had bad experiences with algebra and statistics, are very afraid of them, or they feel they are complicated. I really want to engage the preservice teachers at a basic level and build up to the things they have seen. In general, it's about building their confidence and their skills of being able to do those problems. For example, probability has fractions in it. Most of them have very little confidence with fractions. They can do it, but they just second guess themselves a lot. I try to build some of their confidence. I try to set it out to be, "See you can do this in an elementary class. See here's an activity that would even work for first graders." I try to have them recognize that it's not high level, super complex ideas that they have to work towards doing. It is something that little kids get. It's something they can do instead of something to be afraid of.

2. How well you think the following participants do in your class academically? Do they struggle with anything? How do they feel about conceptual/procedural?

Nicolette—She is performing really well academically in my class. I think she has an A. She wouldn't believe me but she has an A. I'm pretty sure.
Does she struggle with any concepts? I think she did initially with probability, but some of that struggle is…seems to be more a lack of confidence than lack of ability because she did really well on the first test. And she told me before the exam, "Wow, it clicked. I really got it." But she still was very nervous and worried about taking the test. So to me, it seems more to be a confidence issue than ability issue.
Is she good with procedural and conceptual? She always struggles initially with both because she really…I have asked her…but it has probably been a while since she has had math. And so the part of her that she used to know, this interferes with the process and once she stops and thinks about, she says, "Oh, okay." And she sees the interplay between the procedural and conceptual together. I don't think she thinks she does see this connection. She is able to do both.
Does it bother her if she doesn't make an A? No, I really don't think she worries about her grade but she was honestly really surprised when she got the first test back and it was an A. It was honestly…She doesn't seem to be the kind of student who attaches way too much to the grade.

Natalya—She doing really well. She is also doing really well. She seems to interact really well with her group members. She seems to be one of my more confident students in her abilities. I think she has an A. Yes, she does.
Does she struggle?—If she does, I don't see it. She is a very confident person. I don't know that I would see that.

Is she good with conceptual and procedural?—Yeah, I think the conceptual, she… I think she agrees in class and goes home and works it out by herself to really clarify it for herself.
Does it bother her not to make an A?—No, I really don't. She's a lot like Nicolette in the sense that she doesn't seem to attach as much importance to the letter.

Tasha—She does really well. She is one of the more confident students in the class, probably the same confidence level as Natalya. She wants a little bit more reassurance that she is right than Natalya does. And she is confident enough to hold her own in a group, which is a good thing.
Not quite as much reassurance as Nicolette. She thinks it is this way. Someone else thinks it is this way. They want someone arbitrarily to say one is right and one is wrong. They want me to pick sides or something. And oftentimes I'm like, "They are both okay because you got the same answer," and they are different ways of doing it. She seems more okay with that answer than some of the other people. They want me to go back to the traditional one way to do this mode of thinking. But they are young…That is one of the challenges in this class to break out of there is only one way to solve a problem and move to there's one correct answer and multiple ways to solve.
She does have an A. I think she has one of my higher A's.
Does she struggle?—I think she struggled a little bit with the conditional probability in the sense where you have marbles in a bag and you pull one out and keep it. I think adjusting the probabilities for pulling the next one seemed to take her a little bit more to sink. Some of it was conceptual in the sense that it wasn't quite clear, initially, why the numbers were changing. And maybe that's because she maybe is more kinesthetic than some of the other students. That is the case where I tend to notice who is kinesthetic and who's not. Kinesthetic students really want to see it to connect that.
Does she want an A?—Her, yes I do. Well, she seems…I think she would be okay to not get an A on an assignment, but I don't think she would be okay if she didn't get an A in the class, just by what I've perceived and that's not always the truth, but..

Taylor—I don't know. Taylor's really quiet. I feel really bad. It is generally the students who are the most outgoing whose names I get really well. I don't know that I have had a lot of contact with Taylor.
Does she have an A? She's got an A in the class. She's just…She's really quiet, comparatively, so I really haven't had much interaction with her.
Procedural/Conceptual? I really haven't had….Everybody at the table pays attention when they ask me things and when…It's not like she is off in la-la land, but I really wouldn't know how much of that connects with her or doesn't.
Does she want an A? I wouldn't know with her. She is not one of those students I have had a lot of interaction with.

3. How do you think the following preservice teachers utilize group work?

Nicolette—I think she does really well. I think she's…There's some people in her group that think procedurally and really don't want to spend the time on the conceptual part. And she is the person that says, "Okay, why did you do that?" She really tries to force

them to stop and walk through the conceptual part with her. So I really…I wish every group had somebody like that. She's really good about making them stop and put some thought into it and walk it through for her because she needs the time and the effort to go through it that way.
Giver/taker? Initially, I would say she would ask for help more than give it. The first couple weeks of class. I think it is more balanced now, but we are also on a topic (data analysis) that is not as uncomfortable. That may influence that. Nicolette asks her group and gets somewhere and she needs me to reassure her that she's right. She's looking to me more for the reassurance of authority than for clarification.

Natalya—I think she does really well. I think sometimes she…She is in a group with a lot of really strong personalities so sometimes she may be more quiet than just because of that she tends…I also know that when she doesn't understand or she has a question, she's not afraid to speak out to the group and to me. She seems to be very comfortable with how her group works and the dynamics set up so generally, she is not one of the ones that has her hand in the air for the group.
Giver/Taker?—She's pretty balanced. She'll ask and then she'll help.

Tasha—She is one of the stronger personalities in the group. She's occasionally the driving force behind how her group works and so that's a really good thing. But occasionally there's the coming over and who's right and who's wrong. They can stop themselves that way with that discussion about well I think it is this way and I think it is this way. And so sometimes, that can be a hindrance. Not a bad one, but…
Would you say Natalya and Tasha are the stronger ones in that group? It's hard to tell because everyone in that group is really vocal except for that one student whose name I don't know, which is probably because everyone else in the group is so vocal. It's kind of hard to tell because that is a really big group of…the most alpha females sitting at a table that I could imagine because I'm pretty sure there are four of them there or at least three that are very…okay. Natalya is a little more quiet in it, but Natalya's comes from her being the oldest in the group. Tasha is a little bit more outgoing.
Giver/Taker?—I think she tends to be more on the giving side. She seems to pick up the concepts relatively quickly so she's the person who is part of the explaining and helping other people understand.

Taylor—Yes, she does work but she is one of the quieter ones. Like I said, I really haven't…She seems to be one of the quieter ones but that….We've learned that could be so many things. That could be lack of confidence. That could be I do know this, and I'm okay. There are so many things that being quiet can symbolize. Based on her grades, I don't think her quiet is a, "I don't get it," but it maybe, "I don't want to speak up," or, "Afraid I'm wrong," kind of thing.
Taker/Giver? I think she would, probably in a lot of cases, she would be the observer of the group. Yeah, she is just kind of quiet. But like I said, that is my perception.

4. How would you respond to these quotes from preservice teachers' interviews and classroom observations?

Nicolette [She has a low self confidence in math. It takes her longer than some of the others.]—Yes, and no (about getting the materials) Some of that comes from that whole confidence issue. She gets it but she doesn't quite…I believe she gets it…It's kind of the low self confidence really impacts everything else so that makes it kind of hard. I also think that she has a higher expectation of herself, and she really wants to make sure she is learning things well. She has a lot invested in that, not just getting through it. She really wants to make sure it makes sense to her before she moves on. If only all my students were like that….
She feels her work is like 50/50—I see that she helps Caleb. I think she helps all of them out more by making them stop and having to work through it.

Natalya [She talks about how her group talks about different things-a lot of material different because grew up in different country.]—That is what I meant when I said she is in a group with a lot of strong personalities. She tends to be quieter, but some of that comes from they're freshman girls and she's right. They're freshman girls. They would be really good for other people's opinions to be okay. Okay, that's okay and that's what you think. That's my opinion and very opinionated and I have the right to my opinion. They're strong personalities. I think that is why she tends to get quieter when they get into that freshman girl mode. She interacts a lot. And it is always her group. She might not notice it, and I only notice it a little but because she is older they tend to look at her to double check. When they explain something, they check everyone's faces and they will check Natalya's, for sure. You know it doesn't happen all the time, but I have seen it happen.

Tasha—[Her background is pretty procedural, except when she came here. She likes learning conceptually and understanding "why."]—Yeah, she wants to really understand what's going on, which is part of the reason I like that, she and generally other student, are the ones saying, "I did it this way," and, "I did it this way." Both of them are trying really hard to get at that conceptual understanding of what's happening, but they are still….There is still that habit of there is only one way of doing things and that is a very hard habit to break. I'm not quite sure I've broken it. She really wants to get at that understanding and sometimes I feel she gets a little frustrated with her group because she wants to take it a little further than they do, but she may do that on her own, where I'm not around to see it.

Taylor [Some of people may not have confidence with math. She really likes math-fun in understanding]—That is what I kind of would say. I haven't had enough interaction and her quiet could be…If she's a math emphasis, the quiet is probably, "I do get this," and some of that may come from she may have tried the explanation part. That is something I have noticed, especially with math emphasis and the very math competent is that we tend to talk about it a little bit higher than anybody else so she may have tried to help and now she is being quieter to listen to a different level of understanding as part of her learning process because I still have to do that. I have to listen to what they say to…because math was never difficult so to me sometimes I learn more from my students because I never had to struggle with it. So I spend a lot of time going, "So tell me, I'm trying to make sure I understand what you are talking about." And so she may be doing some of that

with her group and that may be why she is quiet because she is trying to hear their understanding.
[She feels bad that there are some strong personalities at her table.]—Oh, yeah, which is why I really like it when Nicolette…Nicolette will say, “Stop, I’m confused.” That’s part of why I really like the fact that Nicolette is not afraid to make them stop and spend some more time on it because there are a couple people in her group who are like, “Well, everyone else is finished but us. Why can’t we be?” To them, they are taking this class to get through it, and you are always going to have those students who are taking this class to get through it. I can’t wait till they teach. I’d love the email about that one. But a lot of these students have the misconception of they are not going to have to teach math.
[Yeah, they think they want to teach the younger kids because they want have to explain as much]--Have you ever been around a six-year-old for longer than 20 minutes? Let me tell you about the “why.” There’s a “why” for everything. There is a “why” after things you are not even sure you’ve ever thought of them out. I don’t have kids, but I have nieces. They are all and have gone through and are going through this why? Well, why? But it is really interesting because you are like, “I have never thought about why. Give me a second.” And there are some of the people there who don’t really care about the “whys.” They want to do what they have to do to get out and the other people at the table really want to get it clear so is a dichotomy, but I haven’t been asked to intercede and so I want. That is another thing about the teaching philosophy. You are going to have to work with people that you don’t agree with and you don’t share opinions with and you have different ideals from so you better learn how to do it now where it is not going to have to cost you your job. So I don’t interfere in a group or how the group functions unless I am asked. And then if I have to interfere, nobody is going to be happy. But that is how it works in the real world. If your boss has to intercede to make your group work, there is generally a punishment involved and you’re adults and you can’t work together. Well, you are in college. You are an adult so figure it out so the entire time I have taught the preservice classes, I have only had to interfere with a group one time. And that was a very particular special case. Generally, once they realize I won’t interfere, they do a good job of figuring out how to make it work. And sometimes that comes from the person who is hurry to get out and they say, “Okay, go ahead and go. We’ll do it. We’ll finish it without you,” and I’m okay with that too because the person who is cutting out early is…They are hurting themselves. They are not hurting the group so I’m okay with that.

5. What do the following preservice teachers feel about manipulatives?

Nicolette—Actually, I think she is very open to using them because I would be willing to bet she never saw them in school. I didn’t notice any hesitancy or nervousness about using them. In general, she seemed to be the person going, “Okay, doing this,” and really trying to create her understanding with them.

Natalya—I think for her, it wouldn’t matter if they were there or not. It helps, but it isn’t necessarily necessary for her to have them. No, I don’t get that she tries to separate herself from them. Believe me, I have students who do that.

Taylor—No, that group was actually really good. The entire group, so I mean, I am guessing they all seem to be really well with using them and working with them.

Second Instructor Interview

Ms. Hernandez Math 200 *4/23/09*

1. How well you think the following participants do in your class academically? Do they struggle with anything? How do they feel about conceptual/procedural? Any changes?

Nicolette—Nicolette currently has an A. I think she is struggling with confidence. She has a pretty big lack of confidence. I think that is her biggest stumbling block and so that comes across as struggling with the material, but she can do it. She just doesn't really believe she can.
Conceptual/procedural?--As far as conceptual goes, she is doing just fine, but she has to have the conceptual down before she even tries to get the procedural down. That is how she works. She has to understand it. She is not willing to just memorize a procedure.
Nicolette did the strongest with the data analysis stuff, but that is pretty common. It is the most familiar: making graphs, creating graphs, and understanding mean/median/mode. It's not a lot of really intense mathematical stuff. And the conceptual stuff is a lot of stuff they have internalized. It is just about getting them to recognize that internalization. They know what the "mean" is. They know what an "average" is, but getting them to verbalize and articulate that understanding. She did really well with that.

Natalya—Natalya has an A. Natalya isn't struggling with anything. She is doing just fine.
Conceptual/procedural?—From what I've seen, she is doing really well with both.
Natalya did really well with all of them. I couldn't pick one over the other.

Tasha—Tasha has an A. Tasha is still struggling, just a little with the conceptual. She is a lot more comfortable with having a procedure.
Tasha did about the same with data analysis and algebra. Probability was the one she struggled with. She has done really well with the other two. Most of the students have already made the decision that probability is hard and so by making that decision they have made it hard. That is just a byproduct of a personal, internal decision they made.

Taylor—Taylor has an A. Taylor is shining. This is material she is comfortable with. She seems to be doing really well with both conceptual and procedural.
This is the material Taylor understands the best of the material.

2. How do you think the following preservice teachers utilize group work? Any changes?

Nicolette—Not much has really changed from our last interview. She's gotten a little bit more assertive about saying, "Stop! Hang on! Give me a minute," which is good.

Natalya—I don't really see a change. She still is pretty quiet, as far as it goes. She will talk, but she is not the leader.

Tasha—Tasha is more a leader of the group. Half that table is really quiet and half that table is really not. It is kind of hard to tell because people who are not quiet tend to possibly overshadow. Tasha works pretty similar to the other interview.

Taylor—Taylor is a lot more talkative from last time and a lot more involved. She is more of a leader with the algebra section.

3. What do the following preservice teachers feel about conceptual material? Their progression in thought about conceptual learning? Any changes?

Nicolette—Nicolette is getting better at the conceptual aspect of the course. Her confidence is her stumbling block and that is just a big thing for her. Her self esteem is getting better about it. "I get this. I understand it. Okay. Alright. Okay. That works." That is her big problem. She understands the material a lot better than she thinks she does, but it is literally her confidence. She just doesn't believe she gets it, but she does. If you talk to her and talk her through or ask her questions, she can answer them. She knows the answer. She just doesn't feel confident.

Natalya—I think she has gotten a little bit more comfortable with using pictures and descriptions to do this. Some of it is through interactions in class and some of it is her expressions when I hand out assignments. Her expressions aren't as distant. She is more willing to engage. She seems to be more comfortable.

Tasha—no change

Taylor—I don't know that the conceptual has changed but her confidence has. I don't know that she has had a lot of problems with the conceptual. She just seems to be more comfortable with it now.

4. How would you respond to these quotes from preservice teachers' interviews and classroom observations? Any changes?

Nicolette-[Some in groups further ahead than others. Some who struggle like me. Taylor always understands and helps.]—This is pretty accurate. Nicolette really, really wants to understand. I don't want to say it is a lack of patience. That is not it. She just wants to understand it, but I think part of it is that she wants to make sure she understands it before she leaves the class. She really wants to make sure that she has got that while I am still there or Taylor is still there or there is somebody there that she can ask for affirmation of what her thought process is. She doesn't want to wait until later. She really wants to make sure there is someone there to say, "Yes, that works," or, "Okay, I see you got it. You are doing great." Just that positive reinforcement is what she wants.
[The tree diagrams are more helpful than the physical pieces.] For most of them, it is more helpful because the dice and cards and stuff like that are everyday things. So for a

lot of them, finding a new way to look at that and being able to see it drawn out really helps a lot. The tree diagram also has a very basic procedure to it and that gives them a comfort place to come from.
[She starts conceptual and then moves to procedure.]—That is how she does it. She wants the conceptual down before she moves on to a procedure.
[I am a 7 in confidence.]—I can see why she would say a 7 because she is talking about elementary students because they don't have preconceived notions, so it is new. So she would be comfortable with it. It's everybody else's preconceived, having to wade through her confidence in there, because everybody else does things so differently before she really has a chance to develop her own way. She is hearing them all talk before she has got it in her head and so for elementary students, that would not be an issue. She would be just fine. It is just having everybody else tell what they think or what they are doing before she is clear. That is where she comes across as lacking the confidence because they start talking, and she is still trying to get it straight.

Natalya—[I think I learn from some, more than others. I am really quiet. I look to some for help like Tasha.] –I agree.
[Some are roommates in my group. I get frustrated with them somedays.]—"I'm older than this and can we just get back to what we were doing?" I see that, but I will also say that that group is very, very good at self policing on staying off topic. I don't have to go over and go, "You have spent 10 minutes talking. It is time to focus on math." They tend to be better about self policing and pulling attention back to what they are doing themselves. I don't know how much of that is Natalya. It wouldn't surprise me if she was the one that said something to Tasha, and Tasha is the one that makes the group get back.
[I help more than get help. I am more thorough.]—She is more thorough, and I think the girl that sits on her other side turns to Natalya. She is a little quieter and so the other three girls, besides Tasha, are a little much for her sometimes. I think she turns to Natalya to help. I think that is where Natalya does the most good is with her because she is really quiet.
[I try to sit back and not be like the mom.]—I feel that way as well.
[I am a visual learner.]—Yeah, she is more comfortable drawing. She is not one of the ones who is, "What do I draw? How do I draw it?" She is really willing to kind of jump in.
[I am visual, conceptual. If I have the procedure, I can usually fair pretty well.]—I agree.

Tasha—[If we see one person behind, we slow down. Natalya, I love. Four are freshman. It is me and Natalya. They look to us a lot. We are older.]—I agree.
[100 manipulatives didn't help. I didn't understand 100 material. With 200, the manipulatives did help.]—Some of it, for her, procedures are just easier for her to start with, but she is doing fine with the conceptual. Maybe it is how we talk about it. Maybe it is how I approach the activities that makes it easier for her to step into that role because I do know it is easy to get frustrated. If you don't understand the first couple of times, you figure you are not going to, and I don't get that feeling from her at all, but I don't know if she felt that way initially. I didn't pick that up.
[I do procedural and conceptual. I get going and don't read the question. I get to going so fast that I miss things.]—I agree. She gets into the grove of what she is doing and then

there is an assumption that everything is the same thing, but I think that falls into that is the easiest. Some of that comes from when you got the conceptual, you don't tend to go through the entire conceptual process for every problem. "Oh, okay. I get it. I understand what I am doing. It makes sense." So you fall onto the procedure because it is quick. It is fast. It is easy. "I got the conceptual. Okay." So she kind of gets going, and she has to stop and go, "That is not what I asked. That is not what it is talking about."

Taylor—[I have a tough time doing the elementary way.] That is not how I see it. I think she does just fine explaining it. I think she is doing just fine explaining things, but I think it is something she just knows how to do and the struggling is the articulating it, but she is doing just fine. It is some of that, maybe she has been so quiet. Maybe stepping into that new role is a little. Some of that is, "Well, everybody else has been doing the explaining. I don't explain as well as they do."
[At the beginning, I didn't listen to the conceptual.] Well, that makes me curious who she had for 100. Did she mean to escape through without having to conceptualize? Did she get into the habit of just tuning out the conceptual part? That is something we should be aware of. If we catch students tuning out, we should have preventive measures to prevent that from happening.
[I am most comfortable with algebra. It just comes to me. I like the whole solving and coming up with answers]—I agree.
[The pictures are more of a check.]—I agree.
[I think more procedurally. The pictures are making me explain more conceptually, even though I get things procedurally.]—I agree. She is having to get the conceptual down to help them. Before the algebra unit, she was so quiet. She was willing to let them talk, but she wasn't really tuning in because she knew what she was doing. That comes from how they are taught before they get here. A lot of K-12 focused on procedures. It is a shift, coming to class and thinking it is going to be easy and we are like, "Oh, no. We are going to start all the way at the beginning and relearn everything entirely different." Some of the students have it and get it. They are not going to regret not knowing the conceptual until they teach. And that is when that is going to be like, "Oh, wow. We talked about this. I don't remember." It is hard to get them to buy into the conceptual because that is not how math has been taught. They tend to fall back on how they learned because that is how it has been for 12 years and why do I need to do it differently. If it is not broken, don't fix it.
[One of the lowest confidences in teaching math at the K-3 level. I am always afraid I am saying something that doesn't make sense.]—I can see that very much. She is very patient, but it comes from that. I can see that. The way she does it in her head is not the way they do it so she doesn't want to share that because she doesn't want to confuse them, but I see her doing just fine. Some of that will come from practice. It helps when she sees Nicolette get something she says. Nicolette is going to say. Nicolette is also an adult in the sense that she has kids. She is not 18, 19, 20 years old. She has less invested in how her peers see her. To Nicolette, and she never talks to them this way, but I remember being an older student and I remember thinking in my head, "They will grow out of it." You don't worry so much about what they think because they will grow out of it. You just have a little bit more confidence in yourself about being a person to be able to say, "Look. Stop. Hey, wait. Back up." Whereas, sometimes I think some of the younger

students, freshman and sophomore, they don’t want to look stupid in front of their peers. They don’t want to look like they don’t get things so maybe they don’t ask about it. There is also that, “I don’t want the rest of my group to think I am dumb.” I don’t want other people I see every day to think, “We have to explain it to her again.” There are those other personality things that come into play. I think it is hard to make it okay.

APPENDIX AA

SUMMARY OF NEW CODE WORDS AND USAGES

Summary of New Code Words and Usages

Code Words	Quotes
Belief about Math Participants explained their evolution in beliefs about mathematics.	Before now, I don't know if it was so much predictable because through high school you keep learning and learning until the last couple of years, you use the same concepts. You are just broadening how to use it so I guess before coming here it was more surprising than predictable. But not even now, it is not so much predictable but I think I have more knowledge and now I am using that knowledge and putting things onto that knowledge and connecting that knowledge to other things, but certain things are still surprising. Stuff I haven't seen are cool. I think the fixed part is probably something as you are growing up you think there is only one right answer so maybe that is why I always felt it was fixed and there is only one way to do it. But then when you work with more kids and you see other people do math and stuff then you are able to see that maybe it is more dynamic than predictable.
Belief about Math Teaching Participants discussed their evolution in beliefs about mathematics teaching.	The change in my beliefs about math teaching occurred because of Math 200 ideas and me actually. I just think things are really cool when I can just look at it and that makes more sense and that would make more sense to an elementary student. I feel like I am going to use some of this stuff later on when I teach. I think they have changed. A lot of the way I grew up, junior/high school, is this is how you do something. This is your hmk. Do it. I think that is the only way I knew math until I got into an educational program where you have to learn the different kinds of perspectives. I learned about the depth of an idea than just why it is. I think I have evolved in my way of thinking.
Confidence Participants mentioned their confidence levels in doing mathematics and/or teaching mathematics.	Probably 8 (with 1 being not confident about teaching math to 10 being very confident teaching math), not quite as confident (about teaching mathematics at the fourth-sixth grade level. It is a little deeper. You know if I could do it for a year. The 8 is probably the unknown factor. I am thinking like of my own three kids who all do really well versus if I had low end (less competent in math) kids. I would be a little bit more hesitant in my abilities with them. It is just because of the lack of experience of really being the teacher because hopefully after actually doing it, it would grow. I am personally kind of dense when it comes to math.

Define Math Participants commented about their beliefs about mathematics.	With math, I think there are right and wrong answers. I think it is dynamic because there are different ways to get there, but you can't do 2+2=5. It equals 4. Math is related to everything else and everything else is related to math. Seeing it through different methods can only help their understanding.
Disconnect Nontraditional participants talked about their problems with connecting to fellow preservice teachers in their class.	It's very apparent that I am at a totally different point in my life. That's just because I'm a nontraditional student. It has nothing to do with the dynamics of the group. It's very hard for me to come to class and sit and talk about what's going on in the dorms. I have a mortgage and two kids. My biggest worries are $300 a week in daycare so I can go to school. Sometimes, I have noticed that I get it but I can't explain it to them because I'll get an answer and I know my answer is right, but I think they will debate me on it so I let it go. That is why I don't like groups because you get some people are right all the time and don't want to be wrong. And then you get others who might be right. I don't know.
Schooling Participants explained the impact of the Math 100/200/300 sequence on their beliefs about math and math teaching.	Probably, a lot of my beliefs and everything have changed since coming to here and in my math classes because before I hated math and I just everything and I wasn't confident in it and through the courses, and Geometry I'm still kind of struggling with but, it was just really fun learning it and just exciting. Prior to coming here, I liked Algebra but the secondary teachers' personality had a lot to play with me not liking math. I think a lot of it comes once I came here to be a teacher.
Working Participants discussed their work experiences in the public schools.	It a little school in Caney. If you were to look at test scores, they were the lowest for the Caney community. But even looking now, I believe we were already two years with that program, you could see the difference in the kids. What was neat about it is kids were moving at their own pace. I had some kids who who already in 9^{th} grade math who were 5^{th} grade in 9^{th} grade math. I worked at an elementary school for 6 years. I helped in fourth grade math and first grade math. I didn't teach the math. I worked with a group. In the first grade, I just worked with one boy. I helped him with counting. He didn't pay attention. I usually took him in another room and I worked with him on counting and adding.