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Exploring Teacher and Student Experiences In a Nature-Based Mathematical Environment

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EXPLORING TEACHER AND STUDENT EXPERIENCES IN A NATURE-BASED MATHEMATICAL ENVIRONMENT

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ABSTRACT

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This research study explored the experiences of this teacher-researcher and eight student-participants in a Montessori-inspired, nature-based mathematical environment. I utilized an action research study framework with a critical event narrative analysis to describe mine and my student-participants' experiences throughout this study, using Uhrmacher et al.'s (2017) instructional arc as a guide. I restoried each participants' lived and told stories as the curriculum seamlessly moved through the arc: intended to operational to received curriculum. The restoried narratives revolved around the identified critical events each participant experienced throughout the mathematical learning experience. The multiple restoried narratives required data collection that included observations, documents, interviews, and photographs to understand and interpret the different and unique experiences of each participant. Findings of this study suggest that students are able to see the mathematical connections in nature with a noted reduction in their mathematical anxiety and increase in their productivity while learning and creating piecewise-defined mathematical equations outside. In addition, this study found that prior experiences influenced this teacher-researcher's curriculum design planning and these student-participants' ability to access their prior mathematical knowledge.

ACKNOWLEDGMENTS

Six weeks after my second son was born, I boarded a flight to Colorado that would take me down the path of educational studies. My wife encouraged me to board the plane, despite having a newborn son and his two-and-a-half year-old brother to take care of, and has since been my biggest cheerleader, advocate, and motivator down this Doctorate of Education road. I could not have completed this journey without her support, picking up the slack when I could not and talking me through my thought processes. She inspires me to be a better writer, a better thinker, and a better educator. Thank you, Lindsay, for the encouragement and the sentence structure lessons. I love you!

I want to also thank my committee members for their investment in my education and career. Dr. Christy McConnell, thank you for always entertaining my ideas, even when they are seem impossible (i.e. communicating with bees). You have pushed my critical thinking about educational practices and philosophies beyond what I even thought possible. The lessons you taught me have influenced my teaching and curriculum design as evidenced in this research study and will continue to influence my future lessons. I am indebted to you and your wisdom. Dr. Derek Gottlieb, thank you for engaging in deep philosophical discussion on the history of education, pedagogy, and theory. In my cohort's first course you instilled in us a "duckrabbit" mentality, and it is a lesson I will remember always, valuing the varying perspectives on educational issues. Dr. Hannah Kang, I appreciate you taking the time to guide me as your graduate research assistant. The experience progressed my research skills and literature review techniques, and it also gave me the opportunity to work with a class of undergraduate students

for the first time. Dr. Chelsie Romulo, you immediately took me under your wing, offered me a spot on your research team, and challenged my writing style, encouraging me to analyze the literature and describe my connections more effectively. I also thank you for allowing me to see to the behind-the-scenes efforts that are needed with a grant-funded research project. To all of you, I appreciate always feeling valued and supported throughout this dissertation process. Your insights, kindness, and thoughtful feedforward suggestions were greatly appreciated and vital to my growth and this dissertation process.

To my current, past, and future students, thank you for serving as my main driving inspiration to improve. I tell you all, all the time, that I am a reflective practitioner, and this research study is, in part, because I wanted some philosophical understanding as to what I should be reflecting on. The other day, the theology guide mentioned that you, the students, "challenge" us, the teachers, every day. And as much as you balked at the idea, it is true. You challenge me to be more creative, more innovative, more engaging, more eloquent…just more. You challenge me to be more, and that is a good thing. Continue challenging me to improve my teaching methods—I am here to learn from you as much as you are here to learn with me. Thank you for always inspiring and challenging me to improve on a daily basis. And, thank you to my current students who participated in this research study. Your insight was, and is, invaluable.

Finally, I want to thank all of my former teachers, professors, and mentors, who encouraged me to think about math differently, to see the mathematical applications all around me, and to pass that mindset on to my own students: discovering the natural beauty of mathematics and the mathematics in the beauty of nature.

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1, 1, 2, 3, 5, 8, 13, 21, 34, and so on to infinity. The Fibonacci sequence, the basis for the Golden Ratio, and the first time I remember thinking math was relevant. I connect this thought with sunflowers, nautilus shells, Da Vinci's Vitruvian Man, and pinecones because these were the examples provided in a middle school math classroom. The sequence is simple enough—the third number is the sum of the two numbers preceding it—and found in nature, which is possibly the most intriguing aspect of this sequence: "The golden ratio is built into natural systems as they grow and evolve" (Ornes, 2019). My fascination with Fibonacci and his sequence continued to increase through my secondary mathematics classes, culminating in Calculus, with a whole unit studying the importance of sequences and series.

I still remember the feeling of connecting math to nature and the impact that it had on me as a young mathematician and as a future math teacher. While teaching the point-slope formula of lines, solving quadratic equations using the quadratic formula, or calculating the area formed under a curve, I always attempt to connect these mathematical principles to nature or real-world applications to show the relevance of the concept. Honestly, this is harder with some mathematical concepts than with others (I'm looking at you exponent rules and imaginary numbers), but Venters (1992) notes the analytical, logical, and problem-solving skills associated with solving and simplifying mathematical expressions and equations using a variety of mathematical tools are skills all students need to possess and apply to all future interests.

The beauty of and the nature connections with mathematics may not come naturally to all students, "but maybe the beauty of math is more like poetry. Not because it speaks to truth, but

because symbols, arranged a certain way, stream through our eyes or ears into our brains, where they collide and bounce off each other until they make some kind of sense, like a puzzle solved" (Ornes, 2019, p. xi). Unfortunately, this is not an intrinsic skill; students need a teacher or a mathematical experience to guide them to these types of connections.

How can I bring my mathematics classroom out into nature? What would happen? What lessons could I do with the students? Would it be meaningful? These are examples of the types of questions that meander through my brain as I challenge myself to give the students experiences making math-nature connections. I want the students to see the multitude of mathematical relationships and processes that exist in the real world, to see the natural beauty of mathematics and the mathematics in the beauty of nature.

CHAPTER I

INTRODUCTION TO THE STUDY

Looking out my classroom's windows, I see the leaves of the interconnecting trees forming fractal patterns; absolute value functions shaping the rooflines of the townhouses across the street; rectangles, squares, and semi-circles forming the outlines of the school's soccer field; and other mathematical connections sitting just outside the classroom window. I wonder how I can get my high school math students to see those connections, too.

At the secondary level, research shows an increase in students attempting to distinguish themselves as "math people" or not (Chestnut et al., 2018): "Underlying the very idea of a 'math person' is the more fundamental notion that doing mathematics requires some sort of innate quality—a spark of brilliance or a 'gift' whose presence determines whether someone is a math person or not" (p. 65). This mythical notion of a brilliant gift affects students' mathematical mindsets (Boaler, 2016), creating fixed mindsets (Dweck, 2010) which is a "barrier to math success" (Chestnut et al., 2018, p. 65).

However, research has also shown that a student's perspective on math and the mathematics classroom mirrors their parents' perspectives and experiences, (Hwang et al., 2019), including those perspectives relating to whether or not doing math is a "gift" or not. Therefore, in the classroom, the teacher (Taylor & Fraser, 2013) and the environment (Gilbert et al., 2014) are important components to either challenge or confirm the student's preconceived notions. The teacher's role in designing the student experiences in a mathematics classroom and the classroom

environment itself are important external components (Chawla, 1998) to consider when researching students' experiences in a mathematics classroom (Gilbert et al., 2014).

Reflecting on my own personal connections with math in nature and my desire for my own students to see these connections, I want to understand these experiences in my math classroom. My research considers both the teacher's role and the classroom environment in designing a mathematical nature-based educational learning experience, analyzing my own intentions, as the teacher-researcher, in the design process as well as the students' experiences in the environment and participating in a mathematics lesson.

Researcher Stance

When my wife and I started thinking about parenting and educational strategies and philosophies, we began researching nonconventional methods to teaching and discovered Dr. Maria Montessori's philosophies and her follow-the-child educational pedagogy (Montessori, 1912). As former public high school teachers who became jaded with the rout system of learning required of us by our administrators, the uninspiring district-provided curriculum, and the lack-luster, motivated-by-grades-only students entering our classrooms, we immediately dove into the literature on the Montessori Method (Montessori, 1912, 1967) and started implementing her strategies in our own classrooms, seeing incredible results.

Montessori (1967) believed that a student becomes an adult "by means of [their] hands, by means of [their] experience, first through play, then through work. The hands are the instrument of human intelligence" (p. 27). Montessori was drawing on the knowledge that the hand is a part of the human body's sense of touch which is "a critical component of [the] sensory system, playing a key part in mediating our ability to explore and dexterously manipulate tools and act upon our environment" (Bensmaia & Manfredi, 2012, p.379). The hands are a key

component to a student's learning and are important at all levels and all ages of education because "the task of getting an adequate *grasp* on the world, intellectually, depends on our doing stuff in it" (Crawford, 2009, p. 164). Students have to have experiences and the ability and opportunity to explore their environment, gaining knowledge and understanding from the experience (Dewey, 1938) and directly from the environment (Demarest, 2015).

My wife and I now both teach at St. Stephen's Montessori (a pseudonym), a small private, Montessori, Catholic school in southeast Texas, providing the avenue to practice and research a variety of pedagogical techniques, grounded in Dr. Montessori's adolescent teaching philosophies (Montessori, 1912, 1967, 2017). St. Stephen's Montessori (SSM) educates students from age 14 months to 18 years old. Students are grouped academically in communities ulitizing a three-year cycle based on their age (Montessori, 1912), and most of the students that I teach in the high school community (grades 10^{th} – 12^{th}) have been in a Montessori educational environment from the beginning of their educational journey. This research study took Montessori's (1912) ideal of a "prepared environment" and applied it to an outdoor classroom, creating a prepared outdoor mathematics classroom and environment conducive to learning mathematics and aptly connecting math to the surrounding environment.

Problem of Practice & Significance

Testing and Accountability

The high-stakes testing culture (Larsen, 2020; Long, 2014; Williams et al., 2018) in the public schools across America has contributed to creating toxic learning environments for students (Fisher, 2016), pushing some students to drop out of high school (Larsen, 2020), and valuing test-taking teaching strategies over learning for learning sakes (Williams et al., 2018). Teachers spend over 30% of their instructional time proctoring tests or teaching test-taking skills

to their students (Long, 2014), leaving little room for teachers to engage students in projects, experiments, and research that is of interest to them, limiting teacher creativity in the classroom (Williams et al., 2018). The time needed for testing and test prep comes from the time students can spend outdoors (Martin et al., 2018), in the arts (Kirkland & Manning, 2011), or any other non-tested subject matter of interest.

In Texas, public high schools are held academically accountable by the Texas Education Agency (TEA) and are evaluated "based on [students'] performance on state standardized tests; graduation rates; and college, career, and military readiness outcomes" ("Academic Accountability," n.d.). However, private schools in Texas do not have the same accountability process; instead, private schools' oversight falls to another organization: The Texas Private School Accreditation Commission (TEPSAC). TEPSAC is not an accrediting organization, but instead is "a confederation of accrediting associations whose primary purpose is to maintain standards of accreditation among its membership" ("FAQ," n.d.). The accrediting associations consider "the school's goals and objectives; compliance with applicable state and federal statutes; effective administration and governance; the teaching of a balanced curriculum; implementation of personnel practices which ensure hiring qualified instructional leaders with college degrees; student achievement; and indicator-based quality of learning" ("FAQ," n.d.) to evaluate and accredit its members. The lack of standardized test performance ratings in the TEPSAC list is advantageous to private schools and private school teachers in Texas because the focus can shift from standardized tests to other learning pursuits (Long, 2014; Williams et al., 2018), such as establishing a prepared, nature-based mathematical classroom.

In addition to the TEPSAC not requiring state-mandated testing, Dr. Montessori's (1912) educational philosophy discusses the need to limit extrinsic rewards, including tests, grades, and

graded assignments. Therefore, SSM does not assign semester or end-of-year grades until a student enters the 10th grade, solely for high school transcript purposes and college application requirements. Without these local and state testing requirements, I have found more time to be creative and personal with my instruction, lessons, and formal and informal assessments. My administrators grant me free reign and autonomy to use a variety of pedagogical practices, teaching methods, and materials that I deem appropriate to accomplish goals set for my classroom, students, or course. The freedom has already allowed me to experiment with different styles and approaches, creating unique integrated approaches to learning and studying mathematics, science, and technology at the high school level. Having this time and freedom to make my math instruction come alive in unique and creative ways is a great privilege I want to capitalize on in this research, formalizing my informal, reflecting-on-my-own-practices inquiry of my own teaching. My hope is that readers can use my research and findings to inject opportunities to make math-nature connections in and outside their math classroom, no matter the confines and limitations, and be inspired to think creatively about pedagogical practices for secondary mathematics.

Authentic Engagement

When thinking of their time in a mathematics classroom, most adults recount the teacher at the front of the room providing examples of how to solve a problem and then completing the assigned homework to practice the skill (Hwang et al., 2019). I recall similar experiences, the majority of the time. There were times when a project or in-class activity had me use the mathematics skills we learned in a new or unusual way. For example, I remember creating a snowman related rates problem using calculus and determining how fast each spherical ball of snow was growing, increasing the snowman's overall size. I also remember taking a section of

comic strip and using proportions and dilation in Geometry to make an oversized comic strip, realizing, for the first time, that I might have some artistic abilities if I used math. Utilizing my own experiences to develop and design curriculum in my classroom (Connelly & Clandinin, 1988) is one way I attempt to facilitate mathematical discourse and to design assessments and activities in authentic and engaging ways.

When students are actively involved and engaged in an activity, there is an "organic connection between education and personal experience" (Dewey, 1938, p. 25). This organic connection provides the necessary synaptic connections to "promote having desirable future experiences" (Dewey, 1938, p. 27). Research also suggests that students feel involved and engaged in activities when they can stimulate more than one of their senses (Itard, 1801), use their hands to complete meaningful work (Crawford, 2009; Montessori, 1967), and learn outside in nature (Demarest, 2015; Froebel, 1821; Sobel, 2008). One way for students to feel more involved and engaged in a mathematics classroom is simply to bring the classroom outside (Demarest, 2015; Otte et al., 2019). In addition to the physical and emotional health benefits nature provides students (Bond et al., 2007; Louv, 2008; Wirth & Rosenow, 2012), bringing the learning outside the classroom helps to develop mathematical growth mindsets (Boaler, 2016; Zeeb et al., 2020) and to lower math anxiety levels (Chernoff & Stone, 2014; Radakovic, 2015; Taylor & Fraser, 2013).

I explored the intersection between mathematics and nature-based environments by designing a math experience in nature, examining my own intentions in designing this nature-based mathematics classroom and the students' experiences participating in the environment. I wanted to gain a better understanding of my role as a mathematics teacher and the students' role as young mathematicians, in addition to the math-nature connections they will presumably make.

I want the students to look out in nature or at any other real-world situation and see the many applications of studying the beauty of mathematics. I also want the experience to be authentic and not trivial (Demarest, 2015; Tekkumru-Kisa et al., 2020), capturing the student's interests to make meaningful math-nature connections (Sobel, 2008).

Research on designing and creating mathematical nature-based learning experiences is growing (Gilbert et al., 2014; Otte et al., 2019; Son et al., 2017) and the focus is on the math skills learned through participation in an outdoors learning environment. I want to create a mathematical nature-based experience where students can be outside in nature, incorporate math and see math-nature connections. Creating experiences and environments where the students have the opportunity to explore math-nature connections is important research to the field of mathematic teaching.

Methodology

The purpose of this qualitative study was to explore the teacher and student experiences in a teacher-designed nature-based mathematical environment. As the teacher-researcher, I used a narrative inquiry approach to examine my own intentions for this mathematical experience and the students' received experiences with the curriculum. The narrative inquiry approach to this study provided the opportunity for me to tell my story and to restory the students' experiences in relation to their own mathematical learning during this nature-based mathematical experience.

As the teacher-researcher and focusing on a specific situation in my own classroom, I utilized an action research methodology to conduct this research. A hallmark of an action research project is on understanding the localized experiences (Bloomberg & Vlope, 2016) of my students. I incorporated narratives in my data analysis to provide "rich descriptions of local

practices [which] are valuable and meet criteria for possible transferability to other settings" (Herr & Anderson, 2015, p. 91).

Research Questions

The aim of this study is to explore each component of the instructional arc: the intended, operational, and received curriculum (Uhrmacher et al., 2017) of a mathematical nature-based educational experience this teacher-researcher designed. The instructional arc asks these questions: "What does the teacher desire/plan to happen? What actually happens? What did the students learn/experience?" (Uhrmacher et al., 2017, p. 24). I have taken these three questions and developed the following four research questions to guide this study:

Q1 What were the teacher-researcher's intentions when I designed this mathematical education experience?

Q2 How did the curriculum come to life?

Q3 How did the students receive and experience the curriculum?

Q4 What connections are students making to other concepts, mathematical or otherwise?

Relating these questions to the instructional arc: Question 1's focus on the intended curriculum delves into my own intentions, Question 2's focus on the operational curriculum looks at understanding what actually happened during the lesson, and Question 3 and 4's focus on the received curriculum explores what the students took away from the lesson (Uhrmacher et al., 2017). As an integral part of this action research project, the instructional arc provides the necessary lens to explore how my teaching intentions and beliefs inform my lesson's design and how they affect student experiences. Overall, I am interested in identifying areas of strengths and growth in my own teaching and reflecting on the students' experiences through the process. For

each of these questions, I will describe below how they inform the design and purpose of this action research study.

Question One

Q1 What were the teacher-researcher's intentions when I designed this mathematical education experience?

Starting with the intended curriculum, I honed in on my planning style, techniques, and processes when I developed this mathematical nature-based educational experience, incorporating the *Teachers as Curriculum Planners* strategies of Connelly and Clandinin (1988). I kept a research journal (Herr & Anderson, 2015) "to document ongoing thinking, decisions, and actions" (p. 91) and collected my lesson plans (Appendix L), notes, and planning documents I used throughout the design process. Analyzing the research journal and documents collected helped me understand and interpret my intentions, a form of autoethnography (Herr & Anderson, 2015), when I designed this curriculum experience for the students.

Question Two

Q2 How did the curriculum come to life?

With this question, I wanted to look at the operational curriculum. I observed the outward expressions, engagement levels, and participation levels of the students. I used my own personal reflections on how I thought the experience went as the math teacher in addition to my perceptions of the students' experiences. The entire experience involved an introductory activity (which lasted 2 days), an investigative activity (which lasted 3 days), and a project presentation (which lasted 1 day). As an active teacher-participant, I recorded, in my research journal, my observations of the students and their interactions with each other, the natural environment, and myself each day. I used note-taking strategies during the lesson and reflective journaling (Connelly & Clandinin, 1988) immediately upon the conclusion of the lesson. These

observations, along with my own journaling during and after each lesson, helped form the overall picture of how the experience transpired.

Question Three

Q3 How did the students receive and experience the curriculum?

Completing the instructional arc, I was interested in comprehending the students' perspective of the experience. One of my main goals when teaching and reflecting on my practices is taking into account the impact the lesson or experience had on the students. With this question, I seek to gain that understanding and to also affirm and value the students' opinions on the experience. Data for this question came primarily from one on one interviews with a purposeful sample of my student participants. I selected purposefully three students to participate in one, one-on-one interview, taking approximately 60 minutes at the conclusion of the experience. Appendix G describes the list of questions I used during the unstructured interview (Creswell, 2015). Each of the nine students completed and turn in a project relating to the experience, a student-reported rubric reflecting on their work, and notes relating to their presentation of the projects. I also collected all of these documents, along with any other student-created or teacher-researcher-created documents created during the experience to analyze and support data gained from the interviews, my observations, and my reflective journaling.

Question Four

Q4 What connections are students making to other concepts, mathematical or otherwise?

Ultimately, I wanted to see if students, after completing a mathematics nature-based learning experience made any connections, specifically any math-nature connections. While the focus would be on any math-nature connections students make, I am interested in figuring out any connections students made to anything that is important to them, and how the experience

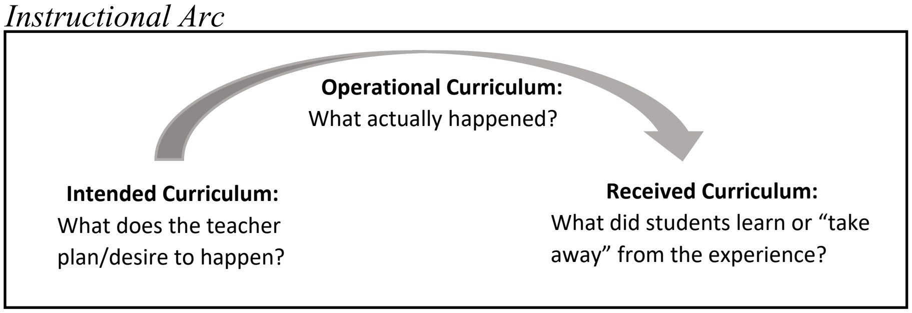
aided in the student making those connections. Data for this question came from observations and documents collected and the same one-on-one interview as the above question, contributing to the overall understanding of the received curriculum portion of the instructional arc.

Instructional Arc

While the research questions posed in this study view the instructional arc as three separate and distinct entities, the instructional arc (Figure 1), however, is more of a continuum. The intended curriculum weaves itself into the operational and received curriculum: "The instructional arc provides us a way [of] seeing what actually happens in [the learning environment], with a focus on discerning congruence or variance between intentions and operations" (Uhrmacher et al., 2017, p. 25). My intentions for the students' experiences may appear in how they receive the curriculum or they may receive something during the process that I had not intended. In either case, comparing the different cases will provide insight into the whole process.

Figure 1

Instructional Arc



Note. Recreated from Uhrmacher et al. (2017).

Conceptual Framework

The advent season is the time before Christmas, typically starting on the fourth Sunday preceding December 25. Advent calendars use the 24 days, December 1 – 24, "since the length of the Advent season changes from year to year, it's easier to pick a fixed number of days for a

calendar that can be reproduced or reused every season" (Wilkinson, 2020, para 7). My wife and I started a tradition of completing a reflective reading each night during the advent season. This past advent season, we found the book *Honest Advent* by Scott Erickson (2020). On the 23rd day, the reading included this line: "Nothing can be truly known through observation. Only through participation" (p. 172). The reading went on to explain how "exterior patterns illuminate the patterns within" and "if you are watching, you'll see them [and] be asked to move from observation to participation" (p. 177). Being that this was an advent reading, Erickson was referring to knowing God and the Divine, but as my wife and I were reading the passage together, I kept bringing my own reflections back to education and the power of students participating in their own educational experiences (Dewey, 1938). I find myself thinking about these connections frequently. Another time, I found myself thinking about how students can make mathematical connections, specifically to nature was while my sons and I were watching the new Disney movie *Soul* and one of the main characters observed a Maple seed falling in its famous helicopter-like twirl. After watching that scene, I thought about how I could use this example as a great way to introduce revolution of solids in Geometry and then again when applying revolution of solids to integration techniques in Calculus. These thoughts form the basis for my constructivist views on education.

My interest in constructivist views on education (Bruner, 1956; Dewey, 1916; Pestalozzi, 1781; Piaget, 1952) led me to explore education outside of the classroom (EOtC) (Otte et al., 2019), and I initially, and naively, believed field trips were the only way to accomplish this type of learning experience. With further research and readings, I discovered topics ranging from environmental education (Chawla, 1998; Orr, 1992) to place-based learning (Demarest, 2015) to transcendent nature experiences (Sobel, 2008) to outdoor classrooms (Eick, 2012), providing

alternatives to field trips as the only EOtC learning. The ability to bring the learning outside of a four-walled classroom has influenced the way I plan and deliver my math lessons to my high school students. As a university-trained public school educator who came to Montessori first as a parent and then as an educator, I feel that the teaching environment at SSM is completely different than the public school environment that I was used to.

The constructivist views, works, and teachings of Dr. Maria Montessori also inspire and influence my math classroom. Her research stressed the importance of incorporating eight student-centered, constructivist principles: avoiding extrinsic rewards, free choice, learning with and from peers, following the students' interests, the adult as the guide, learning in context, movement and cognition, and the prepared environment and prepared mind (Montessori, 1912, 1967, 2017). I consider and incorporated these principles when I design curriculum experiences for the students, including in the design of this nature-based mathematical experience.

As I began to investigate my own intentions as a math teacher, I continually found myself drawn to nature, the abundant math-nature connections in our school's local environment, and the desire to understand these deeply, authentically, and purposefully. Boaler (2016) relates learning to synaptic activity: "If you learn something deeply, the synaptic activity will create lasting connections in your brain, forming structural pathways, but if you visit an idea only once or in a superficial way, the synaptic connection can 'wash away' like pathways made in the sand" (p. 1). My hope is that with this mathematical experience, the students will continue to make math-nature connections in any path or career they take in their post-secondary future, having created the appropriate synaptic connections to do so with this nature-based math experience.

CHAPTER II

LITERATURE REVIEW

Introduction and Search Terms

Math teachers in classrooms across the country tout to their students, "Math is everywhere!" My former students heard me say these exact words, too, while teaching from inside my four-walled classroom with no windows to the outside world, relying on the students' prior (and future) experiences to make those connections. Now, in a different classroom, one surrounded by open windows, lush landscapes, and natural elements both inside and outside the classroom, I still tout these words to the students, but I am able to point to the natural world to give some context to these connections.

The purpose of this qualitative study was to explore the teacher and student experiences in a teacher-designed nature-based mathematical environment. As the teacher-researcher, I used a narrative inquiry approach to examine my own intentions for this mathematical experience and the students' received experiences with the curriculum. The narrative inquiry approach to this study provided the opportunity for me to tell my story and to restory the students' experiences in relation to their own mathematical learning during this nature-based mathematical experience.

My research focuses on these experiences, connecting nature and mathematics, as a part of the classroom's learning environment. As I looked at relevant literature surrounding these connections, I used the following search terms, "math" and "mathematics;" "nature," "outdoors," and "environmental education;" and "secondary," "middle school," and "high school." Through the university library, I used the Education database to collect books and articles discussing

nature, math, or the intersection of the two. Even though my dissertation research will use the experiences in my secondary, high school classroom and of the students, I felt articles from the elementary and pre-Kindergarten levels were useful to round out my research.

My initial research queries led me to incorporate other search terms related to my study: "environmental education," "outdoor classrooms," "mindsets" (growth, fixed, and mathematical), and "math anxiety." The inclusion of these topics allowed for a more robust literature review, integrating these subtopics into the larger picture of both nature studies and mathematics.

This literature review will focus on the importance of nature in educational environments and experiences, the importance of studying mathematics and having a growth mindset, and the interconnectedness between math and nature. My research focuses on the experiences students have and the connections students make, if any, in a mathematics nature-based educational learning environment.

A mathematics nature-based educational learning environment needs to consider both the external and internal environments (Chawla, 1998). The external components consist of the teacher and the location of the classroom, while the internal includes components such as the student's math anxiety level and the student's mathematical mindset. Nature provides benefits both externally and internally. Math teachers who utilize outdoor classrooms and incorporate nature into their learning environments help learners decrease their math anxiety and increase their mathematical growth mindsets.

Importance of Nature

I think it is important to first look at the benefits nature can bring to students in any learning environment, not just a mathematics one.

Learner Benefits

Spending time in nature promotes the healthy development of all of a child's senses (Hanscom, 2016; Louv, 2008) and is essential to students' physical and emotional health (Bond et al., 2007; Louv, 2008; Wirth & Rosenow, 2012). Hanscom (2016) contributes to the idea that spending quality time in nature provides children the opportunity "to take risks, overcome fears, make new friends, regulate emotions, and create imaginary worlds" (p. 3). With the claim that "one transcendent experience in nature is worth a thousand nature facts" (Sobel, 2008, p. 13), incorporating transcendent nature experiences in the design of a mathematics nature-based educational learning environment is essential to its overall success; however, research on this topic at the secondary level is limited.

Most research on the importance of nature in educational settings focuses on students in Pre-K, Kindergarten, and Elementary (Beere & Kingham, 2017; Eick, 2012; Fjørtoft, 2001; Galizio et al., 2009; Hanscom, 2016; Louv, 2008; Otte et al., 2019; Sampson, 2015). On the other hand, little research is conducted at the secondary level (Martin, 2004; McHatton et al., 2014; Moos & Honkomp, 2011) due to the increased focus on high-stakes testing (Long, 2014). With the growing importance to reestablish a "connection between [all] children and nature" (Schenetti & Guerra, 2018, p. 32), the need for more research at the secondary level is evident. Hanscom (2016), even though her research is with primarily younger children, saw older children benefitting from being outside as well: "One way to [bring the older child outside] is by incorporating the outdoors into the curriculum…time outdoors can mean a world of difference in a child's educational experience" (p. 169). A mathematics nature-based educational learning environment that brings students outdoors forms these alternate foundations and landscapes, encouraging mathematical inquiry and making math-nature connections (McLennan, 2017).

Once a teacher brings students outdoors, teachers need to consider looking at research on ecological place-based education and outdoor classrooms to understand fully how to design a mathematics nature-based educational learning environment.

Ecological Place-Based Education

Research on environmental education begins to highlight the intersection of older children and educational experiences, describing one intersection point as ecological place-based education (Demarest, 2015). Additionally, ecological place-based experiences can "increase student engagement and understanding through multidisciplinary, experiential, and intergenerational learning that is not only relevant but potentially contributes to the well-being of community life" (Gruenewald, 2003, p. 7). The introduction of environmental education or ecological place-based education in an older child's educational career provides for "the necessity (read opportunity) to reconsider, rethink, reform, restore, and rebuild our world and worldviews" (Orr, 1992, p. 40) and provides "pathways toward sustainable living beyond the classroom walls" (Moroye & Ingman, 2013, p. 590). Sobel (2008) adds that "if we want to develop environmental values, we should try to optimize the opportunity for [these types of] transcendent nature experiences in middle childhood" (p. 18).

Walker et al. (2017) define environmental education (EE) as "topics such as sustainability, ecology, and citizen responsibility" (p. 185). Their description adds "citizen responsibility" to previous definitions of EE, invoking the desire to encourage students to think about their own personal environments (Demarest, 2015), to start taking ownership of those experiences (Sobel, 2008), and to initiate action towards improvement (Walker et al., 2017). This sense of ownership and action should increase in the adolescent years, where students attempt to try to find their place and their role in society (Montessori, 1948/2007).

Demarest (2015) echoes similar responsibilities in her students: “It is not just knowing something but also doing something about it” (p. 101). Demarest also encourages teachers to think about and use “authentic engagement” over the sometimes frivolous bouts of nature inclusion. Authentic engagement “involves the intricate, creative process of putting ‘obstacles’ in the cognitive path of students that become the context for new understanding” (Demarest, 2015, p. 28). Tekkumru-Kisa et al. (2020) add that authentic learning and authentic engagement is evident when “students must learn to think in new and more demanding ways – ways that are more authentic to the disciplines they are learning” (p. 606). One way to increase a learner’s sense of ownership and action is incorporating the ideals of outdoor classrooms into the design of a mathematics nature-based educational learning environment.

Outdoor Classrooms

Nature-based educational learning environments, or outdoor classrooms (Eick, 2012), are growing in population and researchers are studying the impact these outdoor classrooms have on student performance, engagement, and success (Demarest, 2015; Haluza-Delay, 2001; Milton & Cleveland, 1995; Sobel, 2008; Walker et al., 2017). As Demarest (2015) asserts, “One of the best ways for children to learn is to bring the classroom outdoors” (p. 166).

In Walker et al. (2017) and Demarest (2015), they describe the importance of both acquiring knowledge from a student’s experience with nature and the motivation to stimulate action towards improvement. Walker et al. (2017) state “knowledge and action” (p. 185) as the two components that makeup EE and warn that “the absence of one component will not only limit the success of [an outdoor classroom] but also can be detrimental to its mission” (p. 186).

In studies where either knowledge or action (or both) are missing from the outdoor classroom, there is an obvious and apparent missing link for students. Haluza-Delay (2001)

describes this missing connection and offers that outdoor classrooms need to be “more oriented to helping participants clearly understand the connection among humans, nature, and the local environment” and to “help participants understand specific actions they can take at home to protect the environment” (p. 48). Sobel (2008) reminds us that students need to be “engaged in real projects that connect the core curriculum to real places and real problem-solving in the community” (p. 3). Chawla and Flanders Cushing (2007) add that “small-scale actions at the level of the classroom, the school yard, and the local environment are most appropriate” (p. 438). Demarest (2015) sums up the benefits to learning outside: “They learn science better, write better, learn history better, and develop reasons for using skills they might have previously seen as drudgery. Alongside these tasks, they also learn how the world works and what their place in it might be” (p. 101).

My research project brought my math students outside into nature to experience the mathematical connections within their local environment. For my research and in my design of a mathematics nature-based educational learning environment, I need to consider the benefits nature brings to the students, to provide opportunities for authentic engagement, and to allow students to acquire new knowledge and stimulate improvement.

The location of the mathematics classroom is one component of the external environment, the teacher is the other. To discuss the components of the internal environment, we need to discuss the importance of mathematics and the fear that is sometimes associated with learning math.

Importance of Mathematics

“Why are we learning this?” and “When will I ever use this again?” are questions, more common than not, that enter the mathematics classroom on a weekly, if not daily, basis. Students

and adults, alike, seem to know inherently that there is a need to understand mathematics, but they do not necessarily know why math is important in their everyday lives or, more importantly, their future lives (Venters, 1992). Chernoff and Stone (2014) offer this description of how we are all, sometimes unknowingly, stewards of mathematics:

> Math is all around us throughout our daily lives, from knowing how far one can travel on the gas remaining in a tank; understanding and using plans for three-dimensional structures; interpreting and questioning charts on the evening news or polls on a Twitter feed; measuring ingredients to create a meal; estimating the volume of luggage versus trunk space; to figuring out a monthly budget or doing annual taxes. (p. 29)

Even though these real-life math examples show how people use math daily, students are still challenged with overcoming a fixed mindset towards mathematics (Chestnut et al., 2018; Dweck, 2010; Hwang et al., 2019; Zeeb et al., 2020) and succumbing to math anxiety (Chernoff & Stone, 2014; Radakovic, 2015; Taylor & Fraser, 2013) in the mathematics classroom.

Fixed Math Mindset

Many students, unfortunately, believe from an early age they are either good at math or bad at math and have developed a “fixed mindset” (Dweck, 2010). Students with fixed mindsets (as opposed to growth mindsets) have difficulties learning and growing. The fixed mindset can affect all realms of academic, social, and emotional learning and is most distinct in the mathematics classroom (Zeeb et al., 2020). Students living in this mathematical fixed mindset have a hard time studying, learning, and enjoying math, much less appreciating the beauty and importance of and the connections mathematics has in our daily lives (Chestnut et al., 2018). Boaler (2016) adds that “students rarely think that they are in math classrooms to appreciate the beauty of mathematics, to ask deep questions, to explore the rich set of connections that make up

the subject, or even to learn about the applicability of the subject; they think they are in math classrooms to perform" (p. 21).

In an effort to create more growth-minded math students, Venters (1992) suggests mathematical educators should capitalize on the importance of students understanding the "analytical skills" and "thinking and decision-making skills" (p. 412) they are developing when *simplifying* expressions and *solving* equations, the two most common directions found in a mathematics textbook. Expressions and equations by themselves appear to have little or no application to our daily lives, but multiple times a day people are making analytical decisions. A mathematical educator's multifaceted role includes showing the "hidden" mathematical connections as well as the obvious ones. After all, "mathematics is a cultural phenomenon; a set of ideas, connections, and relationships that we can use to make sense of the world" (Boaler, 2016, p. 23). Incorporating nature into the design of a mathematics nature-based learning environment assists students in making these connections (Otte et al., 2019) and helps encourage growth mathematical mindsets (Boaler, 2016).

If combating a fixed mathematical mindset was not enough, teachers also have to consider a student's math anxiety level when designing appropriate classroom experiences and learning environments.

Math Anxiety

In my own experiences, I have seen math anxiety be a crippling emotion that takes some of my best mathematics students and gives them the self-doubt and worry to believe they are incapable of thinking mathematically. This "lack of confidence, panic, poor performance, [or] avoidance" (Chernoff & Stone, 2014, p. 30) prevents students from taking the necessary

educational risks found in a mathematics classroom (Radakovic, 2015), and a student could conclude they were a poor math student.

Taylor and Fraser (2013) took their research on math anxiety and looked at the different educational environments, finding the teacher, an external component in the learning environment design, is the "key element" (p. 301) in determining a student's math anxiety level. According to their research, most of the research efforts focus on the negative impacts of negative teachers in the classroom, but one could ascertain positive teachers provide positive implications for students with math anxiety. A positive teacher can reduce "students' mathematics anxiety through creating a positive classroom environment" (p. 310). One way to create a positive classroom environment is to build a learning environment based on inquiry (Boaler, 2016; Demarest, 2015; Sobel, 2008) and to bring the learning environment outside: "Learning outside provides more opportunities for risk taking, problem solving, moving the whole body, using the imagination, overcoming fears, engaging in teamwork, and tolerating and integrating new sensory experiences" (Hanscom, 2016, p. 166).

According to McHatton et al. (2014), "success in school is based on a variety of factors including students' individual characteristics, past experiences, and perceptions of the classroom as an environment conducive to learning" (p. 40) and nature-based learning environments add to the success of the student. There is a lack of research at the intersection of mathematics education and learning environments (Taylor & Fraser, 2013), and my research intends to look at one of these intersection points, mathematics in a nature-based educational learning environment.

A Nature-Based Mathematics Educational Learning Environment

> The experiences that people remember as significant in motivating their care and concern for the natural world may be characterized as exchanges between an external and internal environment: an external environment composed of the qualities of physical surrounding, and social mediators of the physical world's meaning; and an internal environment of the [student]'s needs, abilities, emotions, and interests (Chawla, 1998, p. 380).

The above quote describes this balance between internal and external environments. The student's mindset and the student's math anxiety levels (the internal), and the location of the classroom and the teacher (the external) need to be included when examining the internal and external environments of a mathematics classroom. Also, the experiences within these environments need to incorporate authentic, real work based on inquiry (Boaler, 2016; Demarest, 2015; Sobel, 2008), connecting "the core curriculum to real places and real problem-solving" (Sobel, 2008, p. 3) by "creatively connect[ing] learners, ideas, and worthwhile outcomes; and model[ing] inquisitive, rigorous, and original thinking" (Demarest, 2015, p. 5) and allowing students the ability "to share their [mathematical] thinking and ideas, and to make conjectures and construct proofs…like young mathematicians at work" (Boaler, 2016, p. 182).

Designing a mathematics nature-based educational learning experience relies on teachers understanding the conventional to integrative curriculum continuum (Virtue et al., 2009), which has four main components: conventional curriculum, where "clear boundaries exist between subject areas;" correlated curriculum, where "certain vocabulary, concepts, and skills are reinforced in multiple subjects;" integrative/interdisciplinary curriculum, where "curricular boundaries between different subjects are blurred;" and integrative curriculum, where "teachers joined with their students to develop [the] learning experience" (p. 5). Ideally, teachers take

"incremental steps toward integrative curriculum" (p. 5) and "must let go of many ingrained patterns of schooling and come to see the new possibilities all around" (Demarest, 2015, p. 103).

Conclusion and Significance of the Study

Math is everywhere, and what better way to show students this than to give them an experience outside in nature. Demarest (2015) & Sobel (2008) stress the importance of these experiences as authentic and real as possible, while still connecting to the core curriculum. Boaler (2016) showed that math is the set of relationships that helps us make sense of the world. All three of these researchers provide models for successful experiences in nature and mathematics, respectively. Demarest (2015) incorporates personal connections, local investigations to deeper subject understanding, holistic understanding, and building civic engagement in her curricular elements for local learning: "The four approaches [to local learning] are seldom used in their pure form but rather serve as constructs and inspiration for the myriad of ways that teachers engage the local" (p. 101). Sobel (2008) has seven design principles for teachers to consider when designing learning experiences and environments: adventure, fantasy and imagination, animal allies, maps and paths, special places, small worlds, hunting and gathering. He encourages teachers to use these principles in a variety of combinations based on curricular objectives. Boaler (2016) discusses what she calls "The 5 C's of Mathematics Engagement" (p. 57) which are curiosity, connection-making, challenge, creativity, and collaboration. I will merge these three models together to create and design the mathematics nature-based learning experience for this research study.

Therefore, I designed and developed a unit of study that infuses nature with the mathematics concept, piecewise-defined functions. I brought the students and my classroom outside where students explored their local environment and connected their math learning to the

nature they found there. My research will lead to a better understanding of the innate intersection between math and nature, fusing the local environment (Demarest, 2015) and transcendent nature experiences (Sobel, 2008) with engaging math classroom and math teaching techniques (Boaler, 2016). Designing the curriculum with these intentions provided the opportunity for the students to experience mathematics in a natural environment, reducing math anxiety and encouraging a mathematical growth mindset.

CHAPTER III

METHODOLOGY

The purpose of this qualitative study was to explore the teacher and student experiences in a teacher-designed nature-based mathematical environment. As the teacher-researcher, I used a narrative inquiry approach to examine my own intentions for this mathematical experience and the students' received experiences with the curriculum. The narrative inquiry approach to this study provided the opportunity for me to tell my story and to restory the students' experiences in relation to their own mathematical learning during this nature-based mathematical experience.

I want the students to see the multitude of mathematical connections and processes that exist in the real world—to see the natural beauty of mathematics and the mathematics in the beauty of nature. My research studied the process of designing an outdoor mathematics classroom where students participated in a mathematical learning experience outside, seeing and understanding the innate connections math has in nature and nature has in math. Since the focus of my research is on a specific situation in my own classroom, and I, as the teacher-researcher, am the lone insider researcher (Herr & Anderson, 2015), I used an action research methodology to conduct this research study. A hallmark of an action research project is on understanding the localized experiences (Bloomberg & Vlope, 2016) of the students and myself and on studying "the outcomes of a program or actions in [a particular] setting" (Herr & Anderson, 2015, p. 42). To promote critical reflection and the development of my personal education practices (Heikkinen et al., 2012; Pushor & Clandinin, 2012), I crafted restoried narratives (Ollerenshaw & Creswell, 2002) as my data analysis tool, providing "rich descriptions of local practices

[which] are valuable and meet criteria for possible transferability to other settings" (Herr & Anderson, 2015, p. 91). Pushor and Clandinin (2012) elaborate on the use of narrative inquiry in action research studies: "It is in moving beyond the definition of narrative inquiry to the idea of the shared relational narrative inquiry space that the connections to action become sharper" (p. 221). I used these restoried narratives to highlight the experiences of myself and the students in this nature-based mathematical learning experience, allowing the reader to experience the events alongside each participant, from their perspective.

The following four research questions guide this study:

Q1 What were the teacher-researcher's intentions in designing this mathematical education experience?

Q2 How did the curriculum come to life?

Q3 How did the students receive and experience the curriculum?

Q4 What connections did students make to other concepts, mathematical or otherwise?

Setting

St. Stephens Montessori (a pseudonym) is a small, private, Montessori, Catholic school in an urban setting in East Texas, serving students age 14 months to 18 years of age. Students learn in academically grouped communities in three-year cycles based on age (Montessori, 1912). I worked with the high school community, which includes students between the ages of 15 to 18, who are in 10th to 12th grade, for this research study. The high school community, at the time of this study, had nine students with two full-time guides for the core subjects and two part-time guides for Theology and Spanish. With the small number of students, the high school has one room at St. Stephens Montessori (SSM) to serve as our classroom, and we have access to an outdoor space for any outdoor work.

Working in a one-room, private, Montessori, Catholic high school is a unique experience unto itself, providing the opportunity for creativity in pedagogical practices. I worked in this environment with Mrs. Darcy (a pseudonym), and we were in constant contact, discussing our curricular plans together to create an integrated curriculum that connects spiritual life to academic disciplines and methods of inquiry. Mrs. Darcy was responsible for content connected to English, Social Studies, and Humanities. I was responsible for content connected to Math, Science, and Technology. The level of partially integrated curriculum (VanTassel-Baska & Wood, 2009), as affected by this collaborative team-teaching approach, is not directly studied in this research study, but is an area of interest of mine for future studies, and, I hypothesize, influences my curricular and pedagogical intentions when designing student experiences, including the one studied in this research.

In addition to SSM's focus on Montessori principles in and outside of the classroom, the school's culture has a strong sense of environmentalism, activism, environmental justice, and sustainability. The first K-12 school in Texas to be LEED (Leadership in Energy and Environmental Design) certified, SSM acknowledges that students play an important role in shaping our world and by educating environmentally-responsible people, SSM is investing in a sustainable future. The students at SSM take this desire to heart and incorporate sustainable ideas into all of their schoolwork. For example, in the 2018-2019 school year, students organized and participated in a symposium on climate change with a mission focused on creating a sustainable future for all, educating the community about solutions to climate change, and empowering each individual to take action in their personal lives. At this symposium entitled *From Knowing to Acting*, students presented their research on a range of climate change solutions to our community ranging from legislative and environmental justice solutions to energy and

agriculture solutions. In the following school year, 2019-2020, students took the charge to move from knowing to acting and hosted a Climate Caucus in hopes of lobbying for sustainable and environmental policies with local political candidates, demonstrating the desire for representatives to understand and believe in climate activism.

The students at SSM embody the desire to understand the environment, climate change, and environmental justice issues, and the high school community takes their role seriously. Students yearn for the ability to understand this role in all aspects of the academic lives, as well as their personal lives, and Mrs. Darcy and I feel it is our job to help guide our students through our academic subjects to see these connections, infusing sustainable practices and nature-inclusion in our teachings.

Methodology: Action Research

McKernan (1988) defines action research as “a form of self-reflective problem solving, which enables practitioners to better understand and solve pressing problems in social settings” (p. 6). With theoretical foundations “grounded in the importance that John Dewy gave to human experience and active learning in the generation of knowledge” (Herr & Anderson, 2015, p. 21), action research is an ideal methodology for my research study. My focus is on a specific situation in my own classroom and on understanding the localized experiences of the students, a hallmark of the action research methodology (Bloomberg & Vlope, 2016).

With my action research study, I provide an example of one way I have designed an outdoor mathematics classroom and nature-based mathematical learning experience. Through the analysis of my own understanding of the “effectiveness and meaningfulness” (Bloomberg & Vlope, 2016) of these types of environments and the experiences students have within them, my

research adds to the limited studies on secondary mathematics learning in outdoor settings (Gilbert et al., 2014; Otte et al., 2019; Son et al., 2017).

As a reflective practitioner (Schon, 1983), attempting to understand how my own experiences have shaped my professional growth and my pedagogical choices (Connelly & Clandinin, 1988), corresponds with the action research methodology, allowing "the researcher to reflect on the research process as well as the findings" (Herr & Anderson, 2015, p. 2). In an effort to reflect critically on my educational practices and on the experiences of the students, the use of narrative inquiry "awakens and provokes thought about things in a new and different way" (Heikkinen et al., 2012). The restorying and crafting of narratives aids in understanding the experiences of myself and the students (Ollerenshaw & Creswell, 2002) in this nature-based mathematical environment.

Narrative Analysis

Action research is cyclical by design and, echoing a learning cycle (Bloomberg & Vlope, 2016), requires the teacher-researcher to plan, implement, observe, and reflect (Kemmis, 1982). This cyclical framework, reliant on observations and reflections, mirrors narrative analysis:

> "We intentionally come into relation with participants, and we, as inquirers, think narratively about our experiences, about our participants' experiences, and about those experiences that become visible as we live alongside, telling our own stories, hearing an other's stories, moving in and acting in the places—the contexts—in which our lives meet" (Clandinin, 2013, p. 23).

The telling, retelling, and hearing of experiences complements the work of the action researcher and provides a systematic way to "explore the participants' experiences" (Call-Cummings et al., 2019, p. 2179). Narratives are a form of storytelling "involving characters with both personal and

social stories. It contributes to research on teaching and learning through its ability to frame the study of human experience" (Mertova & Webster, 2020, p. 11). I used the storied lives (McMillan, 2016) of my participants to understand, as fully and completely as possible, their experiences, including my own, through this mathematical nature-based educational learning experience.

Participants

This action research study focused on the development, design, and implementation of a specific mathematics curriculum. McConnell et al. (2020) remind lesson planners and curriculum developers that "curriculum is a lived experience mediated by teachers and explored by students" (p. 1). The lived experiences of the students and myself influence both this action research study and the curriculum studied within it.

Teacher-Research Participant

As the teacher-researcher in this study, I considered myself an active participant (Creswell, 2013) and an insider (Herr & Anderson, 2015) in the study. As the sole math teacher, I am responsible for the design and delivery of lessons and experiences to the students on a daily basis, and I "cannot be separated from the setting in which [my practice] takes place" (Herr & Anderson, 2015, p. 41).

Student Participants

The high school community had a class size of nine students (n = 9), composed of six female students and three male students, three of whom represent minority subpopulations. To conduct research with students, I followed the University of Northern Colorado Institutional Review Board (IRB) procedures and received IRB approval (Appendix F) prior to the start of my study. After receiving approval, I received permission from SSM to conduct the study on their

campus (Appendix E). To inform the students and potential participants about the study, I followed my participant recruitment script (Appendix A) and obtained parental consent (Appendix D) followed by student assent (Appendix C) forms from each student participant under the age of 18. I also obtained consent forms (Appendix B) for the students who were over the age of 18 at the time of the study.

Even though all students participated in the lesson and coursework assigned relating to this research project, participation in the study was voluntary. My goal was to observe all participating students during the experience. I used pseudonyms for all students. Students chose their own pseudonyms.

At the conclusion of the experience, and based on my observations of critical events and the students, I chose purposefully three students to interview. I chose only three students because the three chosen students would provide a rich insight into the experiences of the students as a whole (Patton, 2015). The use of purposive sampling provides the opportunity to select information-rich cases (Devers & Frankel, 2000; Patton, 2015), delving deeper into the cases that "yield insight and in-depth understanding" (Patton, 2015, p. 264). The previous parental consent and assent forms described the possibility of selection in the interview process. Participation in the interview was voluntary. One of the originally selected students opted out of participating in the interview process, creating the need to choose purposefully a fourth student to serve as the third interviewee. My goal for these interviews was to add more details to the retelling of the stories and experiences after participating in this mathematical nature-based educational learning experience.

A Brief Overview of Student Assessments

At SSM, Mrs. Darcy and I assess our students in a non-conventional manner. We do not assign grades to students' daily or major work, as is consistent with conventional school models. Students complete projects, presentations, and portfolios, and Mrs. Darcy and I assess those products using a scaled rubric: exceed expectations, met expectations, and did not meet expectations. At the end of each quarter, Mrs. Darcy and I review rubrics and complete a guide assessment for each student. The guide assessment evaluates each student in five areas: knowledge and skills, quality of work, engagement, preparedness, and pace. In the guide assessment, we provide narrative feedback of strengths and areas for improvement in each subject area. The narratives convert into letter grades based on a scaled rubric from 0 to 4 points with priority weight given to the knowledge and skills and quality of work sections (See Appendix H for a sample guide assessment).

I provide this brief overview of assessments at SSM to allay any ethical concerns regarding graded work with this research study. The work students participated in, in this research study, was in line with familiar coursework and assessment guidelines and expectations.

Data Collection

According to McMillan (2016), each story and narrative involves two elements: the told stories and the lived stories. The difference between the two is that "told stories emanate directly from the participant, through both formal and informal conversations, whereas lived stories may involve a broader description based on context, supporting documents such as memos or photographs, [ ] and researcher observations as he or she 'lives with' the participant" (p. 321). My data collection plan focused on collecting elements from both the student-participants' told and lived lives and stories, restorying their experiences. Restorying involves "gathering stories,

analyzing them for key elements of the story (e.g., time, place, plot, and scene), and then rewriting the story to place it within a chronological sequence" (Ollerenshaw & Creswell, 2002, p. 332). The "data sources, which inform these narrative sketches, include surveys, observations, documentation and interviews that enhance the time, place, and description of the critical event" (Mertova & Webster, 2020, p. 58). I collected observations, documents, audiovisual materials, and interview data to help construct and restory the participants' narratives. Table 1 provides the general plan and theoretical framework for data collection and analysis followed in this research study.

Table 1

General Plan and Theoretical Framework for Data Collection and Analysis

Theoretical/conceptual Framework (may be the same for each question)	Research Question(s)	Participants (number, characteristics, selection criteria, selection method)	Data Collection Tools and Sources (e.g., survey, interview, observation)	Data Analysis Procedures
Teachers as Curriculum Planners (Connelly & Clandinin, 1988) Intended Curriculum & Instructional Arc (Uhrmacher et al., 2017)	RQ 1: What are the teacher-researcher's intentions in designing this mathematical education experience?	1 teacher (self)	*Journaling* *Documents* (lesson plans, notes, and/or planning documents)	*Narrative analysis/inquiry*
Erdkinder (Montessori, 1948/2007) Transcendent Nature Experiences (Sobel, 2008) Mathematical Mindsets (Boaler, 2016) Operational Curriculum & Instructional Arc (Uhrmacher et al., 2017)	RQ 2: How does the curriculum come to life?	9 students (whole class population) 1 teacher (self)	*Observations* *Journaling/Field Notes* *Photographs*	*Narrative analysis/inquiry*

Table 1, ***continued***

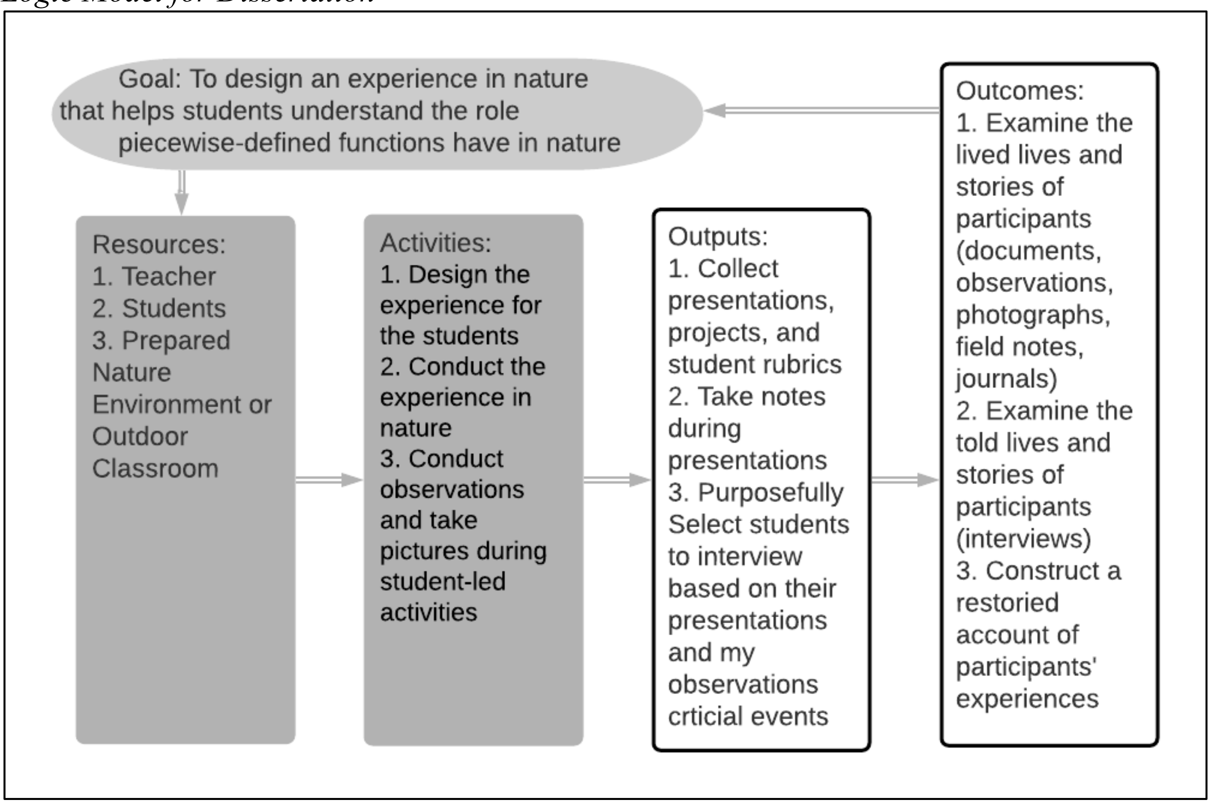
Theoretical/conceptual Framework (may be the same for each question)	**Research Question(s)**	**Participants (number, characteristics, selection criteria, selection method)**	**Data Collection Tools and Sources (e.g., survey, interview, observation)**	**Data Analysis Procedures**
Received Curriculum & Instructional Arc (Uhrmacher et al., 2017)	RQ 3: How do the students receive and experience the curriculum?	3 students (purposefully selected based on critical event observations from the group of students whose parents/guardian have consented to the study) 9 students (whole class)	*One on One Interview* the 3 purposefully selected students, individually *Documents* (projects, presentations, and/or rubrics) *Observations* *Photographs*	*Narrative interviews* *Narrative analysis/inquiry*
Received Curriculum (Uhrmacher et al., 2017) Schema (Piaget, 1952)	RQ4: What connections do students make to other concepts, mathematical or otherwise?	9 students (whole class)	*One on One Interviews* with each of the 3 selected students, individually – the same sample from RQ3 *Documents* (projects, presentations, and/or rubrics) *Observations*	*Narrative interviews* *Narrative analysis/inquiry*

Logic Model

The logic model, Figure 2, organizes each stage of my research. I felt a logic model was appropriate for my research because each step relies on the previous step. My ultimate goal is to design an outdoor experience where students can learn math. After designing the experience, I conducted the lesson with the students. While conducting the lessons, I observed the student-participants during all student-led aspects of the lesson and captured any critical moments with a photograph. In addition to my observations during the lesson, I reflected on the day's lesson, immediately upon completing the lesson, each day during the experience.

Figure 2

Logic Model for Dissertation



Students concluded the experience by constructing a project and a presentation to demonstrate their understanding and mastery of the mathematical focus, piecewise-defined functions, incorporating the nature elements of the outdoor classroom in their demonstration. I used the student-created product as documents in my research, collectively with my observations, field notes, and photographs of students, to examine the lived stories of the student-participants. To examine the told stories of my participants, I interviewed purposefully-selected students to explore critical events observed during the lesson. I also used my observations of conversations I had with students during the different aspects of the project as documented in my field notes and reflective journaling.

Teacher-Researcher Data

As the teacher-researcher, and to address my own intentions in the design of this mathematical nature-based educational learning experience, I kept a daily research journal (Herr & Anderson, 2015) of my thoughts, actions, and reflections throughout the development process, the delivery of the lessons, and at the conclusion of the entire experience. Connelly and Clandinin (1988) explain, "Journals are ongoing records of practices and reflections on those actions" (p. 34), and this journal keeping process provided the opportunity to understand my own intentions in the choices I made during the design process.

In addition to journal keeping, I collected documents related to the development of this experience, including my lesson plans (Appendix L), assignment expectations, notes, and rubric expectations. The collection of these documents provided an additional perspective on my "personal practical knowledge" (Connelly & Clandinin, 1988, p. 41) and added depth of knowledge to interpret my own intentions.

Student Data

Observations

To begin to understand how the curriculum came to life, along with my own personal journal reflections, I conducted observations of the students and myself every day during the experience using descriptive and analytic field notes (Bernard, 2002; McMillan, 2016). I observed and took note of the different combinations of interactions between all participants, including all participants' interactions with the natural environment. Connelly and Clandinin (1988) describe this process as "participant observation" and explain that "participant observation can be loosely defined as participating in the ongoing work in [the] classroom while engaging in making observations on the student and teacher activities, conversations, materials, and so on, as well as on your own activities" (p. 54).

Observation Timeline. I used the observation timeline described in Table 2.

Table 2

Observation Timeline

Day 1 – Introduction to Math in Nature	15 minutes of teacher-led instruction 30 minutes observing students
Day 2 – Setting up the Project – Situating the students in nature	5 minutes of teacher-led instruction 30 minutes of observing students 5 minutes of teacher-led instruction 15 minutes of observing students
Day 3 – Choosing a Location & Sketching	10 minutes of teacher-led instruction 50 minutes of observing students
Day 4 – Independent Work – Converting Sketch to Mathematical Equations, part 1	10 minutes of questions and answers 50 minutes of observing students
Day 5 – Independent Work – Converting Sketch to Mathematical Equations, part 2	10 minutes of questions and answers 50 minutes of observing students
Day 6 – Student Presentations	60 minutes of observing student presentations

Observation Protocol. I noted in my observations the interactions of the students with each other and with nature. To assist in my observation process, I used the observation protocol (Yost et al., 2009) as described in Table 3.

Table 3

Observation Protocol Questions

Outdoor Classroom Atmosphere and Prepared Environment
The outdoor classroom encourages focused learning.
The outdoor classroom atmosphere is orderly and has an established routine.
Students are engaged in learning while in the outdoor classroom.
Knowledge Acquisition
Students connect prior math knowledge or experience with the math content of the lesson.
Students engage with nature
Students revisit nature at the end of the lesson.
Students connect the new math content with nature.
Advancing Knowledge
Students ask a variety of questions during the lesson.
Students engage in higher order thinking during the lesson.
Application of Knowledge
Students consider multiple points of view.
Students direct themselves in their own learning.
Students connect their work to real-life experiences.

Note. The above questions were modified from Yost et al. (2009) to match the needs, scope, and limits of my research

Audiovisual Materials

I documented the experience with a series of photographs and collected them for analysis using a photo-elicitation process (Kettle, 2010) to create narratives from my reflections and memories and to "provide an opportunity [for me] to share directly [my] perception of reality" (Creswell, 2015, p. 223). I used my phone camera to capture students engaged in the experience, in work, or in communication with other students or nature. The photographs of the students include the environment and the backs of students' heads to maintain anonymity. The audiovisual materials, along with the narratives I created in association with them, assisted in understanding how the curriculum came to life during this experience.

Documents

Each day students were asked to complete a reflection, a journal entry, or show their mathematical thinking. I collected all of these student-created documents throughout the project, including the final student-produced presentation at the conclusion of the experience. The documents I collected "represent a good source for text (word) data," providing "the advantage of being in the language and words of the participants, who have usually given thoughtful attention to them" (Creswell, 2015, p. 222). It also provided me a window into each student's thought process during each stage of the project. I redacted all identifiable information from these documents, had the students title them with their pseudonyms, and kept them in a locked file cabinet in a locked interior closet of a locked classroom. Each document provided valuable information to understand how the curriculum came to life, the experienced or received curriculum from the students' perspective, and the connections each student made.

Interviews

The interviews were one-on-one and unstructured, using an initial question, content questions, probe questions, and a final question (McMillan, 2016). My hope was for the unstructured format of the interview to provide a direct path into told stories of the interviewees. Appendix G lists the set of questions I used in these unstructured interviews. The interviews were held in a location and at a time convenient for the student. Each interview was approximately 30 minutes. I audio-recorded each interview using my phone's voice memo app and transcribed the interviews using a transcription app on my phone. I then saved the audio file and its transcription to a password-protected file on a password-protected computer and deleted permanently the audio file and transcription from my phone and its apps.

Students at SSM are accustomed to talking with their teachers about their reflections and experiences in classes, within the high school community, and at home. Mrs. Darcy and I, in addition to their instructional teachers, are also their advisors. One aspect of advising includes meeting with each of our advisees on a weekly basis. Due to this communicative nature already set in our classroom community, I felt one interview at the conclusion of the experience would fully encompass the told lives of the three purposefully selected participants and assist in the restorying of their experience.

The interview provided an opportunity "to capture the thoughts and feelings of participants in their own language, using words, phrases, and meanings that reflect their perspectives" (McMillan, 2016, p. 344). The unstructured, open-ended interview "invites the research participant to engage in storytelling" (Mertova & Webster, 2020, p. 70). These interview responses provided valuable insight into how the curriculum came to life, the experienced or received curriculum, and the connections students made.

Summary of Student Data Collection

The assortment of student data collection tools used in this research study relate well to each other. The in-person observations informed which photos were relevant and how I recaptured the moments on those photos, while the documents and interviews provided depth and detail to the observations and photos. Additionally, the interviews and documents provided a first-hand perspective from the participants into their thoughts, motivations, and understanding. Together, each data collection tool aides in the restorying of the participants' experiences, both their lived stories and their told stories.

Data Analysis

Critical Event Narrative Analysis

To understand my participants' experiences holistically, I needed to understand which events in the experience were impactful and meaningful to them. Mertova and Webster (2020) provide one way narrative analysis can focus on these impactful events: critical event narrative analysis. A researcher using a critical event narrative analysis utilizes a "holistic view of their investigation and enables them to classify occurrences into critical and supporting events, which are often overlooked or not revealed through traditional empirical methods" (p. 58). The "human ability to distil the most important events in any story is essential to the use of critical events in a method of analysis of narratives" (p. 59). In this regard, the narratives developed in this research provided the insight into what participants viewed as critical events through their participation in this experience and why participants viewed them as such.

A critical event, by definition, "reveals a change of understanding or worldview by the storyteller" (Mertova & Webster, 2020, p. 60). The collection of data on both the told and lived stories (McMillan, 2016) allows the critical events of participants to reveal themselves naturally through the narratives and stories. These "collected stories can be categorized into critical events, like events, and other events" (Mertova & Webster, 2020, p. 64). Using this categorization process, enabled me to focus in on the critical events of this nature-based mathematical experience and to analyze the components that were successful and those that would encourage a pedagogical change.

Identification of Critical Events and Themes

Saldaña (2016) describes the process of looking for common themes or experiences between participants as the "five R's: routines, rituals, rules, roles, and relationships" (p. 6).

Therefore, I looked for repeated words and phrases across of the collected data, indicating a critical event, like event, or other event (Mertova & Webster, 2020) in common between the participants in this research study. These repeated words and phrases pointed out the routines, rituals, rules, roles, or relationships each participant identified as important during their experience. I used these to determine the critical events. The bulk of the data came from the students' responses to the self-assessment (Appendix K) and their journal entries. Deeply understanding these documents aided my ability to structure the narratives and the restorying process.

To craft each restoried narrative, I gathered all of the collected data for each participant and read through each piece again. I then identified the key areas that had the most impact on the students' experience and retold their experience in a cohesive and chronological fashion, incorporating vignettes, pictures, graphs, and diagrams to enhance the restoried narrative. Ollerenshaw and Creswell (2002) describe the restorying process as "gathering stories, analyzing them for key elements of the story (e.g., time, place, plot, and scene), and then rewriting the story to place it within a chronological sequence" (p. 332).

Validity & Bias

Quantitative research relies on the validity of the inferences and the reliability of the scores, not the instruments used, and quantitative researchers build trust in their readers based on the validity and reliability of those scores (McMillan, 2016). On the other hand, qualitative research is not asking questions answered through number crunching or statistical formulas. Qualitative researchers are more interested in the experiences of their participants and in believing that the participants actively construct their own reality (McMillan, 2016). Therefore, I, as the teacher-researcher, must detail the process in which I bring meaning and understanding to

these lived experiences, situations, and specific interactions. In this research study, I documented and detailed the entirety of the process and showed how I saw the meaning and understanding in the told and lived experiences of these students, and my own experiences, during this mathematical nature-based experience.

This action research study used a narrative analysis process that involved analyzing critical events to develop and describe emergent themes in the research (Mertova & Webster, 2020). Herr and Anderson (2015) recommend that “bias and subjectivity are natural and acceptable in action research as long as they are critically examined rather than ignored” (p. 73). One way I examined critically the narratives and my findings was to conduct validation meetings (Lomax et al., 1996) with two colleagues who served as knowledgeable outsiders (Suter, 2012) for this research.

As I crafted the restoried narratives and my reflections after conducting the photo-elicitation process, I utilized my co-teacher, Mrs. Darcy (a pseudonym), and my wife, a veteran English and Humanities teacher, as the knowledgeable outsiders for these validation meetings. Due to their relationship with myself and the students, Mrs. Darcy and my wife provided a unique independent outside perspective, enhancing the validity of my research (Suter, 2012). After evaluating and reading through my reflections and narratives, they each determined whether the entry sounded like the student’s voice or too much like my voice. Based on their evaluations, I kept the entry as written or returned to my field notes and other documents to better account for the student’s perspective, ensuring my reconstructions “accurately reflected the reality of [my participants]” (Suter, 2012, p. 378).

Heikkinen et al. (2012) provide five principles for validity in action research: historical continuity, reflexivity, dialectics, workability and ethics, and evocativeness. Historical continuity

relies on the action researcher to recognize “the historical evolution of action” (p. 8). I documented the progression of my ideas and design process in constructing this nature-based mathematical experience to maintain the historical continuity in this research. Reflexivity encourages the action researcher to “be aware of the impact of his/her personal experiences while interacting with the other participants” (p. 9). As the teacher-researcher, I am aware of the impact my role has on the students, and I documented all times when I felt my role, as teacher, potentially impacted the students. Dialectics recommends the action researcher “gives space to different voices and interpretations of the same event” (p. 9). By setting up and using validation meetings, I provided this avenue of other interpretations. Validity of workability and ethics is concerned with whether the action research study “has given rise to changes in social action” and “to consider the practical consequences of research on both study subjects and researchers as well as the scientific community, society and all mankind” (p. 9-10). The hope is that there is a change to the social action of mathematics teachers – that they can utilize a variety of pedagogical skills to reach their students, including capitalizing on the benefits nature provides to mathematical students. The last principle of validity in action research is on evocativeness, the desire for research to touch readers “at an emotional level” because “most significant learning experiences are always both cognitive and affective in nature” (p. 10). I believe this research shows both the cognitive and affective experiences of students due in part to the fact that the experience is in nature and the restorying of the students’ experience provides the opportunity to live the events alongside each participant.

Limitations

Each research study has limitations, and I expect this one will, too. The sample is small, limited to the nine students in the high school. While each student is different, they all represent

middle- to upper-class families who can afford private school tuition. The outside location is dependent on weather conditions, and inclement weather may adversely affect the experiences of all participations. Another limitation to consider is the school schedule. Math meets three times a week on Mondays, Wednesdays, and Thursdays for approximately 60 minutes. The experience took place over multiple days, the possibility of absent students limited the study and the participants' experiences. These limitations, I believe, will have minimal impact on the overall purpose of this study, which is to understand participants' experiences in this one type of learning environment, a nature-based mathematical classroom, and to investigate the effectiveness of the instructional methods used in the experience.

Summary

In this action research project, I used the instructional arc, utilizing the intended, operational, and received curriculum to study the experiences of the students and myself in a self-designed nature-based mathematical experience. I used the narrative analysis process, specifically a critical event narrative analysis, to bring the storied lives of my participants and myself to life. The use of journals, documents, photography, and interviews strengthen the narrative and add dimensions to the participants' storied lives.

CHAPTER IV

FINDINGS

Overview

The purpose of this qualitative study was to explore the teacher and student experiences in a teacher-designed nature-based mathematical environment. As the teacher-researcher, I used a narrative inquiry approach to examine my own intentions for this mathematical experience and the students' received experiences with the curriculum. The narrative inquiry approach to this study provided the opportunity for me to tell my story and to restory the students' experiences in relation to their own mathematical learning during this nature-based mathematical experience.

I collected and analyzed data from student and teacher-created documents, field observations, entries from a photo-elicitation process, and transcripts from one-on-one, open-ended, unstructured interviews to restory my participants' experiences. I used a critical event approach to capture the main events in each participants' narratives, highlighting those events that changed, altered, or modified the participants' understanding or worldview (Mertova & Webster, 2020).

In this chapter, I present the narratives of each participant organized around the critical events associated with each research question. The narratives are presented in this chapter to allow the reader to envision themselves alongside the participants as they underwent the experience, allowing for a deeper understanding of each participants' experience (Connelly & Clandinin, 1988). I provide a brief discussion of the findings as a whole related to each research

question in this chapter; however, a deeper discussion and analysis of the findings is included in Chapter 5.

The research questions, based on the instructional arc (Uhrmacher et al., 2017), were:

Q1 What were the teacher-researcher's intentions when I designed this mathematical education experience?

Q2 How did the curriculum come to life?

Q3 How did the students receive and experience the curriculum?

Q4 What connections are students making to other concepts, mathematical or otherwise?

The organization for this chapter centers on the instructional arc, utilizing the four research questions as the guide through the arc's three components: intended curriculum, operational curriculum, and received curriculum. I organized the narratives using the research questions and the critical events that help answer each research question. For the first research question, I used my own narrative because I am the teacher-research for this study, and my previous experiences as a student and teacher defined and determined the intended curriculum for this project. I utilized restoried narratives for each participant for each critical event for the three other research questions. I incorporated this model for narrative storytelling to present the narratives in a way for the reader to experience the participants' perspectives in how the events unfolded (Connelly & Clandinin, 1988). I alphabetized the narratives by pseudonym purely for organizational structure. The reader should not infer any other meaning from the presentation order of narratives. I conclude the chapter with a summary of the key points and overall findings. Chapter 5 explores the findings in more explicit details.

Findings of Study

Demographics

The participants in this study were myself and nine of my high school math students. The use of convenience sampling is appropriate for an action research study because action research puts teachers at the center of their own study, studying their own practice, with their own students (Vezzosi, 2006).

All nine high school students attend St. Steven's Montessori (SSM), a small, private, Montessori, Catholic school, located in an urban area in southeast Texas. Omar, JoJo, Lina, and Cal are seniors (12^{th} grade students); Ari, Meriam, Blanche, and Shirley are juniors (11^{th} grade students); and Burt is a sophomore (10^{th} grade student). There are three male student participants and six female student participants. Eight students are Caucasian students with two students who are also Hispanic, and one student who is multiracial. All of the students are considered members of middle to upper class families and 0% of the students qualify for the state's free or reduced lunch program. None of the students qualify for our school's tuition assistance program. Five participants are children of faculty or staff members. Staff members receive free tuition for one child. The age range of the student participants is 15 – 18 years old and their experience as a student in a Montessori classroom ranges from three years to 14 years.

Lina was absent for days two and three of the experience. She missed two big aspects of the study: the deep sensory nature experience and the nature scene sketching opportunity. I did work with her to make-up the work she missed as part of her coursework; however, it was outside of the confines of this study. I did not include narrative data collected from Lina due to possible outside influences affecting her participation.

I am a Caucasian male, thirty-seven years old, with 14 years of education experience in both the public and private sectors of education. My main teaching area is high school mathematics, and, at the time of this study, I was responsible for teaching curriculum related to science, mathematics, and technology in my role at SSM. This was my third year at SSM. During my 14-year career, I have also held campus and district-level administrative positions in both curriculum development and program coordination.

Critical Event Narrative Analysis

Using a critical event narrative analysis model (Mertova & Webster, 2020), critical events were identified as those that impacted, altered, or changed the participants' understanding or worldview of the intended, operational, or received curriculum (Uhrmacher et al., 2017). In this study, intended curriculum was defined as those activities completed during the planning and designing stages of curriculum development, including understanding the inspirations behind this teacher-researcher's intentions; operational curriculum was defined as those activities completed with the students during the nature-based mathematical experience; and received curriculum was defined as those activities completed with the students at the conclusion of the nature-based mathematical experience. An event such as this would have changed the participant's perception of his or her mathematical understanding as it relates to math-nature connections. Critical events are identified after the event and happen in an unplanned and unstructured manner (Mertova & Webster, 2020).

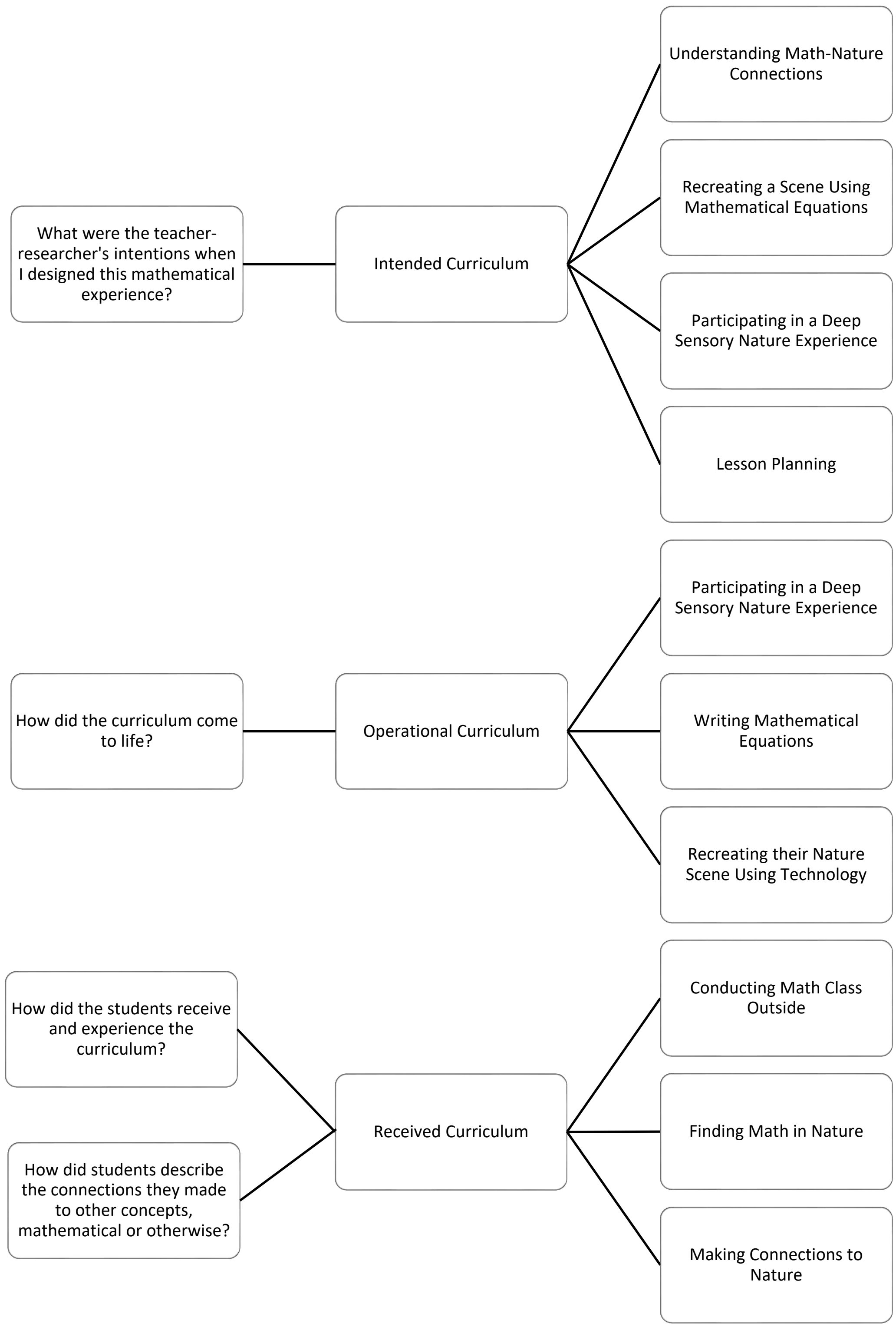
The events that emerged from document analysis, observations, photo-elicitation, and interviews were characterized as critical, like, or other (Mertova & Webster, 2020). A critical event has a large impact on participants and is identified as an event that has a unique illustrative and confirmatory nature (Miller, 2010). Like events are different, and not as reflective as critical

events, highlighting or confirming the critical event (Mertova & Webster, 2020). Events not related to critical or like events are considered other events in critical event analysis and are usually descriptive of the critical or like event or provide background information (Mertova & Webster, 2020).

I found in this research study that the critical events, specifically for the operational curriculum, aligned with the lesson plan activity. For example, I had the students complete a deep sensory experience, and the act of completing the deep sensory experience became a critical event in each student-participant's narrative. I identified other critical events similarly. For the intended curriculum, I identified the critical events in my past which were influential in my lesson design. For the received curriculum, I captured and identified the critical events after reflecting on and reading through the student-participants' final presentations, observed class discussions, and their journal entries about the experience as a whole. Figure 3 connects the critical events to the corresponding curriculum area on the instructional arc and the research question they help answer in this study.

Figure 3

Critical Events to Instructional Arc and Research Question Diagram



Identification of Critical Events

Using the instructional arc to guide my thinking, I identified critical events that influenced the intended curriculum, the operational curriculum, and the received curriculum as seen in Figure 3. The intended curriculum critical events focus on events that influenced me and changed my thinking and development of this study. I used prior experiences with math or nature or both to help finalize my lesson plans for this study. Understanding math-nature connections was a huge motivator for this study and my intentions, drawing on my experience in a conventional middle school math class, remembering the positive experiences I had connecting math to nature. The desire for the students to have a similar experience created the foundation for exploring the local environment around this outdoor classroom. I also remembered the time where I enjoyed the challenge and creativity of having to recreate a scene using mathematical equations in my high school Algebra II class. This was another component of the project that I wanted to include in this experience, but I wanted the scene to be chosen purposefully. I wanted nature to be the students' inspiration and serve as a muse for this project. I again drew from a past personal experience when I participated in a deep sensory nature experience in a college environmental education class. The feeling I had connecting with nature brought a new sense of understanding about my environment. I knew that I needed to include this aspect into the project, helping students situate themselves in their local environment, draw from their senses, and connect with the nature around them. After I identified the overall experiences I wanted students to engage in, I completed the lesson planning process to visually understand and identify the components necessary to complete this experience with two main objectives in mind: I wanted students to identify a variety of mathematical functions in nature, and I wanted them to mathematically define those natural elements using a variety of equations.

Once the intended curriculum had been set, the students and I moved our mathematics classroom to the outdoor classroom space and began with the introductory lesson. The operational curriculum began to unfold throughout the six-day lesson. Students participated in a deep sensory nature experience and used their senses to identify the different components in the local environment and the mathematical functions that define them. During the deep sensory experience, all of the students had accomplished the first goal of identifying mathematical curves in nature. After the students sketched their nature-inspired scenes, they began the more challenging task of writing mathematical equations for the scenes. On average, students wrote 52.5 equations (max: 113; min: 14) to capture their scene mathematically. Equations ranged in difficulty from linear functions to sinusoidal functions. This accomplished the second goal of wanting the students to define nature mathematically, using equations. Once the students had their equations, with restricted domains, the magic happened. Students recreated their nature scene using technology, the Desmos online graphing calculator, and saw their hand-written equations form the scene they had chosen from nature. Students began to see more than just a math class, they began to see the importance that math plays in nature and vice versa.

It was evident, initially, from the operational curriculum, and then confirmed through the received curriculum, that students will continue to understand and see these math-nature connections. Students continued to discuss the deep sensory experience and the types of mathematical curves they kept noticing with me and with their friends. They began to notice these connections in the small and big things throughout their daily lives. If they had not already, they began to see math as practical and worthy of the time it takes to understand its beauty, just like nature’s beauty. Additionally, other parts of the received curriculum came from the operational and intended curriculum. Students enjoyed that fact that I conducted math class

outside, especially when the weather was nice. They continuously were finding math in nature and making connections to nature that had mathematical and non-mathematical roots.

Students who completed this project changed the way they saw nature and mathematics permanently. The intended curriculum, the operational curriculum, and the received curriculum mirrored each other as the experience unfolded, and the student participants and teacher-researcher benefited through their involvement and participation in this nature-based mathematical experience.

Restoried Narratives

Question One

Q1 What were the teacher-researcher's intentions when I designed this mathematical education experience?

I developed and designed a six-day nature-based experience with the purpose of the students to be able to identify accurately and correctly different mathematical curves they find in nature and to be able to write the equations associated with the curves they identified. The critical and like events influencing my design process and my overall intentions for this experience related to the following events: a) understanding math-nature connections, b) recreating a scene using mathematical equations, c) participating in a deep sensory nature experience, and d) lesson planning.

Understanding Math-Nature Connections

I remember the nautilus shell, its mesmerizing spiral, and the pattern hidden in plain sight—my first experience with the Fibonacci sequence, a powerful memory that sticks with me today. Ms. Inman showed me and my fellow 7th grade pre-Algebra classmates this shell as she introduced the concept of sequences. Tracing my finger around its expanding circular shell, I

tactilely felt the Fibonacci sequence coming alive: 1, 1, 2, 3, 5, 8…. It was an amazing experience to see math come alive.

As a tribute to Ms. Inman and my future students, I keep math-nature trinkets on my desk for all to investigate and experience. My collection includes a nautilus shell and a pine cone my son and I found on a nature walk, a fossilized butterfly purchased at a Reptile Convention I attended, a log sculpture a student made for me with the inscription y=a*log(x), and a few other memorabilia. While teaching, I like to bring these trinkets out to show to the students so that they can readily see some math-nature connections, hopefully spurring math to come alive in them, as well.

For this experience, I used the nautilus shell and the pinecone as examples. I pulled these out for the students a few weeks ago as we investigated the concepts of sequences and series, and I decided to pull them out again for this experience as well. My intention was for the students to connect quickly both of these nature elements to the Fibonacci sequence. I wanted them to make the connection to their prior knowledge when I introduced this math-nature experience.

In addition to my own personal experience with making math-nature connections as a student, I have experience as a teacher creating math-nature projects for the students to complete as part of their math coursework. One in particular, eight of the nine participants completed for me last school year. Students chose a type of function from a list I provided to them, for example, one student chose the quadratic function. With their chosen function, I asked the students to conduct some research on their function, its formula, its mathematical importance, and to provide real-life examples of where the function occurs in nature. I would consider the project I created for this study an extension of that project. In both projects, I wanted the students to see functions in real-life. The previous project had the students focusing on a single function;

the project for this study has the students focusing on a variety of functions and curves they can find not only in real-life, but in also nature, specifically.

I utilized the outdoor classroom space and the adjoining soccer field and forest areas to introduce this project and to provide the opportunity for students to easily access and make math-nature connections.

Recreating a Scene Using Mathematical Equations

Mrs. Walters's, as a part of her Honors Algebra II class (note: "Honors" equates to "Pre-AP" classes), always assigned this project where students drew a picture by hand on graph paper and then had to recreate their picture using mathematical equations and their graphing calculators. I remember taking her class as a high school sophomore and looking forward to this project. I do not enjoy drawing, sketching, or doing art in non-mathematical situations, but I was always up to the task when a teacher related it to a math project.

For example, I remember a dilation project I completed in Geometry where I had to enlarge one cell, 11cm x 11cm, of a comic strip to the size of an 11in x 11in poster. Using a grid system on the original comic strip, I replicated the comic cell by cell using the same grid system on the enlarged poster board using the dilation scale of 1 cm = 1 inch. Then cell by cell, I was able to redraw the Calvin and Hobbes comic strip I had chosen on the poster board, and it looked just like the original. I, of course, have reused this project with the students when I investigate the topic of dilation with the students.

Projects that involved math and art were something that I looked forward to, and Mrs. Walters's Piecewise Function Calculator project was no exception. She tasked all students to draw any picture that we wanted, but the picture had to contain a defined list of curves and functions. I recall my drawing containing skyscrapers, a sun, and some clouds. Once we drew

our picture, we needed to create the mathematical equations associated with each aspect of the picture and identify the domain restrictions. We then inputted those equations and domain restrictions into our graphing calculators to see the picture we drew come alive inside the graphing calculator. I still remember the feeling when I hit the "graph" button on my calculator and saw my mathematical work turn into the same drawing on the paper.

The project Mrs. Walters assigned to me as a student served as the foundation and inspiration for the nature-based mathematical experience in this study. Instead of students creating their own picture, they would instead find inspiration from their local outdoor environment, find the mathematical equations for the scene they chose, and recreate the scene using an online graphing calculator program, Desmos. I chose to use Desmos instead of the graphing calculator because, in my opinion, Desmos can draw the equations with restricted domains more accurately than the hand-held graphing calculators can. My intent was to provide students the avenue to see and identify the multitude of curves and functions readily accessible in nature and to review writing all types of mathematical equations with restricted domains. In essence, students created a piecewise-defined set of equations that mathematically described a scene they find in nature.

Participating in a Deep Sensory Nature Experience

In an Environmental Education course I took as a doctoral student, Dr. McConnell (2019) tasked her students with spending time outside in a deep sensory experience to learn and understand about an outdoor place using only their senses. I embraced the task and walked to a bird sanctuary reserve near my parents' beach house. I sat amongst the foliage and used all of my senses to understand better the area. As part of the assigned project, I was inspired to write a poem about my senses and my understanding:

the ocean air a salty brine
the wind echoed offline

the wildflowers gyrate
the bees enticed to pollinate

the birds playing and chatting
the waves rhythmically clapping

the turtles basking on rocks
the fish avoiding the docks

the dirt earthy and bare
the evidence of animals burrowed there

the sun shiny and warm
the collection of all gently inform

the observer of wildlife
the witness to real life
(Tucker, 2019)

This sensory experience resonated with me during the planning process and in designing this nature-based mathematical experience, and I knew that I wanted the students to participate in a similar deep sensory experience as well. I wanted the students to use not only their senses to learn and understand their outdoor local environment; I wanted them to use those senses to make math connections, as well. I want the students to conduct this deep sensory experience before they start writing the associated mathematical equations. I want the students to have an opportunity to pause and take time to focus on appreciating nature and seeing the natural and mathematical associations, phenomena, and aspects they can witness by sitting in the space, silently, using only their senses.

Lesson Planning

While designing this experience for the students, I integrated Demarest's (2015) approaches to place-based curriculum design, focusing on the aspects of personal connections,

holistic understanding, and local investigations and Sobel's (2008) special places and small world investigation techniques for transcendent nature experiences with Boaler's (2016) 5 C's of Mathematics Engagement: curiosity, connection-making, challenge, creativity, and collaboration. By integrating the concepts of place-based curriculum, transcendent nature experiences, and mathematics engagement, I determined my two main purposes and student objectives for this nature-based mathematical experience: I first wanted the students to be able to identify mathematical curves in nature and then to be able to write the equations for those mathematical curves. With these ideas in mind, I designed this nature-based mathematical experience (see Appendix L for a detailed lesson plan).

Day 1 – Introduction. On the first day of the experience, I introduced the topic by having students recall prior knowledge, such as sequences and series that are modeled in nature using the nautilus shell and pinecones [Connection-making]. I then had the students to find a leaf that had fallen on the ground or from a tree in the foliage near our outdoor classroom [Local Investigations]. Students sketched the outline of their leaf on graph paper and described anything peculiar that they noticed about the leaf's outline on a coordinate grid (hopefully leading them to say something like, "these pieces look like lines," [Curiosity]. Students then work together to identify the linear segments they identified on each other's sketches and determine the slopes of each segment [Collaboration]. Students then identified the restricted domains and equations of those segments, creating a piecewise-defined curve for their leaf's outline [Challenge].

Day 2 – Sensory Experience. On the second day, I planned for students to share their findings from day 1, focusing on the linear segments, restricted domain, and the equations they found [Collaboration]. After a brief explanation of expectations, students participated in a deep sensory experience [Small World Investigation]. Students found a spot that interested them

outside in the outdoor classroom, the soccer field, or the forested area on the edge of the soccer field [Local Investigations]. Students spent 15 minutes silently observing their space [Special Places]. Using only their senses, they engaged with the space, focusing on two questions during their observation of nature: What do you see, hear, feel, taste, and smell [Curiosity]? What math connections do you see, feel, taste, hear, and smell [Challenge]? Once the 15 minutes is over, students wrote a journal entry on their observations, nature and mathematical [Personal Connections].

Day 3 – Investigation. Students began their investigation on the third day. Students went back to the spot they completed their deep sensory experience or chose a different spot that they felt like they might connect with [Special Places]. On a new sheet of graph paper, students drew a coordinate plane with x and y-axes and then sketched their scene onto the coordinate plane [Creativity]. While sketching, I asked the students to identify as many different curves as they can in their visual scene; they did not have to draw all the curves they identified in their sketch [Challenge]. Appendix I shows the curve identification frequency sheet students used for this part of the experience. Students spent the remaining class time refining their sketches, connecting with nature, and adding details to their sketches [Creativity].

Day 4 - Equation Writing. Students now needed to convert their sketches into mathematical equations [Challenge]. I worried about the need for a whole-class equation review, so I created an equation writing tip sheet (Appendix J) that we discussed as a whole class before students began writing equations on their own [Connection-Making]. I also conducted mini-review lessons on how to write the different types of equations or curves as the need arises to individual and groups of students. Students also used each other as equation writing resources and memory jump-starters [Collaboration].

Day 5 – Entering Equations into Desmos. Once students completed their equations, I introduced the students to Desmos, an online graphing calculator program located at http://desmos.com/calculator. Students use this program to input the equations they created along with the restricted domains they identified for each curve [Challenge]. Desmos's algorithm for piecewise-defined functions is different from how students write them on paper, so I provided one-on-one mini-lessons, as each student completed their equation writing, to show them how to program the equations into Desmos [Challenge]. During the mini-lesson, I demonstrated the settings controls so that students can alter the colors of the curves, if they wish [Creativity]. Once students completed their drawing, they use the share button to send their Desmos graph to me.

Day 6 – Presentations. On our final day with this experience, students present their sketch and their Desmos drawing to the class, discuss their findings and outcomes, and answer the question, "What would you add or change [Curiosity]?" During the presentation, the plan was to have the students be in the location where they drew the sketch, but it was raining on our presentation day, and students had to point to the spot from our classroom window. After their presentations, students completed a self-assessment of their work and their overall experience with the project.

Findings of Question One

The critical events influencing my intentions and thus the intended curriculum (Uhrmacher et al., 2017), share a common thread: previous experiences I had in my former classes that had an impact on my educational journey. Reflecting on this, I evidently value the impact lessons have on students, and I want to ensure that lessons I impart on the students hold similar meaning and influence. My own experiences shaped my understanding of the world

(Dewey, 1938), and I used my understanding of the world to influence my lesson designs (Connelly & Clandinin, 1988), specifically in the design of this nature-based mathematical experience.

Question Two

Q2 How did the curriculum come to life?

I looked at data on the operational curriculum to answer this question. I also wanted to compare my intentions, the intended curriculum, to what transpired during the lesson, the operational curriculum. I was most interested in understanding the students' experiences with the project overall and how each student participated in each type of activity in the lesson.

I collected documents from the students, daily. Documents included the students' leaf sketches, deep sensory experience journal entries, curve identification frequency tables (Appendix I), nature scene sketches, equation writing scratch sheets, writing equations tip sheet (Appendix J), online access to their Desmos drawings, and self-assessments (Appendix K). I also used a photo elicitation process, my observation field notes, and reflective journaling to complete the data collection process to answer this question. The collected data assisted in accurately restorying each participant's narrative based on the critical events observed during the operational curriculum.

After analysis of the data, the critical and like events centered on the following: a) participating in a deep sensory mathematical nature experience, b) writing mathematical equations, and c) recreating their nature scene using technology.

Participating in a Deep Sensory Mathematical Nature Experience

Overall, the deep sensory experience accomplished its goal, connecting the students with nature. All participants mentioned that they enjoyed the experience, wanted me to incorporate

more of these experiences in future mathematics work, and were easily able to connect the experience to their mathematical studies.

During the 15-minute experience, two students, not involved in this study, and who were from another lower-grade-level class, witnessed us outside, sitting silently and spread out across the soccer field, and became inquisitive. Through the fence that separated us, they asked, “What are you doing, Mr. Tucker? Are they all in trouble or something?”

I responded, “They are involved in a deep sensory experience. We are spending 15 minutes seeing, smelling, tasting, hearing, and feeling nature and any math connections that come our way.”

As they ran towards their guide, I overheard them state to each other, “We need to tell our teacher to do this!”

Participant 1 – “Ari” – In the Veins of Leaves. Finding a spot in the middle of the field, allowing the sunlight to warm her body—a stark contrast from the frigid A/C temps inside the Texas school building—she began to relax and take in her surroundings (Figure 4). The noise of the cars passing by on the nearby freeway and of the occupants in the nearby apartment complex began to fade away, and the chirping bird and rustling tree leaves provide her the white noise she needs for her meditative focus. She “found the smell of the grass to be quite relaxing, and [she] enjoyed picking blades of grass and little flowers.” The tactile activity further enhanced her meditative focus.

Figure 4

Ari's Deep Sensory Experience Location

As Ari's focus increased, she began to see the many mathematical functions present in the nature around her, spotting "absolute value, linear, rational, and quadratic functions in the soccer goals, fences, houses across the street, blades of grass, shapes and veins of the leaves next to [her], and in the trees in the field." She saw 39 instances of linear functions, her highest frequency function, in mostly man-made items: fences, soccer goal posts, and house architecture; however, trees and blades of grass were other examples she saw of linear functions. Other man-made functions she saw were in the soccer goal net (exponential and step functions), house decorations (quadratic functions) and house rooflines (absolute value functions). Trees, however, provided an opportunity to witness other functions in nature. She saw quadratic functions in the trees overall canopy shape, and cubic, square root, and rational functions in the way the tree branches connect with the trunks of their trees.

As Ari relaxed, she realized she did not have to force herself to find the mathematical curves; she was spotting them frequently. It was "impossible not to see them."

Participant 2 – "Blanche" – Grass in All Directions. Blanche focused her deep sensory experience on the microenvironment instead of the macro-environment. Initially, she became overwhelmed and experienced a sensory-overload when focusing on all the possibilities

surrounding her. The trees provided an excess amount of stimuli as well as the apartment buildings at the edge of the soccer field. She decided to focus her attention on a small patch of grass in the soccer field. Lying on her stomach, she deeply investigated one small area of grass that turned out to be a lot more interesting (Figure 5).

Figure 5

Blanche's Deep Sensory Experience Location

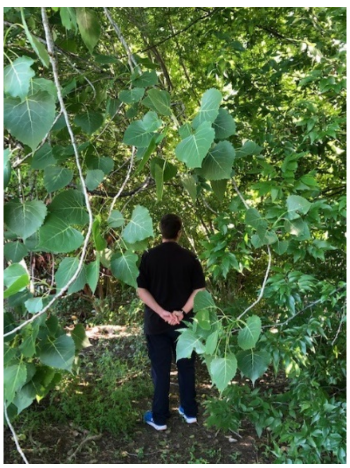
Blanche could still hear the sound of the highway and smell smoke from a nearby charcoal smoker, but she found herself amazed at how she could locate "all the function shapes just by looking at a small patch of grass" because of how the "grass points in all different directions, making all the different shapes and things." As she continued her investigation, she began feeling around the grass, and came across some prickly sticker burrs. Playing with the pointed spines of the burrs, having the spines prick her fingers or stay stuck in the ground, she realized the existence of different layers in the grass: the layer of leaves making up the soil, the layer of animals moving the grass and the soil, and the layer of the grass blades. All the different layers providing an avenue into mathematical inquiry and connection: "the more time I spent looking at the grass, the more I noticed details about it, and the more mathematical questions that came to mind."

Participant 3 – "Burt" – Upwards and Asymptotic. Placing himself underneath the canopy of a tree in the thick foliage bordering the soccer field (Figure 6), Burt had an up, close,

and personal look at some of the trees on SSM's campus—one tree in particular. The different ways the branches sprouted off the tree's trunk formed these beautiful functions. He was able to identify immediately linear, quadratic, square roots, logarithmic, absolute value, and cubic functions just in the way that one tree's branches connected to its trunk. When he inspected the trunk more closely, he realized the pattern the bark created was sinusoidal in nature. He attempted to find a hyperbolic curve in the trees branches, but ultimately came away unsuccessful: "nothing had hyperbolic or [was] visually hyperbolic."

Figure 6

Burt's Deep Sensory Experience Location



When Burt closed his eyes and took in the sounds and the smells of the woods, his olfactory senses took him back to a memory when he was fishing, his favorite hobby, in an area where two creeks met and then "went upwards, as if separated by an asymtop [sic]." Even though he was not able to identify any rational functions, or asymptotes, a main component of rational functions, while in the tree's canopy, his memories found a connection for him.

Even though Burt still had "the taste of leaves, dirt, and wood in [his] mouth" at the conclusion of his sensory experience, he was pleased to realize that "there were all sorts of mathmatical [sic] functions or peicewises [sic] of them" in the woods.

Participant 4 – "Cal" – The Uniformity of Trees. Spotting a spectator's bleacher on the other end of the soccer field, Cal quickly claimed his location (Figure 7). From this vantage point, he was able to see the vastness of the soccer field, its markings, and the manicured landscaping between him and the opposite fence. The "very uniform spaced trees" forming a possible step function in his mind, with their equally spaced distances creating the signature bar shapes found in step functions. He then began to notice the soccer field, the same field where he played tens, if not hundreds, of soccer matches in previous school years, come to life with "both linear and parabolic shapes at different points on the field," making special note of one area of the field, the penalty arc, that "could be a negative parabolic function."

Figure 7

Cal's Deep Sensory Experience Location

The cloudless blue sky allowed the full sun to heat the grass, the bench, and Cal to a slightly uncomfortable degree. Cal noticed the very linear way the sun cast light onto the grass. The same linear functions he attributes to the height of the grass. Noting the difference in the

temperature between the sunny grass and the shady grass, Cal moved off the bench into the shaded grass and continued his exploration. He noticed the different bugs and insects crawling and flying around him. He also recognized that each bug and insect has their own unique shape, and he began to see mathematical curves in a multitude of places.

Participant 5 – "JoJo" – The Square Root of Eucalyptus. Enjoying the crisp, clear air of the gorgeous spring day, JoJo watched as the bugs began crawling around her (Figure 8). A small, jumping spider crawled around her and a ladybug started to crawl on her. She continued to observe the nature around her as the ladybug continued to traverse her body, and she noticed the green, curvy blades of grass. The grass blades forming mostly linear functions; however, she did see blades of grass stuck together in a way where "some even looked like absolute values."

Figure 8

JoJo's Deep Sensory Experience Location

The different shades of green in the trees' leaves quickly made room for JoJo to notice the leaves forming all different shapes and a variety of functions: cubic, square root, and absolute value. She then spotted the recently dead Eucalyptus tree in the back of the field. The dead tree reminded JoJo of the recent bout of frigid temperatures that came into our area a few months ago

and decimated much of our area's not-used-to-freezing-temperatures foliage. The branches of the tree, "curving in square root patterns," are all that remains of this particular Eucalyptus tree.

The stillness and peacefulness as "everyone was immersed in their own world" complemented the "breezy and shady" spot where JoJo sat. She quite enjoyed finding the mathematical in the presumably mundane, like how the "little flags waving with the wind were linear segments curving sinusoidally with the steady breeze."

Participant 6 – "Meriam" – Sensing Different Functions. Meditating in the calming and relaxing warmth of the sun, Meriam sat in the middle of the soccer field and took in her surroundings (Figure 9). The cool breeze along with the warming sun provided the ideal setting for her to study nature. She "studied the linear grass around [her] and also noticed the parabolic-shaped painted [stripes] into the grass on the soccer field." Taking in her surroundings, she also made notes of seeing the variety of trees, flowers, and shrubbery; different sized buildings; the combination of gates and fences; and the benches and bleachers that were visible from her point of view. When she utilized all of her senses, she discovered the absolute value shapes in the corners of the goal posts, the quadratically-curved windows, and the square root shapes of tree branches: "there are so many shapes and sizes that can be found simply in nature."

Figure 9

Meriam's Deep Sensory Experience Location

While enjoying the feeling of sitting and studying mathematics in nature, Meriam realized “how much mathematics can be found in nature when you take the time to look for it.” She used the time to observe and take in all her surroundings, focusing “all my senses to pinpoint different functions that were around me,” and concluding that she “can integrate math with things I see in the natural world.”

Participant 7 – “Omar” – Geometrically Dissecting Logs. Sitting atop a pile of abandoned logs (Figure 10), Omar found himself in the company of “musical bees, cool mushrooms, and rough wood”. The rough texture of the logs is what drew Omar to this particular location. He also enjoyed that the pile of logs provided a natural chair he could use. Focusing his attention to the logs themselves, he was determined to “mathematically dissect” the logs. He first noticed the step function he found in the natural stripping of tree bark. He then marveled in how the logs’ age rings, emphasizing the old age of these abandoned logs, were both the positive and negative forms of a square root function. Continuing his dissection process, he defined the ends of the logs as types of rational and exponential curves.

Figure 10

Omar’s Deep Sensory Experience Location

The calming feeling of being in nature provided Omar with the opportunity to use his mathematical thinking in conjunction with his passion for environmental science to "dissect" the pile of logs. Through that act of noticing "a lot of geometric symmetry" and "deconstruct[ing] the stuff around me into the basic shapes that composed them," he felt comfortable looking "at the individual parts of an object rather than always looking at an object, and trying to understand an object, in its entirety."

Participant 8 – "Shirley" – Possibly Parabolic, Possibly Just Moist. Shirley, finding "everything caked in a layer of dew, making everything all wet and moist," was uncomfortable with the experience. Her chosen location (Figure 11), not yet touched with the mid-morning sun, was still wet from the evening rain and morning precipitation. It took her some time to move past the "wet seat I was sitting on," but when she did she started to first notice the sounds of the birds and the busy highway, both providing a musical accompaniment to her thoughts. She then started noticing shapes at first: triangles and squares. She eventually started identifying linear and absolute value functions in the architecture of the town houses across the street.

Figure 11

Shirley's Deep Sensory Experience Location

A curved stick eventually caught her eye. Picking up the stick and using her hands to manipulate the twig, she noticed its "possible parabolic or maybe square root shape." While not confident in her response, she does recognize the stick has a curve feature that is some other shape than her previously found linear or absolute value shapes. She also recognized a non-linear curve in the shapes of the windowpanes in the town houses. She correctly identifies the semicircle window as a square root curve, but does so with hesitation using a parenthetical note: "probably not."

Participant 9 – the Teacher-Researcher – Mr. Tucker – Mathematical Viewpoint. I could not have asked for a better day to conduct a deep sensory experience with the students. A cool, crisp breeze that kept the outdoor temperature comfortable for a spring Texas morning accompanied the warm, not too hot, air.

Before releasing the students into the field to conduct their experience, I conducted a brief review of the different curves and their general shapes with the students. Volunteers provided and pointed out examples of the curves from the view of our outdoor classroom. I explained the sensory experience—15 minutes where they were just observing with their senses. I posed two questions to them to think about when they are in their space: What do you see, hear, feel, taste, and smell? What math connections do you see, feel, taste, hear, and smell? Then they were off.

Students spread all over the soccer field, some finding places to sit on benches, some in the middle of the field itself, one to the farthest corner to sit on top of a pile of logs, and one adventurous sole to stand within the thick foliage at the edge of the field. After observing each student in each of the locations, they revealed themselves as engaged, studious meditators and

learners. They appeared to be taking it all in, the sounds, the smells, the tastes, the feels, and the sounds—and what a great offering the environment gave to each participant on this day.

At the end of the 15 minutes, students reconvened in the outdoor classroom area and continued the silent mathematical journey by writing a journal entry on their observations.

As we entered the classroom inside, Mrs. Darcy, the other high school teacher at SSM, asked, “How are you all liking math outside?”

Burt responded, “I wouldn’t call what we did math.”

“Why not?” Mrs. Darcy inquired.

“Because we didn’t use *x* or numbers.”

“Did you use math vocabulary?”

“Well…yeah.” Burt then concluded, “I guess math is more than just variables, numbers, and equations.”

Burt’s comments here are not a surprise. He was not solving an equation or graphing a line or something else stereotypically associated with learning math. He was placed in a real situation and asked to make mathematical connections. He did so unconsciously aware that he was “doing math.” It is always fascinates me, as a math teacher, to see how passively and discretely math can make its way into a student’s day and how the simple act of appreciating nature and its beauty leads to changing someone’s mathematical viewpoint.

Writing Mathematical Equations

Burt had previously pointed out during the deep sensory experience that we had not used variables, numbers, or arithmetic, yet; however, this was the part of the project and experience where students dove into the more traditional math work: writing equations of lines. Students returned to their deep sensory experience location to sketch a scene of their choosing onto graph

paper. While students sketched, they used their frequency table (Appendix I) to record the types of functions and curves they identified, incorporating the lesson they participated in prior to the deep sensory experience. In this lesson, the students and I sat in the outdoor space and looked around, identify objects that formed the different types of curves (linear, quadratic, cubic, square root, absolute value, rational, logarithmic, exponential, step, and sinusoidal).

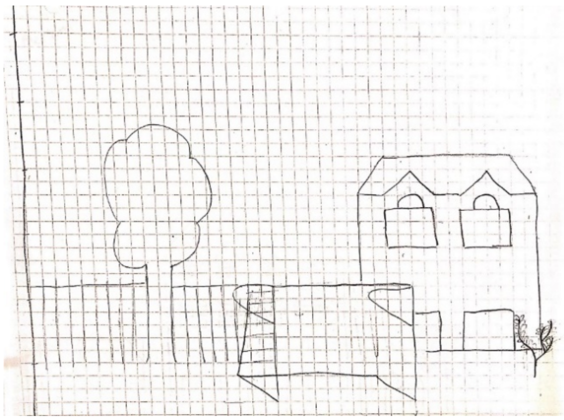
Using a coordinate plane on graph paper, the students turned all the different parts of their sketch into mathematical equations. Students had to identify correctly the type of curve or function they wanted to use to represent that portion of the sketch and then correctly identify ordered pairs, or points, to use to create their equations. Students were able to easily identify the type of curve they were using from the previous exercises we completed, and the challenging work was found in converting the pieces into equations. Students used the Writing Equations Tip Sheet (Appendix J) to aid their success in this task. Students also asked questions of each other and to me, providing me the opportunity to provide mini-lessons to students on the various functions and equations they identified.

The students' sketches used only a specified portion of each equation, therefore students also had to identify correctly the restricted domains, creating a set of piecewise-defined equations. I noticed that all of the students started with writing the linear equations, first, and then moved towards the more demanding curves (quadratic, cubic, sinusoidal, etc.). While this was challenging work, students asked for help, helped each other, and received reassurance that they were conducting the mathematical steps correctly, they preserved and were successful.

Ari – Circles in Trees. Ari's nature scene (Figure 12) utilized the fence structure, two nearby trees, a soccer goal, and one of part of the town house structure across the street.

Figure 12

Ari's Nature Scene on Graph Paper



Ari focused her attention first on the linear segments. She realized all of the horizontal line segments could be represented using the simple formula, y = number. The number being the corresponding y-value. She thought the vertical line segments would also be just as easy, but she could not remember how they were represented. Asking Blanche for help, together they realized the vertical line segments were equally simple. Instead of y = number, like the horizontal line segments, it was x = number: “that's because the horizontal lines intersect the y-axis and the vertical lines intersect the x-axis, ugh...duh!”

After discovering the difference between horizontal and vertical lines, she moved to the diagonal line segments and utilized the point-slope form of linear equations. She was used to the slope-intercept form and needed to ask me for clarification on the difference between the two. I reminded her that the y-intercept was also a point where the x-value of the ordered pair was zero. Studying the point-slope form, she saw quickly how the slope-intercept form and the point-slope form of linear equations are related.

She continued her work, ultimately creating 53 piecewise-defined equations. As she worked out the equations for her quadratic, cubic, and absolute value functions, she began to see a pattern.

Turning to her shoulder partners, Blanche and Meriam, Ari asked, “Did you know these are all pretty much the same process?”

“What do you mean?” Meriam questioned.

“All you need is the vertex and any other point and it turns into a plug and chug kind of situation,” Ari exclaimed.

“Show me.”

Ari began to teach Meriam how to produce these types of equations on her own.

Ari then worked on her last equation. She recognized it both as a circle and as a square root function. Recalling her conic section project on circles, she remembered the equations for circles was $(x - h)^2 + (y - k)^2 = r^2$. She determined the center point of the circle, (h, k), and measured the radius, r, and was able to successfully create an equation for a circle.

Once she created her equation, she did call me over for reassurance that she completed it correctly. She explained her process to me, and I reassured her that she had created a circle equation, but she would need to verify if it what she created was the correct circle equation for her sketch. She would be able to do this once she graphed her equation in Desmos.

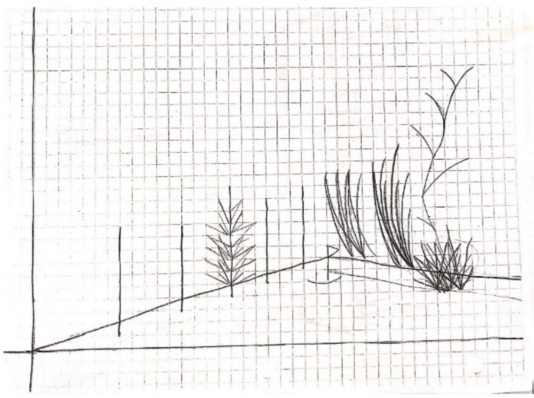
Before I left, I asked her, “I see you have the circle equation marked as a square root equation. How are the circle equation and the square root equation related?”

“Because when you solve for y, the circle turns into positive and negative square roots,” she responded proudly (and accurately).

Blanche – Quadratics Everywhere. In her sensory experience, Blanche focused on a small patch of grass. For her nature sketch, she brought her attention to the trees and the tall grasses in the far corner of the field (Figure 13).

Figure 13

Blanche's Nature Scene on Graph Paper



Similar to Ari, she first determined the linear segments, starting with the vertical equations. She remembered that vertical equations start as $x =$ and reminded her fellow classmates when they asked the table for help with vertical equations. This happened frequently enough that after a while Blanche demanded the table's attention: "Guys! Vertical lines are x equals whatever the x-value is. Stop asking!" She spent the majority of her time after that working on determining the slopes of the linear segments forming the tree on the left side of her sketch and the two bushes on the right. Her concentration on this seemingly tedious task was a partial cause for her outburst; she felt distracted by constantly hearing the same question.

Blanche moved on to the curvier bushes towards the center of her scene. Recognizing these as the left half of a parabola, she started to feel puzzled and confused "because it had been awhile since I had used that skill [writing quadratic equations], it wasn't coming to me quite as easily as I thought it would." Blanche overheard me giving a review of writing quadratic equations to Cal and Omar at the other end of the table "and then I remembered how. It made

sense again." Once she remembered how, she quickly got to work writing the quadratic functions associated with those curves. Then she soon realized, "I don't actually have to do the math for every single equation. Cause when I was graphing my tree, which took a while, and I was like, oh, I don't actually have to do the math for every single one of them. I can just change one or two numbers and it'll move the way that I want it to. And that is when I felt like I was in the flow."

Burt – Squirrels are not Math. As the youngest participant in this study and, as this is his first year in the three-year high school cycle at SSM, Burt's mathematical discourse initially consisted of multiple "I'm lost" or "I'm confused" statements. He is quick to throw up his hands if he is not able to understand the mathematical situation quickly. While gifted mathematically, he is working on his mathematical mindset.

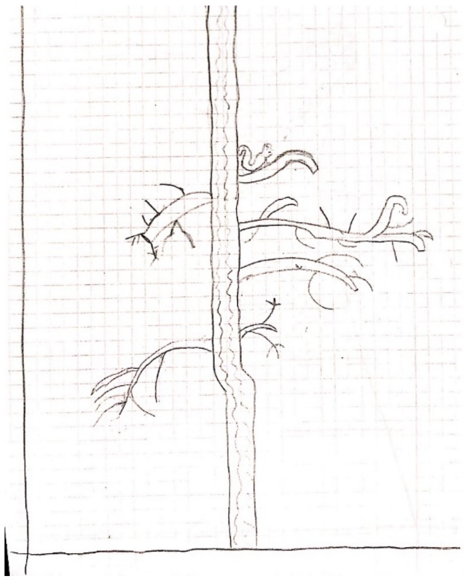
After he consulted with Shirley, he still was confused and asked me for some assistance.

"What's going on? What are you confused about?" I asked.

"I know that this tree trunk I drew is curved here, but it is straight here," Burt said, pointing to the appropriate areas on his sketch (Figure 14).

Figure 14

Burt's Nature Scene on Graph Paper



Distracted, I asked, "Did you add a squirrel in your tree?"

"Yes. He was running all over the tree when I was doing my sketch, so I decided to include him, but I am definitely not going to write equations for him."

"That's ok, if you don't want to." I then probed Burt, "But what equations do you see in him?"

"Well, his tail has some parabolas, but I don't know what to do with his body or head."

"Ok. Well think about it and will you let me know?"

"Sure!"

"Let's return our focus back to your original question. Sorry. What do you notice?" I inquired.

"Great! Now what are you going to do?"

"I don't know. That's why I asked you for help." He said with frustration.

"Ok. If you know that this part is cubic and this part is linear, do you have to write one equation for the whole thing?"

"I guess not. Could I just write a cubic equation for this part and then a linear equation for the other parts?"

"Absolutely!"

"Well, what about these vertical lines. How do you write those equations?" Burt asked.

"I believe [Blanche] already made that abundantly clear," I teased.

"Oh yeah…X equals."

"Anything else?"

"Can you remind me what the inflection point is?"

"Sure! One way to think about the inflection point for your cubic function is as the point on the curve when it changes directions. Do you see that point on your graph?"

"Right here?" Burt pointed to the inflection point on one side of his sketched tree trunk.

"Yep. Can you name that point?"

"About (12.5, 16.5)?"

"Close, but remember which values come first in the ordered pair?"

"Right! (16.5, 12.5). Can I use approximations like these?"

"Absolutely. It is your drawing." I said.

After this encounter, Burt was able to create successfully the piecewise-defined equations for his tree trunk. He continued to ask for assistance for the curves that made up the branches, using a combination of sinusoidal and square root curves to complete his sketch. Once he started writing the equations and understanding the process, he recalled the act of writing equations as

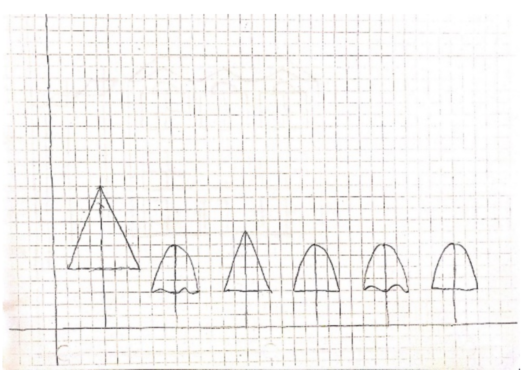
immersive: “I just started like, just writing them down and was like, well, let’s just keep going. I got them all done in like 30 minutes.”

Cal – The Natural Curves of Trees. Knowing exactly what he wanted to do for his sketch, Cal used the uniformly spaced trees as the focus for his nature sketch (Figure 15). He chose to move to a shadier location on the day we went out to complete the sketch: “I’m still looking at the same scene from last time, but I wanted to be in the shade. It’s too hot this afternoon, and this tree coverage gives me adequate shade with the same view of the trees I want to sketch.”

Cal, similar to all the other students, initially started with the line segments in his sketch. He was the first to ask about the equation for vertical lines. Once he created his vertical tree trunks, he focused on the tree’s canopies. He recognized two of the tops as absolute value curves and the others as parabolic. The bottom portions of the canopies were mostly linear, except for two. He recognized those as sinusoidal.

Figure 15

Cal’s Nature Scene on Graph Paper



Cal, not confident in his math skills, organized his mathematical work to help streamline his thought process. He started four columns and labeled them linear, absolute value, quadratic, and sinusoidal. He would complete one equation in each column and wait for my reassurance

before moving on to the next equation. Even though his classmates reassured him he was doing the work correctly, he needed me to look at his work. He wanted to talk through his thought process with me for each equation type. We reviewed writing quadratic equations initially, discussing the importance of identifying the vertex and the direction the parabola opens. He remembered and applied the lesson to absolute values. He required more assistance when it came to the sinusoidal function, even though the previous project he completed several weeks ago was on writing his own sinusoidal word problem and solving it. After I reviewed the components with Cal, his memory was jogged enough to continue his work.

Cal thought that the equation writing process went "by really quickly and easily," even though he "hadn't touched them [writing equations] in a while, so I needed a bit of, I needed some refreshers on like how to write or get the equation formatted on some of them." Even needing the additional help and reassurance, he was the first one to finish writing all of his equations for his scene. He then continued improving his skills by assisting the fellow classmate next to him, Omar.

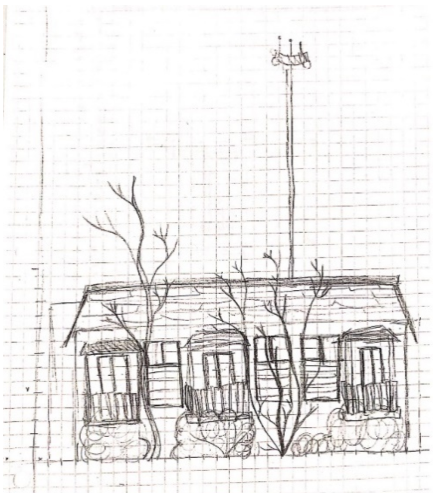
JoJo – Vertical Sinusoids. JoJo, a self-proclaimed creative, drew an intricate sketch of her view of the apartment buildings, shrubbery, and barren trees at the far end of the soccer field (Figure 16).

Realizing she had a herculean task of creating equations for all of the curves in her sketch, JoJo chose an area on her drawing to focus on. She felt most comfortable with sinusoidal curves and linear functions because she "recalled the sinusoidal graphs we did [the sinusoidal word problem project]" and "all the work with linear equations and knowing how to figure out slope." Therefore, she started with those equations, starting with the lines making up the

windows, doors, and entry stoops. She sought the help of her neighbors Blanche and Lina frequently, wanting the reassurance she was completing the equations correctly.

Figure 16

JoJo's Nature Scene on Graph Paper



After determining the equations of 70 different linear segments of all types: horizontal, vertical, and sloped, she moved on to the sinusoidal curves. She noticed that the tree trunk of the left tree had a sinusoidal wave to it, if you tilted your head.

"Mr. Tucker, how do I make the sinusoidal go vertically?"

"Show me, please." She showed me what she was thinking, and we began the following thought provoking conversation.

"[JoJo], how would you write a vertical linear equation?" I query.

"Ok. So how do you think a vertical sinusoid would start if a vertical linear started with x equals?"

"Um, I guess x equals, too?" JoJo guessed.

“Absolutely. But instead of a number, we have to describe the behavior. How would you describe the behavior?”

“Sinusoidally”

“Yep. So instead of what you are used to as y equals cos x, we would write it as—”

“X equals cos y!” She exclaimed.

With that, JoJo was off. She successfully created two vertical sinusoidal curves before she “realized that it would take too much time to recreate the whole drawing [she] made,” so she decided her project was complete after the 72nd equation she created.

Meriam – Slope is Rise over Run. Focusing on linear and absolute value curves, Meriam sketched out a scene where the townhouses were the focus (Figure 17).

Figure 17

Meriam’s Nature Scene on Graph Paper

Requiring review lessons on both linear and absolute value equations from myself and her classmates, Meriam’s confidence level kept her equations within those two categories. To help aid her growing mathematical confidence, she wanted to keep her mathematical work organized. Labelling each quadrant of her paper: vertical lines, horizontal lines, sloped lines, and absolute value functions, and she worked her equations in that order.

As she was working on the equations for the vertical lines, she turned to her neighbor, Ari, and asked for a reminder on how to determine the restricted domain and a refresher on the correct direction for the inequality symbol. Ari was happy to help. The two of them are frequently collaborating on mathematical assignments.

When she moved on to horizontal lines, I peeked over Meriam's shoulder to check her continued understanding. She was right on track. When she noticed me, she took the opportunity to ask for a quick reminder on how to find slope from a graph. With a gentle reminder on how slope is equivalent to the number of units one point *rises* (or the vertical movement) to another point divided by the number of units the same first point *runs* (or the horizontal movement) to the same second point, she remember that slope is "rise over run."

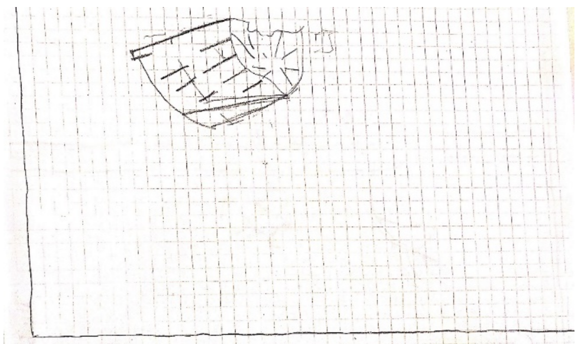
As Meriam continued to work, she ended up arriving at a point where she started looking around, she was not doing any work, and she was not asking any questions. She was lost. I went over to investigate. She needed a review on absolute value equations. She knew how to identify the vertex, but she did not know what to do next. She and I walked through the process together, completing one of her absolute equations. Gaining confidence, she completed the four others on her own.

While she was working on her absolute value equations, she heard Ari exclaim that cubic, quadratics, and absolute value functions all followed the same process. Because Meriam was feeling confident about her absolute value function knowledge, she seized the opportunity to learn more and ask Ari to show her what she meant. Even though Meriam did not write a cubic or a quadratic equation as part of her sketch, she shared in the collective insight on writing equations Ari shared with her.

Omar – The Importance of Point-Slope Form. Crouched next to the pile of logs he sat on for his deep sensory experience, Omar sketched a single log (Figure 18). "I didn't realize I had my graph paper upside down when I completed my sketch," he admitted to me when we started the writing equation process.

Figure 18

Omar's Nature Scene on Graph Paper



Not wanting to use the point-slope form of linear equations, instead he wanted to use the slope-intercept form. However, none of his line segments crossed the y-axis (a data point needed when using the slope-intercept form), so he found and used a ruler to draw extensions on each line segment until it crossed the y-axis. After several minutes, he got to a line segment that did not visibly cross the y-axis on his graph paper, and he asked me, "What do I do if my line doesn't cross the y-axis?" It was at this moment he was finally ready to use and see the importance of the point-slope form of linear equations. He had seen linear equations most of his middle school and high school mathematics career, but in his last month of high school he found its importance. I gave him a quick refresher on how to use the point-slope form and he said, "I could've been doing this the whole time?"

“Yep!” I replied, brimming with mathematical joy and excitement—the joy and excitement educators can experience when they have a breakthrough with a stubborn student.

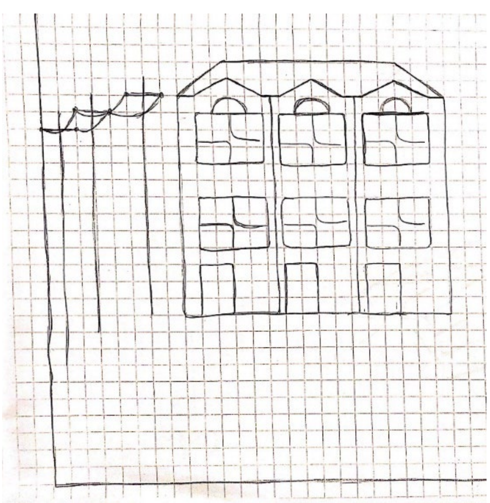
He quickly abandoned the ruler and completed the rest of his linear segments.

Next came the hard part, Omar needed to solve a system of equations to determine the equation for the exponential curve he drew for the right side of the log. He was able to set up the system, but wanted a quick refresher on how to solve for both variables. After discussing the three possible processes: the graphing method, substitution method, or elimination method, Omar chose to use the substitution method. I watched as he solved his system and determined the exponential equation. Even though he solved it correctly, he opted to redraw his log using linear segments instead because “it just didn’t look right.”

Shirley – I Don’t Get It. Attempting to incorporate more than just linear functions, Shirley sketched out the townhouses and the telephone pole wires (Figure 19). In her original sketch, rational, quadratic, absolute value functions are visible. She opted to omit the rational and quadratic portions of her sketch when determining the equations.

Figure 19

Shirley’s Nature Scene on Graph Paper



Struggling through this part of the project, Shirley did not want to write the equations. As I attempted to talk her through the process, she became quite agitated: “I just don’t get it!”

“What don’t you get?” I calmly ask. I am trying to gather quickly additional information to find a way to connect to her lack of understanding

“Everything!”

“Can you show me a specific example?”

“No!” Shirley huffed.

“Can you try, please? I am here to help.”

“Just go help someone else. I’ll figure it out on my own.”

“I see that you are asking for space. I will circle the table and see how everyone else is doing with their work. I’m going to come back to check on you. Will you work on finding a specific example that I can help you with?” At this request, Shirley grunted a “sure,” and I gave her some time to think. After circling the table, I came back and asked her if she had found a specific example.

“Um, like, how do I make the equation line thingy go from here to here? It’s like five lines all connected together. There’s no way that’s possible.” Shirley provided the avenue to start the conversation. We discussed the ability of looking at each piece separately. She would need to find five equations for the five different lines—she did not like the idea of that at all; however, she began to see the task in smaller, more manageable tasks.

At the end of the project, Shirley crafted 48 different equations. They were all linear or absolute value equations, having erased the rational and quadratic components on her original drawing, but she wrote 48 equations.

Recreating their Nature Scene Using Technology

As the students finished writing their equations, I began to introduce the online graphing calculator software, Desmos (http://desmos.com/calculator). In class, we discussed the proper way to write a piecewise-defined function on paper is to define the function first and then provide the restricted domain, separated by a comma, and curly braces around the different pieces, as in Equation 1, for example.

$$(1) \qquad f(x) = \begin{cases} 3x, -2 \leq x \leq 0 \\ -x^3 \quad\ , \ x \geq 0 \end{cases}$$

In this example the first line defines the function as $y = 3x$, but only for the x-values between -2 and 5, inclusive. The second line defines the function as $y = -x^3$, but only for the x-values between 7 and 12, inclusive. The students know how to do this, this is what they are accustomed to, and this is how they all wrote their equations.

However, Desmos does not use this same algorithm for graphing piecewise-defined functions. Desmos wants each piece entered as a separate equation and with the restricted domain first and the function second, separated by a colon, as in Equation 2 and Equation 3. Equation 1 is mathematically equivalent to the two equations, Equation 2 and Equation 3.

$$(2) \qquad y1 = \{-2 \leq x \leq 5{:}\, 3x\}$$

$$(3) \qquad y2 = \{7 \leq x \leq 12{:} -x^3\}$$

Not a hard fix, but I did not mention this initially. Students rebuked at the thought of having to redo their equations. I quickly reminded them it was just a change in the notation and that they did not have to rewrite all of their equations. They would only need to input the equations correctly into the Desmos program. Soothed, for the moment, the students got to work plugging in their equations into their computers.

The other issue that came up was when the students attempted to input their equations for linear equations. Students were using the point-slope form of linear equations, Equation 4, and had written their equations in the same form. Desmos would not graph the equation in this form, so students needed to convert the equation to "y equals" similar to how they are use to when graphing lines in their graphing calculators. Shirley, Burt, Cal, and Meriam, needed a reminder of how to perform this mathematical operation. They soon saw the pattern of needing to move y_1 as the only step. If y_1 was initially positive, then it became negative on the other side of the equation and vice versa. While several students initially balked at the additional work to writing their equations, they eased up after noticing the pattern.

$$(4) \qquad y - y_1 = m(x - x_1),$$

All of the students have personal laptop computers, so the students continued to utilize the outdoor classroom space to complete this portion of the activity. Students are used to working inside the classroom when working with their computers, but students embraced the change. JoJo's computer, however, was not able to access the school's Wi-Fi network while we were in the outside classroom, so when she began writing her equations into the Desmos program, she moved inside.

As students began to input more of their equations into the Desmos program, I heard audible gasps from all sides of the table. Their hand-drawn sketches were coming alive right before their eyes. Students were able to see the piecewise-defined equations they created connect with each other and form their individualized picture. Some students noticed that their equations did not match up exactly; the tree branch did not connect to the trunk of the tree for example, and were easily able to adjust their restricted domain, their slope, or another aspect of their equation

to fix the issue. The self-checking aspect of Desmos provided students with the opportunity for immediate feedback on their own equations.

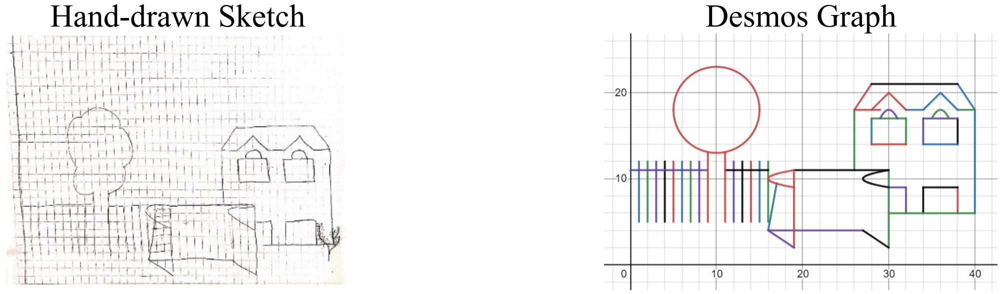
The side-by-side comparison of the students' hand-drawn sketches and their computer-aided images provides a visual to understand the ecstatic feeling these students had when they watched their sketch come to life, equation by equation.

Ari. "It was really cool to see the equations that I had written on paper slowly turning into the picture that I had drawn." Figure 20 shows Ari's hand-drawn sketch compared to her Desmos-produced graph.

Figure 20

Ari's Side-by-Side Comparison

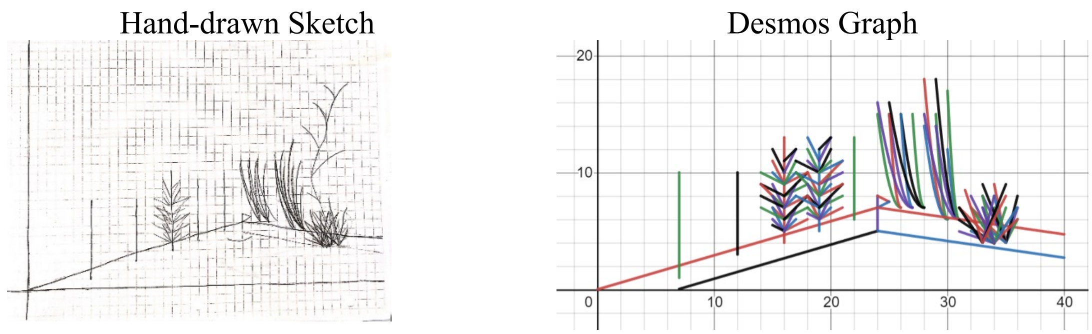
Hand-drawn Sketch   Desmos Graph



Blanche. "I thought it was a lot easier to come up with the equations for each function (and a lot faster) once I was using the online Desmos calculator." Figure 21 shows Blanche's hand-drawn sketch compared to her Desmos-produced graph.

Figure 21

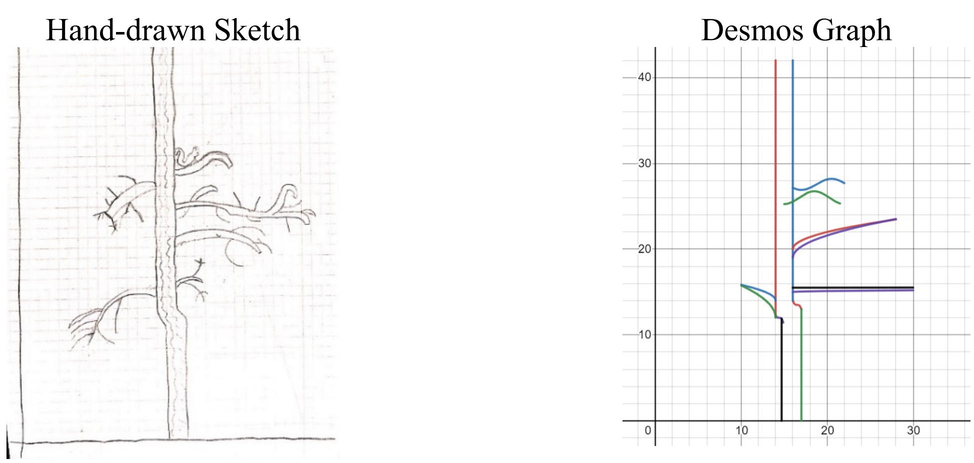
Blanche's Side-by-Side Comparison



Burt. "I was surprised when the lines actually formed into a tree. I didn't think that would actually happen, but it did." Figure 22 shows Burt's hand-drawn sketch compared to his Desmos-produced graph.

Figure 22

Burt's Side-by-Side Comparison

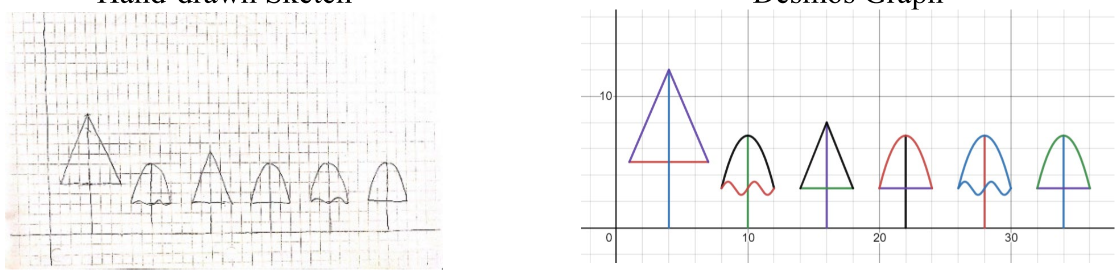


Cal. "Most of the time when you're making a graph, you're going out and collecting data, but this time we were building the graph from our sketches. Our sketch was the data, and it was nice seeing the different equations in different colors." Figure 23 shows Ari's hand-drawn sketch compared to his Desmos-produced graph.

Figure 23

Cal's Side-by-Side Comparison

Hand-drawn Sketch Desmos Graph

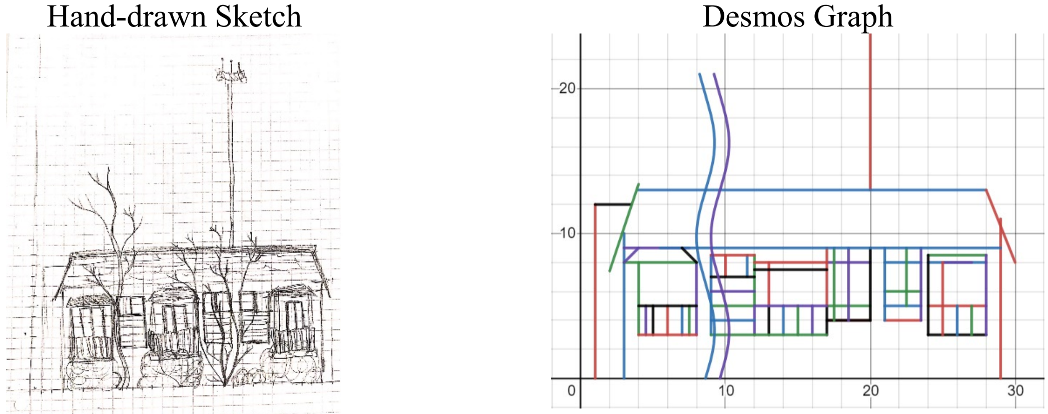


JoJo. "It was cool when I plugged my x sinidisual (sic) function in because I had never created an x sinidual (sic) function, and it was cool when the lines connected and everything started coming together and looking like a building." Figure 24 shows JoJo's hand-drawn sketch compared to her Desmos-produced graph.

Figure 24

JoJo's Side-by-Side Comparison

Hand-drawn Sketch Desmos Graph

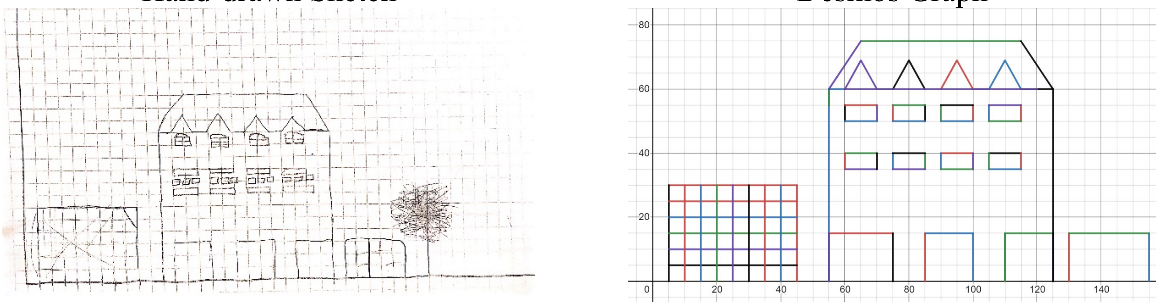


Meriam. "I enjoyed using Desmos to create my graph and it was very satisfying to watch my equations form my picture. There were a few times where I had to change or adjust my numbers to get it where I wanted, though." Figure 25 shows Meriam's hand-drawn sketch compared to her Desmos-produced graph.

Figure 25

Meriam's Side-by-Side Comparison

Hand-drawn Sketch Desmos Graph

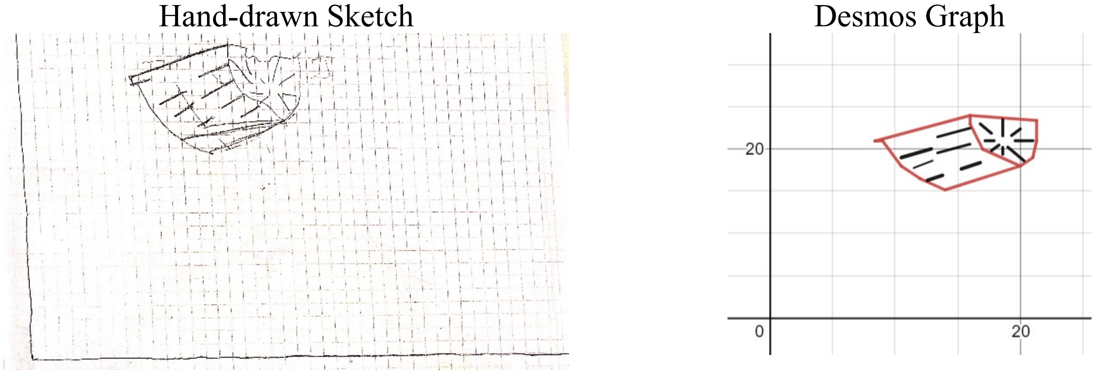


Omar. "Changing the colors of the lines in Desmos helped me keep track of them more easily." Figure 26 shows Omar's hand-drawn sketch compared to his Desmos-produced graph.

Figure 26

Omar's Side-by-Side Comparison

Hand-drawn Sketch Desmos Graph

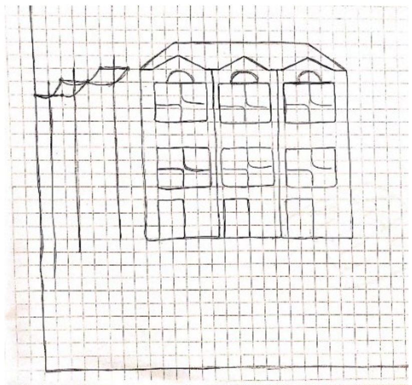


Shirley. "Why would we need to set it up one way only to rearrange it later for Desmos?" Figure 27 shows Shirley's hand-drawn sketch compared to her Desmos-produced graph.

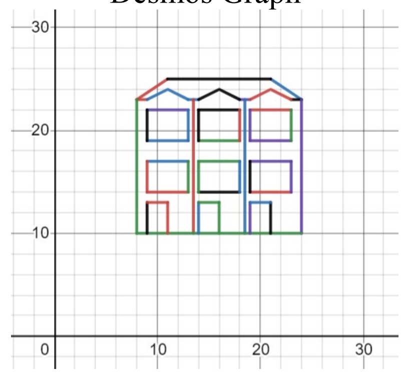
Figure 27

Shirley's Side-by-Side Comparison

Hand-drawn Sketch



Desmos Graph



Findings on Question Two

The critical events for this question lined up with the intentions of the lesson plan. Students participated in a deep sensory experience, sketched a scene found in nature and converted their drawing into mathematical equations, and finally inputted their mathematical equations into an online graphing program to recreate their sketch and their nature scene. These critical events encapsulated the operational curriculum (Uhrmacher et al., 2017). From each of the restoried experiences it was evident that each participant had a different experience while completing the project. Some emerged as math-nature connoisseurs and some budded into the experience by seeing the mathematical connections outside of the classroom, but still in man-made objects, not natural objects.

Dewey (1938) wrote, "I assume that amid all uncertainties there is one permanent frame of reference; namely, the organic connection between education and personal experience" (p. 25). The connection between education and personal experience was evident and prevalent during this nature-based mathematical experience—each student experienced unique educational growth through their personal experiences with this project.

Despite the fact that students had their own unique personal experiences, the community had some shared experiences as well. Each participant was able to make mathematical connections to nature and objects outside of the classroom, they were able to create piecewise-defined functions from those objects, and then they all experienced this “magical moment” when they saw their mathematical equations come to life using the online Desmos graphing utility. The combination of the shared community experiences in combination with each participant’s personal experiences provides the opportunity to consider the operational curriculum a success and aligned with the intended curriculum.

Question Three

Q3 How did the students receive and experience the curriculum?

The received curriculum is the learned information participants remember and take away from their participation in the experience. I was interested to know how the students felt about the whole experience, what they were taking away from the experience, and if there were any aspects of this experience that I should incorporate into future math lessons.

Data for this question came mostly from the self-assessment document (Appendix K) I asked each participant to complete. Students completed this self-assessment after their presentation. During their presentation, I took observation notes and completed a reflective journal entry to gather additional data to support their overall experience with this project. I also used the student’s deep sensory nature experience journal entry to complete each participant’s narrative for this section.

I also purposefully interviewed Blanche, Burt, and Cal to delve deeper into their told lives and to add some further depth and understanding to their experiences. I wanted to learn more about Blanche’s experience with the microenvironment, the small patch of grass, during

her deep sensory experience and her seemingly easy experience writing equations. I chose Burt to hear more about his experience choosing to go into the thick foliage and his experience struggling with writing the equations. I chose Cal to represent the seniors and hear about his experience writing his equations quickly, finishing all of his equations and his Desmos drawing first, before any of the other students.

I initially had selected Shirley as a possible participant to interview, instead of Blanche, as her experience was visibly different from the other participants. She declined to participate in the interview. I still was able to capture a narrative for Shirley through my observations, field notes, reflective journaling, and the collected documents she submitted.

The critical and like events centered on the following events: a) conducting math class outside and b) finding math in nature.

Conducting Math Class Outside

Planning to teach math outside is always a gamble. Late Spring in Texas can either be really nice with dry air and temperatures in the mid to low-70s, really hot and unpleasant with 100% humidity and temperatures up in the 90s, rain showers, anything in between, or a combination of all of the above. For this experience, we lucked out. Four out of the six days were pleasant with temperatures hovering around 75 °F, low humidity, and a cool breeze. Students felt relaxed and comfortable outside on these days. One day, the temperatures soared into the 90s, and we took shelter in the shade of the trees. The only day we were unable to be outside was on the sixth and final day—presentation day—due to the rainfall. Instead of visiting and standing in the location where the students completed their sketches for the presentations, the presenter needed to point out where they were standing through our classroom windows and the audience members had to visualize the presenter’s perspective. While I did get the sense that the

presentations lost some of their meaning because we were not able to actually visit and stand in the sketched locations during the presentations, I did not see any other effects on the results of this study.

Most readers associate the words “high school” with a massive building with the ability to handle thousands of students. SSM is a small, private, Montessori, Catholic school with a total of approximately 250 students, ages 18 months to 18 years old. The high school “wing” has one indoor classroom and one outdoor classroom for its students. There are also gym, theater, and soccer field spaces available for students and guides (Dr. Montessori preferred this term over “teacher”) to use throughout the day, as needed, but these areas are shared with other members of the school community and are not exclusive to the high school. This study utilized the outdoor classroom space and the shared soccer field.

Ari – Outdoor Math is Enjoyable. Ari discovered that “on the days when it was breezy and cool outside, the outdoor class made math class very enjoyable and [she] enjoyed doing the work;” however, “on the day when it was hotter, it was a bit more difficult to concentrate and harder to focus on the work.” Even with her mixed feelings on conducting math class outside, she was adamant that “we should incorporate this [being outside] into every math class.”

Blanche – Comparisons Make Sense. Enjoying the “break from the indoor classroom,” Blanche especially appreciated the opportunity to remove her facemask. SSM’s COVID-19 protocol required all students to wear a facemask or face covering at all times, while inside the building and therefore using the outside classroom provided a mask-break for the students. She also appreciated the opportunity to “find things that we were talking about in class to places outside of class, in the real world, um, and taking it from there.” Comparing her mathematical learning outside versus inside, she “felt like things suddenly made sense compared to how [she]

had been thinking about them before. Like, um, being able to find it [mathematical examples], not just in the places that human beings have created. It's also going to be in just places where things are natural outside."

Burt – Enjoys Working Outside. Prone to distractions both in an indoor and outdoor classroom, Burt felt the outdoor classroom "was a very relaxed environment which really helped calm [his] brain and allowed [him] to focus more on [his] work and get distracted less." Burt attributed the increased focus and improved productivity on the fact that he "just enjoys working outside." He does not know what it is about the outside, but he assumes "it's kind of common and just feels natural. It just kinda makes [his] thoughts flow better." Burt is prone to getting himself distracted with a sight or sound that may draw his attention, but he finds the distractions outside are just about the same as the distractions inside.

Cal – No More Whiteboards. Cal enjoyed moving math class outside, straying away from the "normal routine of math class." In Cal's mind, he views the conventional math class as "regimented" and "organized," learning and seeing the "different mathematical processes on the whiteboard." Instead of "staring at a whiteboard," he was able to "move around and be outside" during this nature-based project: "When you're outside, it feels more like you can relax a bit and stretch out." Even though he could relax and stretch more, Cal found the outdoor classroom to be "much louder than inside" and "the small bugs were a bit of a hindrance."

JoJo – The Sound of Nature and Peace. JoJo loves beings outside. She was able to block out the "sometimes distracting highway and car noises" to focus on the "noises of nature" that helped her focus on her work. The "stillness and peacefulness" she felt while outside only helped her productivity and her focus.

Meriam – Outside Math is Helpful and Fun. Meditating under the warm, relaxing sun brought Meriam a sense of calm and comfort, “aiding [her] understanding of math.” She was able to “pinpoint to the different examples” simply by using her “senses and looking around in nature.” Conducting math outside was both “fun and helpful” for Meriam because she could understand the math concepts and saw “how easy it is to find them [mathematical examples] in nature if I just take the time to look for them.”

Omar – Stretching His Legs. Except for the “one day that was really hot and humid,” Omar “really enjoyed being outside.” He found that it was “easier to focus because [he] didn’t feel cramped.” As the tallest participant, he sometimes feels sitting at a table and chair can be restricting and uncomfortable. Learning outside gave him the opportunity to, literally, stretch his legs, become more comfortable, and was able to focus on his work.

Shirley – Wet and Moist. Shirley wrote that being outside was “really fun and enjoyable,” but she also mentions she was “hot while outside” and “the sun was in my eyes and hindered my ability to focus.” She also mentioned, “everything was all wet and moist” because “everything was basically caked in a layer of dew.” She was hot and was not able to focus, she was quick to angry outbursts about not understanding what to do, “I just don’t get it!”, and was quickly feeling frustrated with any perceived additional work, the need to alter the equations to put them into the Desmos program. The heat of the sun was not as comfortable to her as it was to her classmates.

Finding Math in Nature

Throughout the project, the focus and purpose was to show the students how readily available math is in nature. Nature uses mathematics and mathematics uses nature. I wanted the

students to see this harmonious interaction for themselves. Bringing them outside was step one, encouraging the students to find this relationship on their own was step two.

Ari – Refreshing Her Brain. Participating in the deep sensory experience was the first time Ari began to notice that the mathematical functions were "coming to [her] very naturally." She was easily able to spot the "linear functions in the buildings, fences, and blades of grass, absolute value functions in the houses, parabolas in the trees, leaves, and windows, and some rational and cubic functions in the grass and trees." She was able to identify a variety of curves in nature and then was able to use her mathematical knowledge to write the equations and graph each type: "I enjoyed refreshing my brain on how to graph each of these different types of functions." The whole experience, working outside, identifying curves, and writing equations helped Ari see the "many applications of math [that] exist in daily life and in nature."

Blanche – Even in a Microenvironment. Blanche immediately started to see all sorts of curve shapes in nature. It became too overwhelming for her that she had to focus on a specific microenvironment, a small patch of grass: "I was able to see the general shapes of all the functions we have discussed in class in the area around me, both on a small scale and on a larger scale." In that one small patch of grass she could "find all of them [the different mathematical curves] because the grass is pointing in all the different directions and making the different shapes and things, and I just thought that was interesting."

Burt – Deeper Understanding. Burt started to appreciate piecewise-defined functions more through the completion of this experience. Nature does not use every equation in its entirety, "nature is made up of piecewise functions." Realizing that his tree trunk was part cubic and part linear, he saw the tree trunk as a whole and also as the parts formed together to make

that whole. Burt now has a "deeper understanding of piecewise functions" that he "would not have received or enjoyed as much if it were inside."

Cal – Not Just in the Classroom. Cal concluded, "math is something that's not just to be used in a classroom. Like you can go outside and see math everywhere instead of just, you know, the hour of math class every day." He also noted that nature became the data points for the creation of our graphs. He previously thought of graphs as having "one, maybe two lines that are representing, you know, like data or something, but for this you see X's and Y's [the ordered pairs that make up data points] everywhere, and you can use these for your graph in a creative way."

JoJo – Observe and Take it All In. JoJo noticed that in this fast-paced, need-for-immediate-gratification life most teenagers live these days, "we do not take the time to just be present in nature so we do not notice the little things." Having the opportunity to "sit down in nature and just observe and take everything in", helped JoJo "realize math is literally everywhere in nature. We don't associate math and nature, but this made [her] realize that mathematical functions are everywhere."

Meriam – The Presence of Many Functions. Being able to see and spot the "many different functions in nature" was an eye-opening experience for Meriam. She recognized "how present math is [in nature] and how easy it is to see the math in nature all around me." She never would have considered herself confident in mathematics; however, Meriam discovered that this process in nature "helped [her] be more confident with working with functions." She went as far as saying that she "even had fun with them [functions] during this project as I furthered my understanding of functions."

Omar – More Appreciation. Omar felt confident with his previous experiences understanding the relationship between math and nature; however, the hands-on application of taking nature and turning them into equations gave him "a better appreciation for the actual presence of math in nature." This experience helped him to "actually take note of [piecewise functions] and see them where [he] had previously not."

Shirley – Objects Have Math. Shirley took the journey out of her comfort zone and immersed herself in nature, sat on a dew-soaked bench in the heat. There was nature all around her, and she focused on the man-made artifacts. She saw the houses, the windows, and an electrical pole: "There were linear lines for the house, parabolas for the windows, absolute values for the roof, and maybe even a step function on the electrical pole, maybe?" While her focus was not on nature, she did manage to make some mathematical relationships to real-life objects not found in her classroom or in her textbook.

Findings on Question Three

This question's focus was on the received curriculum (Uhrmacher et al., 2017). Looking back at my intentions for this experience, I wanted students to make mathematical connections to nature and use piecewise-defined functions to mathematically describe a scene they found in nature. Each student received these intentions in different ways; however, the findings related to this question point towards every student making mathematical connections in nature, seeing a variety of mathematically-defined curves all around them. Conducting these first-hand experiences outside the conventional four-walled classroom provided the appropriate environment for students to make these connections in an environment that naturally has a reduced math anxiety level (Hanscom, 2016). The environment's natural reduction of math anxiety gave students the opportunity to relax in this nature-based mathematics classroom

(Taylor & Fraser, 2013) and make these mathematical connections, and, in some cases these connections were made subconsciously, as we'll see more clearly in the types of connections students made.

Question Four

Q4 What connections are students making to other concepts, mathematical or otherwise?

Another component in the received curriculum is the types of connections students make to the information they received during the learning process. For this study, I was specifically interested in understanding any mathematical connections students made to nature. I also was interested in any non-mathematical connections students made by completing a nature-based project.

Data for this question came from my observations, field notes, reflective journal, and the student's deep sensory experience journal entry and their self-assessment. I also used data collected from my interviews with Blanche, Burt, and Cal.

The critical and like event for this question centered on the event, making connections to nature.

Making Connections to Nature

Participants, for the most part, enjoyed their time outside, working in and with nature. They were able to successfully find and identify objects in nature that appeared to have characteristics of a variety of mathematical curves and see the relationship between mathematics and nature.

Walker et al. (2017) and Demarest (2015) express the importance of both knowledge and action in planning outdoor experiences. In an effort to extend the student's knowledge of the subject matter towards a plan of action, I asked the same question to each presenter at the

conclusion of their presentation: "Thinking about the scene you sketched, wrote equations for, and reproduced in Desmos, what would you add or change?" I left the question vague on purpose to see what each of them would come up with and their answers surprised me and ranged from engineering design to Hügelkultur, the use of decaying wood debris to start a raised plant bed.

Ari – Many Applications In Daily Life. Eventually not having to look very hard, Ari began to "spot the functions throughout." She "didn't have to force [herself] to look for them," because she easily was able to find the math-nature connections during her deep sensory experience. Spotting the functions in not only the man-made objects, but also the natural elements that made up the space. The project overall, "starting with identifying them [the curves in nature], drawing them, then identifying the curves of our drawings and writing equations, and finally plugging the equations into Desmos" provided Ari with "a very holistic understanding of functions in nature."

When asked what she would add or change to her scene, Ari commented on how the fence had no additional gate besides the main entrance. She wants to propose installing another gate to the school administrators, using this project as the basis for her reasoning: "Sitting in the space, I realized there was no way out. If I was at that end [the far end] of the field, I could get not quickly and easily make it to this gate on this end, and I don't think I would be able to scale the fence…well, maybe if the need arises."

Ari consistently finds the "many applications of how math exists in her daily life."

Blanche – Combating Deforestation. Blanche continued to compare the micro and macro environments surrounding her and described "the biggest connection [she] made was just seeing the general shapes of all the functions we discussed in the area around me, both on a small scale and on a larger scale," being able to "make connections between what we can see in nature

and how we can represent that using functions." Blanche never felt functions were something that she would "go and look for a lot outside. 'Oh! Let me find a parabola…' But when you're actually thinking about it, um, and you're, you're looking at things and trying to find them, it becomes a whole lot more easier and natural and now, um, I cannot *not* see them. You know?"

When asked what she would add or change to her scene, Blanche pulled from her environmental science studies and "remember[s] looking at trees and starting a little spiel in my head about trees and deforestation, and I realized we need more trees. Let's plant some trees." A majority of the trees had died due to the freezing temperatures we experience a few months before this study. In addition to us planting more trees, she would recommend to "replace the dead ones while we're at it."

Burt – Clean Up Our Trees. Already connecting the deep sensory experience to fishing using his olfactory senses, Burt began making connections almost immediately. He noticed that the trees, their branches, and their leaves "were all sorts of mathematical [sic] functions or peicewises [sic] of them." While not yet a huge fan of piecewise-defined functions at the beginning of the project, he was able to make the connection that this mathematical idea was present where he stood and he developed a "deeper understanding of piecewise functions."

Burt noticed that there was a lot of trash all in the trees and the foliage. Typically, the trash is blown in from the surrounding areas or is left on the field during soccer games and matches. In addition to the school using the field, the school rents out the soccer field to club soccer and lacrosse programs. So when asked what he would add or change to the environment, he quickly stated, "We need more trash cans. We should first, however, do a trash cleanup drive thingy. I don't remember what they are actually called, but the thing where we get a group of people to volunteer to pick up trash. And then we should install more trash cans."

Cal – Ships Maneuver Parabolically. After identifying the parabolic shapes of the tree canopies, Cal could not stop finding parabolas. He would come to class and casually say, "I thought of another one today, Mr. Tucker – the bows of ships. They are parabolas, right? I mean some are pointy at the tips, so they start off absolute [the shape an absolute value function makes], but then they curve more parabolically…is that even a word?" His enthusiasm to find additional examples was grounded in his belief that "math functions exist everywhere in nature." Elaborating on this idea, Cal spoke about how "there's a lot more math around you or you can use a lot more math for the things around you than you would normally think. Math is a much wider, or it can be applied to a much wider field than just statistics and deriving equations and, you know, advancing mathematics, things like that."

Cal did not appreciate the uniformity of the trees and spoke about this as the one thing that he would add or change. Thinking like a landscape architect, he suggested, "we definitely need more, smaller, trees or even bushes to add height and distance variation and complexity to what is already there."

JoJo – Math is Literally Everywhere. JoJo wants to understand the real life applications to learning and studying math. She admits that math is not one of her favorite subjects "because [she] feels like there is no practical application for it. Yet this activity felt very practical and I like when things we learn have real life applications." She realized that "math is literally everywhere in nature" and how she does not always "associate math and nature, but this [project] me realize they [mathematical curves] are everywhere and present in our day to day lives." JoJo also made the comment that this project reminded her to slow down and "spend more time outside observing things."

When asked what she would add or change to her scene, JoJo also suggested to plant more trees and to replace the dead trees; however, her reasoning was for privacy: “I don’t want someone in the apartments or the town houses to just see me through their windows. We definitely need more trees to block their views.”

Meriam – The Mesmerizing Patterns of Leaves. Initially Meriam was only able to identify the curves in the man-made objects: the paint on the field, the roofs of the buildings, and the soccer goals. The more involved the project became, she had the “chance to sit down in nature” and “connect math to the nature around me.” She began to see the similar curves she previously identified in the man-made objects in the natural elements, as well. The linear lines of the paint made were the “same lines forming the trunk of a tree”; the absolute value curves in the roofs could be seen in “the veining of leaves”; and the parabolic shapes of the soccer goal were the “branches of the trees reaching towards the light.”

Meriam definitely created a strong connection with nature during this project, specifically with trees and their leaves. She discussed her desire to add more trees so that she can “sit and stare up through the leaves and see the light. It is so mesmerizing. Have you ever noticed the patterns the light makes?”

Omar – Mathematical Models and Hügelkultur. Omar immediately started thinking about mathematical modeling situations while he was out in nature. While observing the bees buzz around him and his pile of logs, he thought about the way bees dance to communicate and wondered, “Maybe I could make a mathematical model for that?” He found another example while examining the multiple sticker burrs in the grass around him and designed an experiment “to see what conditions those horrible things [sticker burrs] prefer.” Not only was Omar

connecting math to his natural surroundings, he was using the natural surroundings to create testable and solvable math problems.

Omar proposed two suggestions for the SSM community to utilize the log pile during his presentation. His first suggestion involved us chopping up the wood "to sell as firewood or for, like, a wood-burning stove. People still have those, you know." His second suggestion utilized a horticultural technique, Hügelkultur: "We can use Hügelkultur to create a raised flower bed—they [the school's groundskeepers] did *that* in the prayer garden, and it looks awesome. The decaying wood is put underneath the grass and gives the plants nutrients."

Shirley – Math is Man-Made. Shirley appreciated the opportunity to draw and sketch in math class. In her journal entry, she did mention connecting the parabolic shape to some of the sticks she saw; however, most of her attention was to the man-made structures she should see. She was able to identify linear and absolute value curves and functions in those objects. Her original sketch even included parabolic and hyperbolic shapes. The act of sketching gave Shirley the opportunity to push her mathematical limit by including more complicated equations and curves, even if she ultimately did not end up finding the equations for those curves.

Shirley struggled during her presentation and when asked what she would add or change she did not initially have an answer. After some coaxing, she said, "electrical poles." These objects she excluded in her Desmos drawing were visible in her original sketch.

Findings on Question Four

Question four's purpose was to understand how these students, after completing this experience, use the knowledge they received and what connections do they make after the lesson was over. From the restoried narratives, the general consensus was that students began to see mathematical models everywhere—from the bows of ships to the dancing of bees. Students took

the experience of learning math outside and began to apply the math they were learning to other systems they observed.

I also wanted to see if students would begin to think about their own personal environments (Demarest, 2015) and of any actions they could take to improve the environment (Walker et al., 2017). Most of the students noticed the practicality of the environment: cleaning up the dead plants and trees, adding to the architecture of the landscape, or adding parts to the existing fence or electrical infrastructure. But one student took it a step further, he thought about a different way to utilize the materials in the environment to beautify, naturally, using the wood logs to create a Hügelkultur flower bed.

The variety of ways students received the curriculum and connected mathematical functions in nature to their own lives points to the different experiences each student-participant had throughout the experience. It also highlighted what each student-participant values in the environment.

Summary of Findings

From the restoried narratives, each student-participant had a separate and unique experience during this nature-based mathematical experience, drawing from their point of view during the deep sensory experience, what they chose to sketch, and the mathematical equations they used to define those sketches. However, all of the student-participants were able to make the connection that nature can be defined mathematically, observing the multitude of mathematical relationships and processes that exist in the real world and, hopefully, seeing the natural beauty of mathematics and the mathematics in the beauty of nature. This was most evident when the students initially inputted the mathematical functions into the online Desmos graphing calculator program and saw their nature-scene come to life. The act of typing in an equation and seeing that

part of nature drawn on the computer helped to concretize the idea that nature can be defined mathematically.

This nature-based mathematical experience was academically successful for the student-participant's. I intended for the students to make mathematical connections with nature, using a sequence of activities inspired by environmental (Demarest, 2015; Sobel, 2008) and mathematical (Boaler, 2016) curriculum designers. The integration of these ideas provided the foundation for the operational curriculum to proceed, as intended, resulting in a received curriculum that was ultimately beneficial to the student-participants.

As the teacher-participant, I appreciated the alignment of the intended, operational, and received curriculums (Uhrmacher et al., 2017). I was able to witness, research, and reflect on my curriculum design from start to finish. As a reflective practitioner (Connelly & Clandinin, 1988), I did note areas where I would alter the experience when I conduct a similar experience. The main thing I would change would be to have the student's choose nature-based objects to include in their sketches, as opposed to man-made objects (the townhouses, fences, and goal posts), providing the opportunity to make more mathematics connections to nature.

CHAPTER V

DISCUSSION AND CONCLUSION

In the first chapter, I introduced the study and discussed the pressures standardized testing puts on teachers and the benefits of teachers finding ways to incorporate authentic engagement in the math classroom, including using nature and outdoor learning environments in their curriculum design. Chapter 1 also includes an introduction to the instructional arc, the intended, operational, and received curriculum, framing the research questions for this study. Chapter 2 includes a review of the literature focusing on math anxiety and growth mathematical mindsets. Chapter 2 also includes a theoretical perspective on how an outdoor learning environment can help decrease math anxiety and increase growth mathematical mindsets. Chapter 3 describes my rationalization for using critical event narrative analysis and includes my data collection methods and design for this action research study. Chapter 4 organizes the participants' restoried narratives (Ollerenshaw & Creswell, 2002) by aligning the narratives with each research question and critical event, providing the reader an insight into each participants' experience (Connelly & Clandinin, 1988). In Chapter 5, I conclude this dissertation by presenting a full evaluation and summary of the narrative study results, discussing emergent themes and implications of this study, and providing recommendations for future research.

In examining the incorporation of nature or outdoor classroom environments in classrooms, research discusses environmental education (Walker et al., 2017), outdoor education (Eick, 2012), place-based education (Demarest, 2015) and transcendent nature experiences (Sobel, 2008). The majority of these studies are interested in pre-school or elementary-aged

children and classrooms. I have spent my entire career in math education at the secondary level, and I wanted to understand how nature research impacts secondary students. I developed this dissertation and the nature-based mathematical project studied in this dissertation to look at these impacts, specifically the experiences I, as the teacher-researcher, had in developing the experience, and the students in completing the nature-based mathematical project. The unique voices of the Montessori adolescents in this study and the corresponding critical events created a rich narrative of common experiences that had implications beyond just this one mathematical classroom. My goal is for math educators, practitioners, and curriculum designers to gain valuable insights from these implications and to find creative ways to incorporate outdoor learning environments in their lesson-planning.

I used the data I collected from documents, observations, a photo elicitation process, interviews, and my reflective journaling to restory the narratives presented in Chapter 4. As each narrative was restoried and written the following themes emerged: a) students were able to make mathematical connections to nature, b) students and the teacher-researcher relied on past experiences to complete this project, c) students experienced an increase in focus and productivity in the outdoor environment, and d) students stayed in their writing equation comfort zone. Each theme exemplifies how the experiences of the teacher and the students during this nature-based mathematical experience moved through the instructional arc: intended curriculum, operational curriculum, and received curriculum (Uhrmacher et al., 2017).

Discussion of Findings

In this discussion section, I plan to present the themes identified above and, using evidence from the participants' narratives and relevant literature, answer the research questions for this study. My hope is to take the reader back through the narratives, originally presented by

research question, using a thematic lens. The research questions that guided this study aligned with the instructional arc's three elements: intended curriculum, operational curriculum, and received curriculum (Figure 28), and they are:

Q1 What were the teacher-researcher's intentions when I designed this mathematical education experience?

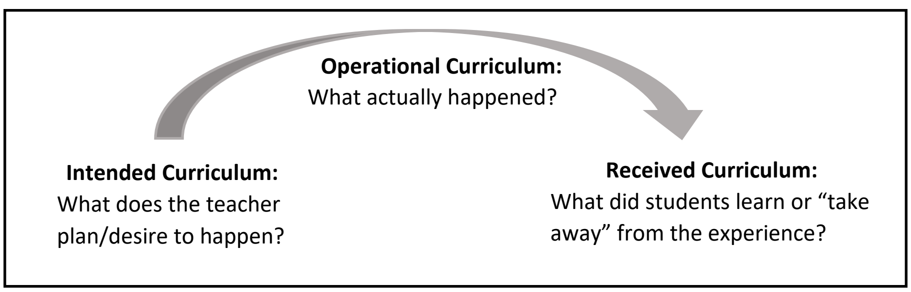
Q2 How did the curriculum come to life?

Q3 How did the students receive and experience the curriculum?

Q4 What connections are students making to other concepts, mathematical or otherwise?

Figure 28

Instructional Arc



Note. Recreated from Uhrmacher et al. (2017).

Students Were Able to Make Mathematical Connections to Nature

The theme of making mathematical connections to nature was evident from the moment of inception of this project—I wanted and intended for the students to make mathematical connections to nature and designed the project accordingly. Incorporating placed-based curriculum design (Demarest, 2015) and transcendent experiences in nature (Sobel, 2008), I intentionally made nature, natural elements, and the outdoor classroom central to this project. I also intended the students to complete a mathematical project while they were in this natural

environment. It was not surprising to see the participants making mathematical connections to nature; however, I was surprised by how quickly participants began to see the mathematical curves used to model natural elements because they were immersed in the natural setting and connected what they were seeing to the mathematical models they were familiar with.

Ari concluded that it was impossible for her not to see the mathematical connections all around her. Blanche became overwhelmed and had to focus her attention to only a single patch of grass and still saw mathematical connections all around her. Burt, diving head first into the thick foliage, immediately saw the branches of the trees forming mathematical curves, while Cal saw the uniformity in the trees spacing. JoJo saw math in the wind as its sinusoidal wave structure became visible in the waving flag. Omar took his mathematical connections even further, creating math-modelling problems of the dancing bees and the prickly sticker burrs.

Meriam and Shirley, however, initially struggled to find the mathematical connections in nature, but were able to spot the mathematical curves in man-made structures. Meriam was able to take her experience with man-made structures and apply those to objects found in nature, specifically the trees and its branches, while Shirley continued to look at just the man-made structures.

Connecting math to nature was my original intention coming into the project and my intention was to put students out in nature and use natural elements as their inspiration to form mathematical connections to functions. Other studies have used non-traditional and unconventional methods to have students understand the mathematical importance of functions: Carducci (2008) used Legos, Mamolo (2019) used digital interactive math comics (DIMaC), Rivas (2015) used elevators, and Radu (2013) used LEED-certified buildings. For this nature-based mathematical experience, I used the natural and, unintentionally, man-made elements in

our outdoor space, for example the trees, grass, and townhouses. I specifically wanted to understand how nature and an outdoor classroom environment influenced the students' ability to identify a variety of mathematical functions in nature and to define mathematically those natural elements using a variety of equations.

Similar to the other studies, the students were successful in developing a deeper understanding and appreciation for functions. Analyzing the received curriculum, students do understand that mathematical functions and curves exist in nature in piecewise-defined forms. Each participant participated in a deep sensory experience in nature and used their senses to see any mathematical connections to the nature they immersed themselves in. After the deep sensory experience, each student went back to the same spot and sketched a scene of the natural and man-made elements that made up the space. Students took their sketches and determined the equations of the curves that existed in their sketch and in nature, creating a piecewise-defined set of mathematical equations defining their sketch. Finally, the student-participants entered the equations they created into the online graphing calculator program, Desmos, to see how accurately the equations they created were to their sketch. As they completed this process, they witnessed not only nature's mathematical models coming to life, but they recognized they have the capacity to define natural elements mathematically: "Students learn by experiencing the real world: learning about nature in a natural environment; about society in the society; about the local environment in the local environment" (Cotič et al., 2020). Taking the students out of our four-walled classroom and immersing them in their natural outdoor environment at school to create a mathematical situation to model had profound impacts on the participants. Not only did they appreciate the natural world they immersed themselves in, they were able to see their slice of the world as mathematically-defined functions, providing evidence towards a Deweyan

education system where students experience meaningful activities they are invested in (Dewey, 1917). As the teacher-researcher, these findings suggest that the students would benefit from exploring other mathematical concepts immersed in nature and from me intentionally imbedding additional nature-based mathematical experiences into their future studies.

Students and the Teacher-Researcher Relied On Past Experiences To Complete This Project

Kolb defined experiential learning as “the process whereby knowledge is created through the transformation of an individual’s experience” (Kolb, 1984, p. 41). I observed in this study that knowledge creation includes not only the individual’s current experience, but also their past experiences, as well.

Ari’s previous work on circles on her conic sections project, helped her recreate a circle equation to represent the canopy of the tree she drew. Cal, JoJo, and Burt’s previous work on sinusoidal functions gave them the confidence to recreate sinusoidal models in this project. The project itself relied on the students’ previous understanding of writing equations of different curves. To help with student’s understanding of the different functions, I embedded opportunities for students to review identifying and writing all types of equations. During the student’s deep sensory experience students used their senses to identify different types of curves they saw and they used a frequency table to keep track of the different curves they noticed. Appendix I shows the frequency table students used to track their observations. I also provided a tip sheet (Appendix J) to remind students of the steps they need to write the different equations they might run across when they start mathematically defining their sketches. I also provided mini-lessons to the students. These were mostly one-on-one, but the occasional “extra set of ears” seemed to want to listen to the explanations. While students needed to rely on their prior knowledge and

understanding of the writing equations, there were plenty of opportunities to help students recall prior understanding to enable the "active building of new knowledge from experience and prior knowledge" (Low & Chew, 2020).

I also noticed my own past experiences influenced my curriculum design and my overall intentions for this project. Connelly and Clandinin (1988) describe how teachers use what they have experienced in the past to develop their curriculum and lesson plans. Lee and Zeppelin (2014) add that teachers reflect on "the way they were taught and the way they teach" (p. 334). Reflecting on Ms. Inman's initial use of nature to explain mathematical concepts to me in middle school, Mrs. Walter's writing equations in a graphing calculator project in high school, and Dr. McConnell's deep sensory experience in grad school, helped determine my goals for my curriculum designing process.

I used my experiences to help create a new experience for the students, hoping they found the experience valuable. Lisciandro et al. (2018) highlight the importance of using past experiences of ourselves and our students to enable a productive learning environment: "Understanding our learners not only includes understanding their academic disparities, but also their past experiences and perceptions towards subjects, in order to develop and teach curriculum" (p. 16).

Students Experienced an Increase in Focus and Productivity in the Outdoor Environment

Richard Louv is a leading researcher on studying nature's impact on children and how they learn about the world. Louv noted in a 2012 article that "time spent in more natural environments (whether it's a park, a wilderness, or a nature-based classroom or play space) stimulates the senses, improves the ability to learn, and helps students connect the dots of the

world" (p. 8). Even though this teacher-researcher knew the benefits of including natural environments into students' educational workplaces, I was still surprised to learn, in this nature-based mathematical environment, the high-school-aged student-participants in this study experienced an increase in focus and productivity by simply being outside.

Burt prefers to be outside, fishing for example, and he noticed an increase in productivity and focus as he determined the mathematical equations of his nature scene. JoJo found that nature sounds to be soothing and provided her with a way to focus on her work. Ari utilized some of the natural tactile stimulants she found to aide her focus; however, she did note that when the weather was too hot, her focus decreased. Omar appreciated the openness of the outside environment and the not-cramped feeling of being inside the building helped his focus.

Not having measured the student's math anxiety prior to this research study, I can only base my findings on my observations. Prior to conducting math class outside, students were tardy, not as engaged, rarely offered up their own ideas or answers in class, and mostly relied on the stronger math students to come up with a solution. When the math classroom moved outside, the students openly engaged in mathematical discourse with me and each other, attempted to answer their own questions before asking for assistance, and genuinely engaged with the mathematical learning more than when they were inside at a whiteboard. This indicated to me that the students' anxiety levels around mathematics decreased when the math class moved outside.

Cotič et al. (2020) and Otte et al. (2019) conducted quantitative research on the use of outdoor classrooms on science and math competencies, respectively. Both studies could not prove a statistical significance of an outdoor classroom having an impact on the competency of the students; however, understanding the students' experiences in these outdoor environments, as

in this qualitative study, provided some insight into a possible source of statistical significance. The increase in focus and productivity of the participants in this study provided the avenue for these student-participants to understand the mathematical competencies on a more meaningful level.

Simply bringing the classroom outside (Demarest, 2015; Otte et al., 2019) provided one way for students to feel more involved and engaged in a mathematics classroom. In addition to the physical and emotional health benefits nature provides students (Bond et al., 2007; Louv, 2008; Wirth & Rosenow, 2012), bringing the learning outside the classroom helped students focus and increase their productivity. Some students even mentioned experiencing lower levels of math anxiety as well. All of the students commented on how much they enjoyed the math classroom outside, on nice days, and requested that I bring them outside more often. I do intend to make more of an effort to move away from the whiteboard mathematics teaching to the experiential mathematics learning that the outside environment lends itself to nicely.

Students Stayed In Their Writing Equation Comfort Zone

Research on nature-based environments typically shows that participants are risk-takers (Hanscom, 2016; Wirth & Rosenow, 2012) and tend to move out of their comfort zones (Anderson, 1999; Burks, 2011; McLennan, 2017). Surprisingly, I observed most of the participants in this study did the exact opposite—they did not take mathematical risks and stayed in their mathematical comfort zones.

Omar chose to complete his project using only linear equations, Meriam and Shirley both only used linear and absolute value equations, Ari and Blanche incorporated quadratic equations in addition to the linear and absolute value equations, while JoJo, Burt, and Cal also used

sinusoidal curves. Linear, absolute value, and quadratic equations are equations students have been working with since eighth or ninth grade, and they are the most comfortable modeling real-life applications with these types of functions. I included sinusoidal functions inside the students' comfort zones because this group of student-participants completed a sinusoidal modeling project immediately preceding this experience.

Students were able to recognize additional curves in nature, such as cubic, exponential, square root, and rational; however, most students opted to omit those from their writing equations portion of this project or from their sketch entirely. Meriam, Blanche, Shirley, and Burt noted time as a factor in their choice to omit certain functions, adjusting their initial sketches to fit the equations they felt comfortable writing. Cal only had three functions (linear, quadratic, and sinusoidal) in his original sketch, by his own design. Omar identified and attempted to determine an exponential equation for one portion of his sketch, but became frustrated with the process and changed the sketch to a show a linear segment. Burt, the youngest participant, on the other hand, because he focused on the shape of a tree trunk and its branches, saw and modeled equations that are more challenging to determine: cubic, square root, and sinusoidal functions with a limited number of linear equations.

I think that students stayed in their comfort zone because the equations they were most familiar with provided each student with the path of least resistance. I expected students to have used their growth mathematical mindset to push themselves out of their comfort zone, but there was not enough evidence during this research. I think more research is needed in understanding how to push students to take mathematical risks and grow into their growth mathematical mindsets.

In my initial planning of this portion of the project, I intended for the nature space to provide a variety of curves for the students to write equations, and the space did. Four students' sketches (Ari, JoJo, Meriam, and Shirley) incorporated mostly man-made objects and not solely natural elements found in the space. Ari and JoJo had natural elements in their sketches in addition to their man-made objects, which allowed them to incorporate non-linear equations into their work.

Hsieh (2017) posits the classroom teacher needs "to take a more active role in pushing everyone to move beyond their comfort zone" (p. 298). In retrospect, instead of allowing the students to sketch whatever they see from their sitting-in-nature location, I should have encouraged the students to sketch natural elements, instead of man-made elements. Making this change to the assignment expectations may have increased the amount of participants who moved out of their comfort zone, attempting the more challenging mathematical models.

Implications

Action research studies conducted by teacher-researchers are "an exploratory process during which the teacher attempts to increase understanding about and improve teaching quality" (Şenyadin & Dikilitaş, 2019, p. 64). With that vane in mind, I structured this section as a way to answer my research questions, reflecting on my understanding of the interconnectedness of the intended, operational, and received curriculum (Uhrmacher et al., 2017) and on ways I can improve my teaching moving forward. My hope is that other teachers can use my reflections as inspiration for ideas on how to improve their teaching practice and find ways to infuse more nature into (or out of) their classrooms.

Question One

Q1 What were the teacher-researcher's intentions when I designed this mathematical education experience?

Reflecting on this question, I found myself using past experiences to draw inspirations for these projects. I had meaningful connections to certain lessons I had as a student that I wanted the students to also experience. These impactful experiences have, evidently, stuck with me, either consciously or subconsciously, and they provide the first nugget to creating an impactful lesson. I also noticed that it wasn't just one lesson that I wanted to emulate, it was multiple: the use of nature materials to demonstrate a mathematical concept, using a graphing calculator to draw a picture with a collection of equations, and a deep sensory experience to feel connected to my local environment. The infusing of these multiple experiences created this newly designed lesson, a heterogeneous mixture of ideas coalescing together to form something unique, and, hopefully, also memorable to the students.

Havu-Nuutinen et al. (2019) discuss the benefits of a collaborative team teaching model (CTTM) with science teachers. Pre-service and in-service teachers collectively discuss successful teaching strategies and practices, and, as a group, create new inquiry-based lessons for their students. I do not think it reasonable for one person to have a plethora of good educational experiences to draw from to create unique and memorable lessons for their students, so I think it would be helpful to use colleagues' experiences, as well. Drawing inspiration from their positive learning experiences, in addition to my own, opens the possibility to design a variety of engaging curriculum options.

I mentioned that the co-teaching model between Mrs. Darcy and I is not directly studied in this research study, but I want to mention, even though we are responsible for different subject areas, we discuss our curriculum design ideas with each other. We discuss possible setbacks and

ways to improve each other's lesson design; however, we could add to our discussions and improve our co-teaching model by drawing on each other's stories and experiences, infusing those inspirations into our ideas bank when initially planning out our lessons.

Overall, my intentions were to have the students understand that nature can be defined mathematically, and I wanted them to have a memorable experience, in nature, to concretize this understanding. I used my own prior experiences to design an experience in nature that involved the students in a series of activities, immersing the students in nature and in the mathematics that they found there.

Question Two

Q2 How did the curriculum come to life?

After determining my intentions and my inspirations for this experience, I was able to put together a lesson plan (Appendix L). I wanted the students to be able to conduct all aspects of the lesson outside and once I brought the students outside, they immediately knew that something different was going to occur. Sitting in the prepared outdoor classroom space, we took in our surroundings and were able to immediately see and make mathematical connections all around us. I wanted to keep this energy up each day, and, thankfully, we had great weather for the first five days. On the sixth day, the presentation day, it was raining, so students were not able to physically take us to their chosen locations, but they were able to indicate their location to the class by pointing to the spot through our classroom windows. It was exciting to see the students immersed in the work each day. Admittedly, their favorite days were the deep sensory experience day and when they started entering their equations into the Desmos online graphing calculator.

Overall, the curriculum came to life by being outside. I do not think we would have had the same experience if the students were not able to fully immerse themselves in their outdoor environment. I am already designing a variety of ways to incorporate more outdoor, nature-centered activities to help explain and teach mathematical concepts. This academic school year, I plan to create an activity where students take pictures in nature to use in their definitions of Geometric terms, providing a concrete and applicable scenario for these Geometric terms. For my Calculus students, I want students to use their local environment to describe naturally occurring rates of change and develop a nature-inspired related rate word problems. The school's outdoor classroom space, backyard, and soccer field area have served and continue to serve as excellent local environments for inspiration and for students to easily access, see, and make math-nature connections.

Students also came to the realization that they felt more productive and less anxious about learning math by simply conducting math class outside. Taylor and Fraser (2013) researched the statistical significance between positive learning environments and lowering math anxiety levels in students and this study showed that an outdoors learning environment was a positive learning environment for these students.

For this research study, the one main thing that I would do differently would be to have the students really focus on naturally occurring objects and not man-made objects. This experience had a tremendous amount of freedom and choice for the students, by design and within the scope of the Montessori Method (Montessori, 1912); however, I do think if I had set that expectation, more of the students would have steered away from the neighboring townhouses and apartments, and more on the foliage and the naturally occurring objects in our local environment. Students were still able to determine mathematical relationships in these man-

made structures, but I think the understanding that mathematical functions and curves occur naturally, without interference from humans, carries more weight. I would be interested in understanding how different the experience would have been for those students who did gravitate towards the man-made structures if they had focused on a natural element.

Question Three

Q3 How did the students receive and experience the curriculum?

The student-participants were active participants throughout the study. Each day, each student came to the class with positive attitudes and were excited about the opportunity to be outside, enjoy the weather, take mask breaks (a time when students were not required to wear face masks), and engage in mathematical discourse. Students felt the deep sensory experience to study mathematics was their favorite. I think this was because they thought they were not really involved in a math lesson; they were absorbing math from their environment.

On the other hand, students mainly grumbled when they had to create the equations to mathematically define their sketches. These two days of class were intense because students were having to recall and remember how to write equations of various types all at the same time. In one breath students were discussing quadratic equations and in the next they were talking through the components of a sinusoidal graph. As the mathematics educator observing this synthesis of mathematical ideas and skills, it was exciting to overhear the multiple conversations between students as they determined the type of equation they needed to use and how to execute it. I also think the struggle students experienced moving from one equation type to the other helped their overall understanding of the different equations. Brown et al. (2014) state that "learning is deeper and more durable when it's effortful. Learning that's easy is like writing in sand, here today and gone tomorrow" (p. 3). Students had to put in the effort to switch between functions, and I believe that will help them remember the different functions later in life.

One student in particular (Omar) had an “aha!” moment when he finally realized the connection between the slope-intercept form of linear equations and the point-slope form. As a second-semester senior, he was already stuck in his ways, but to see him make this connection that the point-slope form is and can be useful was a powerful moment. He was used to extending the line until it crossed the y-axis, identifying the y-intercept he needed to plug into the slope-intercept form. He ran across some lines on his sketch where he could not extend the line all the way to the y-axis and needed help. After working with him and reminding him about the point-slope form, he realized that he could have been saving himself time and effort by using the point-slope form, instead of feeling the need to extend his line every time. This is something that Omar will carry with him, having the experience and struggling a little with the solution was more impactful and memorable to him than me trying to convince him of the same thing without the practical application.

Question Four

Q4 What connections are students making to other concepts, mathematical or otherwise?

Barret-Zahn (2021) asserts that good teaching “starts with knowing and understanding your students as people with gifts, challenges, connections to the community, and life history” (p. 6), and I think that it also involves understanding that each student brings with them their own passions and interests. Evidence in this study points to students applying what they learned during this experience and connecting it to things that they are interested in. Cal was constantly thinking and talking about the different types of parabolic shapes he began noticing on the bows of ships. Omar was beginning to see mathematical models in some of the most mundane situations. Blanche connected the project to preventing deforestation. Students are going to use their experiences in school, whether it’s a math class or any other class, and apply those

experiences to what they are personally interested in. I think it is up to us, as educators, to help students begin to see those connections in a multitude of arenas, similar to Noddings' (2018) care theory.

When I repeat this experience with a new group of students in the future, I think it would be helpful to understand and incorporate more of their interests into the project. I also want to include ways for students to showcase their thoughts and connections to their passions and interests into their presentations—have students describe more with what they learned and took away from the experience. I think this would have made the experience even more impactful and memorable to the students.

Limitations

Limitations in this study surrounded the trustworthiness of participants and the transferability to other groups of students. Participants were the students and may not have been completely honest during interviews. The students are used to giving and providing feedback to Mrs. Darcy, my co-teacher in the high school classroom, and myself on all topics, academic, community, and personal; however, they may have answered my questions with comments they thought I would want to hear. In addition to creating an environment where each student felt comfortable and understood there was no right or wrong answer to the question, I used my field notes, observations, and other collected data to corroborate their told stories. I used the told stories to fill in any gaps from the restorying of the observed lived stories of the participants. However, my field observations of the students appeared to have aligned with their responses.

This study utilized a small, private, Montessori, Catholic school where teachers are granted a wide berth of autonomy to create curriculum in accordance with the state, Montessori, and archdiocesan guidelines. I had the flexibility and support of my administration to create this

nature-based math environment and experiment with little to no oversight. I am aware that not all schools have similar systems in place; therefore, this study may not be transferable to all educators. To support the transferability, I used rich descriptions of each participants' stories based on the critical events in the study, including how I conducted each aspect of the project. My hope is for math educators to have the possibility to use a similar approach with their students.

Recommendations for Practice and Future Research

Action research "is a form of investigation designed for use by teachers to attempt to solve problems and improve professional practices in their own classrooms" (Parsons & Brown, 2002). In this action research study, focusing on the experiences of one teacher and eight students in a nature-based mathematics environment, I investigated the use of an outdoor classroom on my mathematics teaching and the students' learning mathematics. As the study unfolded, evidence supporting the inclusion of nature-based mathematical opportunities in teaching practices surfaced. By moving the students outside of the classroom walls, students showed an increase in their focus and productivity levels. Students also enjoyed being outside, when the weather was nice, and that enjoyment spread to their study of math. I know that I will be working to find ways to embed more outdoor classroom activities into my future mathematics curriculum designs.

Students seemed to connect the most with the deep sensory experience, where they spent 15 minutes looking for mathematical curves in nature with all of their senses. I can have students participate in similar deep sensory experiences with different objectives. Some examples that come to mind include having the students looking for patterns or geometric shapes for

Geometry-based studies, variety of positive, negative, zero, and no slopes for Algebra-based studies, or rates of change scenarios for Calculus-based studies.

Otte et al. (2019) coined the phrase education outside the classroom (EOtC) and their research looked at all different environments outside the classroom where learning can occur, including outdoor classroom spaces. Researchers have shown the benefits of nature and outdoor education in students (Cotič et al., 2020; Greenwood, 2013; Hanscom, 2016; Larimore, 2018; Louv, 2012; McLennan, 2017; Son et al., 2017; Wirth & Rosenow, 2012). However, there is a need for more research in understanding how math-nature connections can enhance secondary mathematic teaching practices. My research study demonstrated one possible way to study the piecewise-defined curves students found in this nature setting.

Recommendations for Other Practitioners

I recognize that this nature-based mathematical experience and environment are unique to my own teaching and my own classroom; however, I can see implications and recommendations for other practitioners deriving from this work. Remembering Demarest (2015) encourages all educators to simply bring the classroom outside, I encourage all educational practitioners to utilize the outdoor spaces available to them. It could be a simple as the front entrance of their school or as involved as walking through the neighboring botanical gardens. Take your students outside and see what connections that you and your students can make to whichever topic you are studying. Allow the outside environment to be yours and your students' muse.

I also encourage other practitioners to integrate design principles to fit the needs of your intentions, objectives, and students. In this study, I took the principles of designing a local-environment educational investigation (Demarest, 2015) and transcendent nature experiences (Sobel, 2008) with the principles to design a mathematically engaging lesson (Boaler, 2016). In

the majority of cases, educators do not come across a one-size-fits-all lesson, idea, or design principle. An important area in advancing ones own practices is to mesh and merge multiple ideas to address the current needs and environment.

Summary and Conclusion

The last two lines of the poem I created after my first deep sensory experience come to my mind often: "the observer of wildlife/the witness to real life" (Tucker, 2019). When one takes the opportunity to observe any experience, scenario, or scene, one can witness so much. Dr. Montessori (1912) discusses the importance of observing the children in the classroom, and it reminds me of the power in simply observing. Observation is its own data collection tool—it must be powerful.

I started this dissertation with an idea of somehow incorporating nature and outdoor classrooms into my own teaching practices. I remember my research advisor and I at one point even discussed the possibility of figuring out how to talk to bees and understand how they talk to the classroom environment, providing a need for the members of the community to harvest the honey or check on the queen. The idea continued to grow, and as I observed my own teaching practices, I realized that this dissertation had to focus on mathematical concepts.

Once I knew my focus, I used the instructional arc to help define the questions I wanted to understand about my own experiences and the experiences of the students. The instructional arc provided the roadmap, starting with the intended curriculum; my own intentions for this nature-based mathematical project became the driver. Next, the operational curriculum focused more on observing the students and understanding their experiences, including how the curriculum came to life. The received curriculum completes the instructional arc and provides

the opportunity to observe what the students got out of the project and how they experienced the project, including the connections they made.

My intentions for this dissertation and for this project were similar; I wanted the students to be involved in nature. I combined my previous experiences in math and environmental education classes to create the project studied in this dissertation. These past experiences proved influential and meaningful to me, initially as a student, and now as a teacher-researcher, designing this project.

The curriculum came to life seemingly without cause for concern. The students enjoyed the fresh air, the time outside the confines of a classroom, and, thankfully, the weather cooperated (for the most part) during this experience. Students found the experience learning math outside enjoyable, while also noticing an increase in focus and productivity. Students were successful in identifying mathematical curves in nature and mathematically defining those curves, recreating their nature scenes using the online graphing program, Desmos. While this teacher-researcher was hoping for the students to push their equation writing out of their comfort zone, students were still able to observe the "mathematical magic" of using piecewise-defined curves to recreate nature.

Students consistently made mathematical connections with nature and some students even started making connections to other real-life objects. The critical event narrative analysis provided the understanding into each students' mathematical connections and sense of mathematical wonder as they completed this nature-based mathematical project.

Based on the results of this study, secondary mathematics teachers should take advantage of the natural spaces in, around, or near their classrooms to help students see the mathematical connections to real-life, encouraging mathematical growth mindsets, as Boaler (2016) states:

Equitable, growth-mindset teaching is harder than more traditional teaching in which teachers lecture and give short closed questions for practice. It involves teaching broad, open, multidimensional mathematics, teaching students to be responsible for each other, and communicating growth-mindset messages to students. This is also the most important and rewarding teaching that the mathematics teacher can do: teachers quickly feel fulfillment and energy from seeing engaged and high-achieving students (p. 140).

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APPENDIX A

PARTICIPANT RECRUITMENT SCRIPT

UNIVERSITY OF
NORTHERN COLORADO

RECRUITMENT SCRIPT
UNIVERSITY OF NORTHERN COLORADO

Researcher: Brian Tucker, Doctoral Candidate, School of Teacher Education
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Advisor: Christine McConnell, Ph.D., christine.mcconnell@unco.edu

I am inviting you all to participate in my doctoral dissertation research study. In this research study, I plan to observe the experiences of you, the students, in a nature-based mathematics class. I will analyze the effects of my role, as your teacher, on your experience in this setting. I will use the information obtained from this study to provide recommendations to teachers and mathematics curriculum developers in an effort to infuse more outdoor class time in mathematics classes.

If you agree to participate in this study, I will ask you to complete the nature-based mathematical experience as part of your mathematics coursework. If you choose not to participate in the research study, you will still complete the work associated with the research study as a part of your coursework, but I will not use your participation in the study. Your participation in this study has no bearing in how I will assess you on any work in relation to any products or presentations conducted as a part of this study and will have no effect on your overall grade.

To help document your experiences, I will take photographs of you engaging with the environment and with each other. I will only take pictures from the back of your head so that your face is not in the picture. In addition to photographs, I will also jot down my observations using field notes. After the conclusion of the experience, I will purposefully select three participants to participate in a one-on-one interview discussing their experiences in greater detail. If you are selected to participate in the interview, the interview will last approximately 60 minutes to complete. I will record the interview using a voice memo app on my phone.

The mathematical experience will last 3-5 class periods and corresponds with our current mathematical coursework. The guidelines and expectations for student involvement are similar to previous projects we have completed in this class.

I will keep all records of this study strictly confidential and keep them in a locked filing cabinet in a locked interior closet of our locked classroom. I will secure all electronic information in a password-protected file on a password-protected computer. I will not include any information in any report I may

publish that could possibly identify you. I encourage you to choose a pseudonym for yourself or you may request that I choose one for you.

I do not anticipate any risks greater than those experienced in daily life through your participation in this study. The cost of participating in this study is the time invested to participate in the interview and no compensation will be provided to you for participating in this study.

Participating in the study is up to you and is completely voluntary. If you change your mind later, you can stop participating at any time. If you decide to stop participating, I will not use the information you have already given us. All data collected from you will be destroyed.

If you are interested in participating in this study, please read through and sign this consent form if you are 18 or older. If you are under 18, you will first need to obtain parent consent from your parent or guardian, and then you can sign a student assent form. I need to receive all signed forms in order for your participation in the study, and I need to receive them before we start the experience.

APPENDIX B

CONSENT FORM FOR HUMAN PARTICIPANTS IN RESEARCH

UNIVERSITY OF
NORTHERN COLORADO

CONSENT FORM FOR HUMAN PARTICIPANTS IN RESEARCH
UNIVERSITY OF NORTHERN COLORADO

Project Title: Exploring Teacher and Student Experiences in a Nature-Based Mathematical Environment
Researcher: Brian Tucker, Doctoral Candidate, School of Teacher Education
Phone: E-mail: tuck0559@bears.unco.edu

Advisor: Christine McConnell, Ph.D., christine.mcconnell@unco.edu

Introduction
You are invited to participate in a research study. In this research study, I am observing the experiences of students in a nature-based mathematics class. I will analyze the effects of my role, as your teacher, on your experience in this setting. The information obtained from this study will be used to provide recommendations to teachers and mathematics curriculum developers in an effort to infuse more outdoor class time in mathematics classes.

What is Involved in the Study?
You are being asked to participate because I am interested in how you feel about math and learning math outside in nature. Please read this consent form and ask me any questions you have about the study. If you agree to participate in this study, I will ask you to complete the nature-based mathematical experience as part of your mathematics coursework. If you choose not to participate in the research study, you will still complete the work associated with the research study as a part of your coursework, but I will not use your participation in the study. Your participation in this study has no bearing in how I will assess you on any work in relation to any products or presentations conducted as a part of this study and will have no effect on your overall grade.

In addition to your participation in the experience, you may be one of three participants who will be purposefully selected to participate in a one-on-one interview discussing the experience in greater detail. The experience will last 3-5 class periods and corresponds with our current coursework. The guidelines and expectations for student involvement are similar to previous projects we have completed in this class. I will take photographs and keep field notes to record my observations of your participation in the experience. The photographs will document your engagement and involvement with the environment and each other. If you are in the photograph, the photograph will only have the back of your head and not your face. If you are selected to participate in the interview, the interview will last approximately 60 minutes to complete. I will record the interview using a voice memo app on my phone.

____
(Participant's Initials)

Confidentiality
The records of this study will be kept strictly confidential. Research records will be kept in a locked filing cabinet in a locked interior closet of a locked classroom, and all electronic information will be coded and secured using a password-protected file on a password-protected computer. I will not include any information in any report I may publish that could possibly identify you. You are welcome to choose a pseudonym for yourself or request that I choose one for you.

Risks and Discomfort
There are no anticipated risks greater than those experienced in daily life.

Payments and Compensation
The cost of participating in this study is the time invested to participate in the interview. No compensation will be provided to you for participating in this study.

Right to Refuse or Withdraw
Being in the study is up to you and is completely voluntary. If you change your mind later, you can stop participating at any time. If you decide to stop participating, I will not use the information you have already given us. All data collected from you will be destroyed, shredded, or erased from electronic sources. Your participation in this study has no bearing in how I will assess you on any work in relation to any products or presentations conducted as a part of this study and will have no effect on your overall grade.

Right to Ask Questions and Report Concerns
This study has been reviewed and approved by the University of Northern Colorado's Institutional Review Board (IRB). The IRB has determined that this study meets the ethical obligations required by federal law and University policies. If you have any concerns about your selection or treatment as a research participant, please contact Nicole Morse, Office of Research & Sponsored Programs, University of Northern Colorado, Greeley, CO; 970-351-1910 or nicole.morse@unco.edu.
If you sign this paper, it means that you have read this and that you want to be in the study. If you don't want to be in the study, don't sign this paper.

Consent of participant

- I have read, understood, and agree to participate
- I consent to being audio recorded
- I consent to being photographed

Name of Participant (print): ____

Signature of Participant: _______________________________ Date: ___________

Signature of Investigator(s): _____________________________ Date: ____________

APPENDIX C

MINOR ASSENT FORM FOR HUMAN PARTICIPANTS IN RESEARCH

ASSENT FORM FOR HUMAN PARTICIPANTS IN RESEARCH
UNIVERSITY OF NORTHERN COLORADO

Exploring Teacher and Student Experiences in a Nature-Based Mathematics Classroom

Students, I'm completing my doctoral program through the University of Northern Colorado. My dissertation project will research student mathematical learning experiences in nature. That means I will study the way students interact with nature during a math lesson. I would like to observe and interview high school (10^{th} – 12^{th} grade) students learning math in nature. If you want, you can be one of the students I observe and interview.

If you want to participate, I'll observe you during the duration of a mathematics lesson in nature and ask your experiences. I will also ask you to think about specific moments during the lesson that were memorable or important to you and also about your mathematical understanding. For each question I will want you to explain and elaborate in your answer, providing any specific information that you feel is important for me to know about. Participation in this process isn't an assessment of any kind and will not impact your overall assessment. There are no right or wrong answers and there won't be any score or grade for your answers. I will record what you say using my phone, but I won't write your name. I will upload the audio recording to a password-protected file folder and delete the file from my password-protected phone. The interview will take about 60 minutes for you to answer my questions about this mathematics lesson in nature. We will work together to determine the best time to talk so that you don't miss out on any other classwork.

Talking with me probably won't help you or hurt you. Your parents have said it's okay for you to talk with me, but you don't have to. It's up to you. Also, if you say "yes" but then change your mind, you can stop any time you want to. Do you have any questions for me about my research? If you want to be in my research and talk with me about health and eating, sign your name below and write today's date next to it. Thanks!

Student ______________________ Date __________

Researcher ______________________ Date __________

APPENDIX D

PARENTAL CONSENT FORM FOR HUMAN PARTICIPATION IN RESEARCH

UNIVERSITY OF NORTHERN COLORADO

PARENTAL CONSENT FORM FOR HUMAN PARTICIPANTS IN RESEARCH
UNIVERSITY OF NORTHERN COLORADO

Researcher: Brian Tucker, Doctoral Candidate, School of Teacher Education
Phone: E-mail: tuck0559@bears.unco.edu

Advisor: Christine McConnell, Ph.D., christine.mcconnell@unco.edu

Introduction
Your child is invited to participate in a research study. In this research study, I am observing the experiences of students in a nature-based mathematics class. I will analyze the effects of my role, as your child's teacher, on their experience in this setting. The information obtained from this study will be used to provide recommendations to teachers and mathematics curriculum developers in an effort to infuse more outdoor class time in mathematics classes.

What is Involved in the Study?
Your child is being asked to participate because I am interested in how they feel about math and learning math outside in nature. Please read this form and ask me any questions you have about the study. If you grant permission and if your child indicates their willingness to participate in this study, I will ask your child to complete the nature-based mathematical experience as part of their mathematics coursework. If you choose not to have your child participate in the research study, they will still complete the work associated with the research study as a part of their coursework, but I will not use their participation in the study. Their participation in this study has no bearing in how I will assess any of their work in relation to any products or presentations they create as a part of this study and will have no effect on their overall grade.

In addition to their participation in the experience, your child may be one of three participants who will be purposefully selected to participate in a one-on-one interview discussing their experience in greater detail. The mathematical experience will last 3-5 class periods and corresponds with our current mathematical coursework. The guidelines and expectations for student involvement are similar to previous projects we have completed in this class. I will take photographs and keep field notes to record my observations of your child's participation in the experience. The photographs will document their engagement and involvement with the

____
(Parent's Initials)

environment and each other. If your child is in the photograph, the photograph will only have the back of your head and not their face. If your child is selected to participate in the interview, the interview will last approximately 60 minutes. I will record the interview using a voice memo app on my phone.

Confidentiality

The records of this study will be kept strictly confidential. Research records will be kept in a locked filing cabinet in a locked interior closet of a locked classroom, and all electronic information will be coded and secured using a password-protected file on a password-protected computer. I will not include any information in any report I may publish that could possibly identify your child. Your child is welcome to choose a pseudonym for them self or request that I choose one for them.

Risks and Discomfort

There are no anticipated risks greater than those experienced in daily life.

Payments and Compensation

The cost of participating in this study is the time invested to participate in the interview. No compensation will be provided to you or your child for participating in this study.

Right to Refuse or Withdraw

Being in the study is up to you, and your child, and is completely voluntary. If you or your child change your mind later, you can have your child stop participating at any time. If you or your child decide to stop participating, I will not use the information your child has already given us. All data collected from your child will be destroyed, shredded, or erased from electronic sources. Your child's participation in this study has no bearing in how I will assess any of their work in relation to any products or presentations they create as a part of this study and will have no effect on their overall grade.

Right to Ask Questions and Report Concerns

This study has been reviewed and approved by the University of Northern Colorado's Institutional Review Board (IRB). The IRB has determined that this study meets the ethical obligations required by federal law and University policies. If you have any concerns about your selection or treatment as a research participant, please contact Nicole Morse, Office of Research & Sponsored Programs, University of Northern Colorado, Greeley, CO; 970-351-1910 or nicole.morse@unco.edu.

If you sign this paper, it means that you have read this and that you grant permission for your child to participate in the study. If you don't want your child to be in the study, don't sign this paper.

____
(Parent's Initials)

Parental Consent

- I have read, understood, and agree to have my child participate
- I consent to having my child audio recorded
- I consent to having my child photographed

Name of Child (print): ____

Signature of Parent/Guardian: _______________________________ Date: ___________

Signature of Investigator(s): _____________________________ Date: ____________

APPENDIX E

RESEARCH SITE PERMISSION LETTER FOR THE UNIVERSITY OF NORTHERN COLORADO

School Name
School Address
School City, State Zip

Date

Dear University of Northern Colorado Institutional Review Board,

The purpose of this letter is to inform you that I give Brian Tucker permission to conduct the research entitled, "*Exploring Teacher and Student Experiences in a Nature-based Mathematical Environment"* on the campus of SCHOOL NAME. This also serves as assurance that this school complies with requirements of the Family Educational Rights and Privacy Act (FERPA) and the Protection of Pupil Rights Amendment (PPRA) (see back for specific requirements) and will ensure that these requirements are followed in the conduct of this research.

Sincerely,

PRINCIPAL NAME
PRINCIPAL TITLE

- The right of a parent of a student to inspect, upon the request of the parent, a survey created by a third party before the survey is administered or distributed by a school to a student. Any applicable procedures for granting a request by a parent for reasonable access to such survey within a reasonable period of time after the request is received.
- Arrangements to protect student privacy that are provided by the agency in the event of the administration or distribution of a survey to a student containing one or more of the following items (including the right of a parent of a student to inspect, upon the request of the parent, any survey containing one or more of such items): Political affiliations or beliefs of the student or the student's parent. Mental or psychological problems of the student or the student's family. Sex behavior or attitudes. Illegal, anti-social, self-incriminating, or demeaning behavior. Critical appraisals of other individuals with whom respondents have close family relationships. Legally recognized privileged or analogous relationships, such as those of lawyers, physicians, and ministers. Religious practices, affiliations, or beliefs of the student or the student's parent. Income (other than that required by law to determine eligibility for participation in a program or for receiving financial assistance under such program).
- The right of a parent of a student to inspect, upon the request of the parent, any instructional material used as part of the educational curriculum for the student. Any applicable procedures for granting a request by a parent for reasonable access to instructional material received.
- The administration of physical examinations or screenings that the school or agency may administer to a student.
- The collection, disclosure, or use of personal information collected from students for the purpose of marketing or for selling that information (or otherwise providing that information to others for that purpose), including arrangements to protect student privacy that are provided by the agency in the event of such collection, disclosure, or use.
- The right of a parent of a student to inspect, upon the request of the parent, any instrument used in the collection of personal information before the instrument is administered or distributed to a student. Any applicable procedures for granting a request by a parent for reasonable access to such instrument within a reasonable period of time after the request is received.

APPENDIX F

INSTITUTIONAL REVIEW BOARD APPROVAL

UNIVERSITY OF
NORTHERN COLORADO

Institutional Review Board

Date: 04/02/2021

Principal Investigator: Brian Tucker

Committee Action: **IRB EXEMPT DETERMINATION – New Protocol**
Action Date: 04/02/2021
Protocol Number: 2101019246
Protocol Title: Exploring Teacher and Student Experiences in a Nature-Based MathematicalClassroom

Expiration Date:

The University of Northern Colorado Institutional Review Board has reviewed your protocol anddetermined your project to be exempt under 45 CFR 46.104(d)(701) for research involving

Category 1 (2018): RESEARCH CONDUCTED IN EDUCATIONAL SETTINGS. Research, conducted in established or commonly accepted educational settings, that specifically involves normal educational
practices that are not likely to adversely impact students' opportunity to learn required educational contentor the assessment of educators who provide instruction. This includes most research on regular and special education instructional strategies, and research on the effectiveness of or the comparison among instructional techniques, curricula, or classroom management methods.

You may begin conducting your research as outlined in your protocol. Your study does not require furtherreview from the IRB, unless changes need to be made to your approved protocol.

As the Principal Investigator (PI), you are still responsible for contacting the UNC IRB office if andwhen:

- You wish to deviate from the described protocol and would like to formally submit a modification request. Prior IRB approval must be obtained before any changes can be implemented (except to eliminate an immediate hazard to research participants).
- You make changes to the research personnel working on this study (add or drop research staff on this protocol).

UNIVERSITY OF NORTHERN COLORADO

Institutional Review Board

- At the end of the study or before you leave The University of Northern Colorado and are no longer a student or employee, to request your protocol be closed. *You cannot continue to reference UNC on any documents (including the informed consent form) or conduct the study under the auspices of UNCif you are no longer a student/employee of this university.
- You have received or have been made aware of any complaints, problems, or adverse events that arerelated or possibly related to participation in the research.

If you have any questions, please contact the Research Compliance Manager, Nicole Morse, at 970-351-1910 or via e-mail at nicole.morse@unco.edu. Additional information concerning the requirements for the protection of human subjects may be found at the Office of Human Research Protection website - http://hhs.gov/ohrp/ and https://www.unco.edu/research/research-integrity-and-compliance/institutional-review-board/.

Sincerely,

Nicole Morse
Research Compliance Manager

University of Northern Colorado: FWA00000784

APPENDIX G

UNSTRUCTURED INTERVIEW QUESTIONS

I plan the interview to be unstructured, using an initial question, content questions, probe questions, and a final question (McMillan, 2016). The unstructured, open-ended interview "invites the research participant to engage in storytelling" (Mertova & Webster, 2020, p. 70) and "to capture the thoughts and feelings of participants in their own language, using words, phrases, and meanings that reflect their perspectives" (McMillan, 2016, p. 344). These interview responses will provide valuable insight into how the curriculum came to life, the experienced or received curriculum, and the connections students made.

Initial Question: Reflecting on the experience, how would you describe the purpose of the experience?

Content Questions: Tell me about a time during the experience when you felt you were: "in the flow?" confused or puzzled? enlightened?

Probe Question: These questions will come up during the interview process, but may include:

- Tell me more.
- How do you know?

Final Question: How will you use this experience to shape your mathematical understanding?

APPENDIX H

SAMPLE GUIDE ASSESSMENT

(I have removed all identifying information from this sample and used a pseudonym)

	4	3	2	1-INC[1]	0-NC[2]
Knowledge and Skill Development (x2)	The student consistently demonstrates mastery of key knowledge and outstanding development of key skills.	The student demonstrates understanding of key knowledge and shows proficiency in key skill development.	The student demonstrates acceptable/satisfactory knowledge of the subject area, and shows an emerging development of key skills.	The student demonstrates less than satisfactory knowledge of the subject area and key skills.	The student demonstrates little or no initiative in acquiring and/or developing necessary skills.
Quality of Work (x2)	The student consistently prepares excellent, creative, intellectually mature, thoroughly prepared work.	The student prepares proficient work.	The student has met the course requirements by preparing acceptable/satisfactory work.	The student's work does not meet the course requirements.	The student rarely or never attempts to produce adequate work.
Engagement (x1.5)	The student is highly engaged in coursework and frequently contributes to class discussions in a thoughtful and respectful way.	The student is engaged in coursework and often contributes to class discussions.	The student is somewhat engaged in coursework and sometimes contributes to class discussions.	The student is rarely engaged in coursework and infrequently contributes to class discussions.	The student does not engage in coursework and does not contribute to class discussions.
Preparedness (x1.5)	The student is reliable, prepared, and demonstrates exemplary organization and skillful application of materials to aid his or her learning.	The student is reliable, prepared, and demonstrates some organizational strategies and uses some materials appropriately to aid his or her learning.	The student is emerging as a reliable, organized, and adequately prepared learner.	The student lacks focus, is unreliable, disorganized, and/or generally unprepared.	The student refuses to focus, stay organized, and is consistently unprepared.
Pace (x1)	. The student fully completes work within expected time frames and by the expected deadlines.	The student fully completes work within expected time frames and by the expected deadlines, most of the time.	The student is inconsistent about completing work within expectations. May have difficulty managing time expectations.	The student lacks awareness of due dates and/or works at a pace that leads to minimal advancement.	The student does not complete or turn in work.
					[1]Incomplete [2]No Credit

Range	32	28-31	25-27	24	21-23	17-20	16	15	14	0-13
Grade	A+	A	A-	B+	B	B-	C+	C	C-	INC
GPA	4.33	4.00	3.67	3.33	3.00	2.67	2.33	2.00	1.67	0.00

	Knowledge & Skills (x2)	Quality of Work (x2)	Engagement (x1.5)	Preparedness (x1.5)	Pace (x1)	Final Grade
Course: Current Events	4	4	3.5	4	4	**31.25 A**
Comments: Jeremiah, you've hit your stride with Current Events this semester both in terms of your presentations and participation in class discussions. Your presentation on Chile was one of your strongest in recent memory because you provided ample historical context and current detail in an organized way. I also liked that you thought about the way the topic was addressed across media and rehearsed what you wanted to say. Let that be your baseline for this year!						
Course: Integrated Humanities	2.5	2.5	3	3.5	4	**23.75 B**
Comments: Jeremiah, you gave a very successful presentation about the Iroquois and the Texas-Indian wars. You're quite comfortable presenting research verbally to an audience and preparing a slideshow. That said, I think your research paper was less successful than I would have expected of you as a senior. My overall impression of the final draft was that it was rushed. I found myself a bit disoriented about how your paper would be organized until several paragraphs into the body, and I was, frankly, distracted by the errors. You need to develop the habit of polishing all written work regardless of the class. It's also important to ensure you're properly citing work in your Works Cited and parentheticals. If a reader can't tell where you've sourced something, you haven't given proper credit and the reader can't go back to the original source. When you move on from here, some professors may decline to read work that hasn't received more scrutiny from you. Because you were done pretty early, I offered to read your draft and give you some feedback. I know you've been busy with college applications and test prep, but next time, it might be a good idea to take me up on that, or at least have a very honest friend do the same.						
Course: Integrated Mathematics	3	3.5	3.5	4	4	**28.25 A**
Comments: Jeremiah, I can tell how much your math skills have improved from year to year, and this year is no exception. You are working hard and are finding interesting connections to mathematical topics. Your conics project on parabolas in architecture was fascinating. I appreciated the connection you made between parabolic-shaped roofs and absolute value-shaped roofs. I can tell that you have been working hard on your follow up work and that you are not afraid to ask questions to further your understanding of the topics discussed in class. I encourage you to continue working hard and asking questions. The math assessment demonstrated your mastery of most of the topics covered. I noted some errors in piecewise-defined functions, but I see that area of knowledge improving, too.						

Course: Integrated Science	3.5	3.5	3.5	4	4	**29.25 A**
Comments: Jeremiah, I have enjoyed seeing your work in Integrated Science this year. Starting with your Telescope presentation to your planet travel brochure to your biome hat and speed dating presentation. All these projects and presentations showcased your hard work and effort. They were informative, thorough, and clear. I encourage you to work on the creative aspects of your presentations to bring these projects and presentations to life. In addition, your lab reports have shown how you make connections between the scientific principles and the real-world. I encourage you to continue identifying and seeing these connections.						
Course: Literature Seminar	3	3	3.5	4	4	**27.25 A-**
Comments: Jeremiah, I've always thought you made a good seminarian, and this semester is no different. You listen attentively to the conversation, aren't afraid to express a dissenting viewpoint, and aren't intimidated when a peer (or guide) pushes or questions you. Although I doubt you'd describe yourself as a fiction lover, you're open to a wide range of styles and perspectives and are willing to take what you can from each reading experience. The one place you could push a little harder is your written responses. You definitely have the weekly response requirements down pat, but they could be tidied up, and you could challenge yourself to think even more deeply in your analysis.						
Course: Theology	3.5	4.0	3.75	4.0	3.75	**30.375 A**
Comments: Jeremiah, a couple of strengths show forward when I think of your work: your intellectual discretion and your excellent and conscious listening skills. Now, what do I mean by "intellectual discretion"? Well, you are in a class that appears to have a mildly shared political and social vision (I am not fully positive here, but at this point my guess is that I am correct). When the expression of this viewpoint bodies forth, you listen and respectfully attend. You do not appear to be in a hurry to either oppose or qualify your peer's (or peers') expression, though I suspect you do not always agree -- at least not fully. Such a step-back point of view is rare generally, but more so in one your age. You do not reject or clam up, two typical response patterns. Rather, if necessary, you respond without being pointed in the rejoinder. Your audience, rightly, I believe, feels that you both like her (them) and respect her (their) opinions. Conscious listening is a skill a large proportion of human beings -- in my experience -- never learn. A real conversation requires that listening and understanding take place before a response can be properly given. You listen, even with the sense that your own point of view may be incomplete or in need of information and insight that the other may be able to provide. Again, the gift is a rare and valuable one in my experience. I make these deductions based on your in-class performance and your writing. Your Letter to God was simple, clear, and heartfelt, an expression of thanksgiving. The Covid Monastery essay nicely set the pattern of feeling a bit distanced while still expressing gratitude for your life (As I have said before I like the Firefighter segment). Perhaps, though, the bad neighbor essay delighted						

me most, though I will not claim objectivity on the issue of thoughtfulness in public discourse, since I think the two of us are closely aligned in this regard. Overall, I am highly gratified by your work. I see true growth in your self expression since last year. I think the colleges and universities you applied to would be very fortunate to include you in the in-coming class of 2021. Please let me know if I can help.						
Course: Web Design	4	4	4	4	4	**32 A+**
Comments: Jeremiah, your ePortfolio was simple and straight-forward. I really felt the simplicity of it worked well with what you were going for. Your theme also worked well. I would encourage you to round out your selection of artifacts to include some math/science ones, but that just comes from me also being your math/science guide. I am excited that you have chosen to stick with Web Design this year and am excited to see where this course will take you.						
Course: Writer's Workshop	3	3	3	3	4	**25 A-**
Comments: Jeremiah, the bar for the college essay is just shy of perfection. The point is to show a sample of your most polished, most efficient writing that also helps a group of strangers understand something essential about you. In writing your essay, I think you've done some of the best written reflection I've seen from you. It's taken you a long time to articulate what it is about the business industry that has been so appealing to you for so long, and I think you developed a specific, satisfying and mature answer. The conclusion to your essay has been challenging, and I wonder whether your will to be done has outweighed the desire to rework the ending. Outside the college essay, your writing has seemed more rushed and a little sloppy at times. I want to emphasize the importance of presentation in your writing. When communicating ideas in emails, letters, or other professional communication, it's important to proofread and consider audience before sending.						

APPENDIX I

CURVE IDENTIFICATION FREQUENCY TABLE

Type of Curve	Frequency	Description
Linear		
Quadratic		
Cubic		
Square Root		
Absolute Value		
Rational/Hyperbola		
Logarithmic		
Exponential		
Step		
Sinusoidal		

APPENDIX J

WRITING EQUATIONS TIP SHEET

Type of Curve	What you Need	The Process to Write Equation
Linear	Slope (m) Point	$y - y_1 = m(x - x_1)$; Find slope (with two points or count) and plug both into the equation
Quadratic	Vertex (*h, k)* Point	$y = a(x - h)^2 + k$; Plug-in vertex and point to find a; then use vertex and a to create your equation
Cubic	Inflection Point (*h, k*) Point	$y = a(x - h)^3 + k$; Plug-in inflection point and point to find a; then use the inflection point and a to create your equation
Square Root	Starting Point (*h, k*) Point	$y = a\sqrt{x - h} + k$; Plug-in starting point and point to find a; then use starting point and a to create your equation
Absolute Value	Vertex (*h, k*) Point	$y = a\|x - h\| + k$; Plug-in vertex and point to find a; then use vertex and a to create your equation
Rational	Point 1 Point 2	$y = \frac{a}{x} + b$; Plug point 1 in to create equation 1; Plug point 2 in to create equation 2; Solve system of equations for a and b; then use a and b to create your equation
Exponential	Point 1 Point 2	$y = a * b^x$; Plug point 1 in to create equation 1; Plug point 2 in to create equation 2; Solve system of equations for a and b; then use a and b to create your equation
Step	Length of bar (b) Vertical dist b/w bars (a) Shifts (h & k)	$y = a * [\![\frac{1}{b}(x - h)]\!] + k$; Plug-in known information. See me for additional help.
Sinusoidal	Maximum Minimum Midline (k) Dist b/w the mid & max (a) Starting x-value (h) Period; b = 2pi/period	$y = a * \cos\big(b(x - h)\big) + k$; Plug-in known information Use cosine, if the starting point is at the maximum Use –cosine, if the starting point is at the minimum Use sine, if the starting point is at the midpoint

APPENDIX K

MATH IN NATURE SELF-ASSESSMENT

Topic	Comments
Outdoor Classroom In what ways did the outdoor classroom help or hinder your learning experience?	
Sensory Experience While you were participating in the sensory experience, describe any math-nature connections you made/saw/created?	
Nature Sketching While you were developing equations from your sketch, what math skills… ...did you recall from previous lessons? ...did you need a refresher/mini-lesson on? ...were new skills that you had never seen before? Which curves or functions did you utilize in your sketch?	
Technology-aided Sketching When you started writing your equations into the Desmos online graphing calculator, describe any revelations or "aha" moments as you saw Desmos recreating your sketch.	
Presentation When you presented your work, describe your comfort level describing the different types of curves and functions you used in your sketch?	
Summary Overall, how do you think this experience helped or hindered your understanding of equations, piecewise-defined functions, and curves found in nature? What, if anything, are you taking away from this experience? Describe something, if anything, from this experience that you can apply to other areas of mathematics, any other field, or your life? Is there anything else you want to share about your experience completing this project?	

APPENDIX L

MATH IN NATURE LESSON PLAN

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Math in Nature, Day 1

Engage:

- Bring out the nautilus shell and pine cone. Show the patterns that exist. Connect to sequences and series unit we just finished.
- Connect other prior knowledge -- function project; sinusoidal project -- what functions have we studied this year (and their shapes)? Review shapes and functions, if necessary.

Explore:

- Label a piece of graph paper with x- and y- axes.
- Find a fallen leaf and bring it back.
- Trace the leaf on your graph paper.
- What do you notice?
- Identify any parts of your leaf that are linear.
- Determine the equations of the lines that make up your leaf's outline.
- Identify the restricted domain (review "restricted domain", if necessary)
- Working with a partner, create a piece-wise defined curve to represent your leaf's outline.
- Show off piece-wise defined curves to define your leaf outlines to your other classmates.

Math in Nature, Day 2

Engage:

- Discussion Question: Take a look behind us, what mathematical shapes do you see?
- Discuss with your neighbor. Discuss with the class.
- Reintroduce appropriate terms and vocabulary.
- Review the different types of functions one might encounter. Have students provide examples that they have seen before.

Extend:

- Explain the Deep Sensory Math Experience: "I want you to find a spot in our school's backyard and have a deep sensory experience with your location. Sit with that space and silently observe the space for 15 minutes. Use only your senses and be engaged with your space. What do you see, hear, feel, taste, and smell? Do you see, feel, taste, hear, smell any math connections?"
- At the conclusion of the 15-minute deep sensory experience, students should spend some time (approx. 10 - 15 minutes) writing down their experiences, thoughts, and observations as a journal entry. Their journal can be in prose, poetry, bullet points, drawings, sketches, or any other way you want to express your experience.

Math in Nature, Day 3

Extend:

- Students return to the spot they went to last time or, if they choose to, find a new spot. This time, take some graph paper, draw a coordinate plane on it, and have them sketch what they see.
- "When you sketch on graph paper, it helps your ability to write equations later, if you can identify clear coordinate points in your sketches. For example, starting a branch of a tree at the point (8,2) will assist you in your project later."
- As they sketch they should take note of the type of curves/functions they are using to create your sketch -- use a frequency table to keep track.
- While they are sketching have them think about the following question: "Is there anything that you would add to the space? Another tree? More flowers? More open space? An outdoor activity?".
- On their frequency table, have them note which functions/curves they used to draw their sketch and any other functions/curves they saw in the space and describe what those curves/functions formed (a tree, a blade of grass, etc.)

Math in Nature, Day 4 & 5

Elaborate, part 1:

- Have students take their sketch and convert each segment into mathematical, piecewise-defined equations or curves. This will take some time depending on the student's sketch.

Explain (optional):

- Provide mini-lessons on writing equations from points, as needed. May need to be one-on-one or whole group, depending on the needs of the students. Students can use the writing equation tip sheet to help them move through different equations more independently.

Elaborate, part 2:

- Once they have their piecewise-defined curves and functions, have them enter their equations into Desmos. Give a lesson on the way Desmos works - how to input piecewise-defined functions and how to recolor the graph. Students should enter all of the equations they created into Desmos to recreate their sketch.

Math in Nature, Day 6

Evaluate:

- Students present their work to the class. Students will take the class to the spot where they stood, show their original sketch, their Desmos sketch, and discuss the types of functions and curves they created to make their sketch. Students will also include in their presentation a discussion on how they want to improve the space they studied.
- After their presentation, students complete a self-assessment of their work, reflecting on how and what they learned.